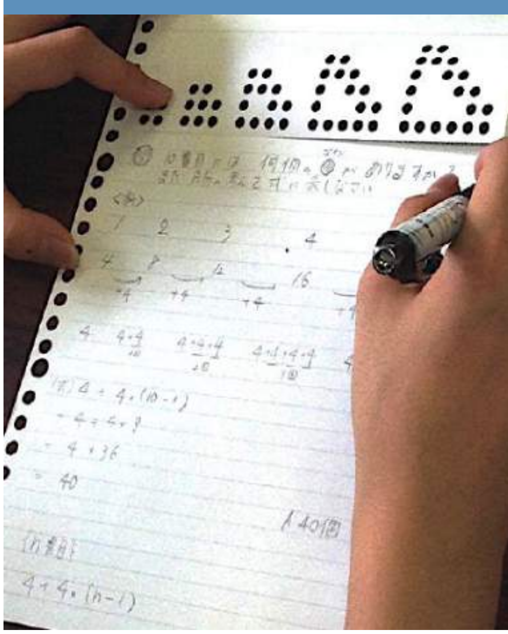
Appendix



Appendix Contents

Section 1

Participant List

First Name	Last Name	Position	Country	Affiliation
Nakachi	Clark-Kasimu	Grade 1 Special Education Teacher	USA	ACORN Woodland Elementary, OUSD
Shelby	Halela	grade 2 teacher	USA	ACORN Woodland Elementary, OUSD
Sarah	Horwitz	4th Grade Teacher	USA	ACORN Woodland Elementary, OUSD
Evanne	Ushman	1st Grade Teacher	USA	ACORN Woodland Elementary, OUSD
David	Allyn	4/5 Grade Teacher	USA	Argonne Elementary, SFUSD
Kim	Towsley	4th/5th grade teacher and Math Lead Co-Teacher	USA	Argonne Elementary, SFUSD
David	Correa	7th Grade Math Teacher	USA	Bret Harte, OUSD
Amber	Richard	AP	USA	Dr. Jorge Prieto Math and Science Academy, CPS
Stephanie	Hironaka	7th Grade Math Teacher	USA	Edna Brewer Middle School, OUSD
Leila	Christenson	2nd grade	USA	Hillcrest Elementary, SFUSD
Karen	Cortez-Ramirez	grade 3 teacher	USA	Hillcrest Elementary, SFUSD
Lisa	Gaglioti	SPED	USA	Hillcrest Elementary, SFUSD
Sara	Liebert	Math Coach and 5th grade Math Teacher	USA	John Muir Elementary School, SFUSD
Martin	Garrett	2nd Grade Teacher	USA	Lawton Alternative School, SFUSD
Stephanie	Moore	Grade 2 teacher	USA	Lawton Alternative School, SFUSD
Meghan	Smith	Grade 2 teacher	USA	O'Keeffe School of Excellence, CPS
Alexandra	Johansen	5th grade teacher	USA	Prieto Academy, CPS
Lauren	Goss	Grade 1 Teacher	USA	San Francisco Community School, SFUSD
Nicole	May	Kindergarten Teacher	USA	San Francisco Community School, SFUSD

Laura	Schmidt-Nojima	IRF/ math coach K-8 at SF Community	USA	San Francisco Community School, SFUSD
Nora	Houseman	Supervisor, Office of Professional Learning & Leadership	USA	SFUSD, Office of Professional Learning & Leadership
Ruth	Trundley	Primary Maths Adviser (team lead)	UK	Babcock LDP
Belle	Cottingham	Mathematics Author and Consultant for KS2/KS3	UK	Consultancy basis in St Albans, Hertfordshire
Edward	Southall	Secondary mathematics teacher and University mathematics teacher trainer	UK	Huddersfield University
Rory	Dearlove	Maths Coordinator, Lead Maths Practitioner and Year 3 Teacher	UK	John Donne Primary School / London Borough of Southwark
Karen	Wilding	Independent Primary Maths Consultant	UK	Karen Wilding Education Ltd
Camilla	Pratt	Senior Lecturer in Primary Mathematics	UK	Leeds Trinity University

Section 2

Faculty List

First Name	Last Name	Position
Akihiko	Takahashi	Associate Professor, Elementary Math and Teacher Education, De Paul University
Toshiakira	Fujii	Director, IMPULS Project
Shinya	Ohta	Math Professor, Tokyo Gakugei University
Tatsuhiko	Seino	Math Professor, Tokyo Gakugei University
Shinnosuke	Narita	Math Professor, Tokyo Gakugei University
Keiichi	Nishimura	Math Professor, Tokyo Gakugei University
Shelley	Friedkin	Senior Research Associate, Mills College
Tad	Watanabe	Translator / Professor of Mathematics Education, Kennesaw State University
Makoto	Yoshida	Translator / Director, Center for Lesson Study at William Paterson University
Soh	Arikuni	Graduate Student, Tokyo Gakugei University
Ryo	Kobayashi	Graduate Student, Tokyo Gakugei University
Ryo	Nagase	Graduate Student, Tokyo Gakugei University
Reo	Namba	Graduate Student, Tokyo Gakugei University
Kazuya	Nishimura	Graduate Student, Tokyo Gakugei University
Mayuko	Oshikawa	Graduate Student, Tokyo Gakugei University
Sora	Nakaitsu	Graduate Student, Tokyo Gakugei University
Jakumi	Fukushima	Graduate Student, Tokyo Gakugei University
Ryohei	Fujiwara	Graduate Student, Tokyo Gakugei University
Taiyo	Watanabe	Graduate Student, Tokyo Gakugei University
Kei	Oyamatsu	Graduate Student, Tokyo Gakugei University
Yoo	Seunghee	Graduate Student, Tokyo Gakugei University
Kiyoko	Ishihara	Program Coordinator
Naoko	Matsuda	Program Coordinator

Section 3

Program Reading

ZDM Mathematics Education
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ORIGINAL ARTICLE

Designing and adapting tasks in lesson planning: a critical process of Lesson Study

Toshiakira Fujii[1]

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Abstract There is no doubt that a lesson plan is a necessary product of Lesson Study. However, the collaborative work among teachers that goes into creating that lesson plan is largely under-appreciated by non-Japanese adopters of Lesson Study, possibly because the effort involved is invisible to outsiders, with our attention going to its most visible part, the live research lesson. This paper makes visible the process of lesson planning and the role and function of the lesson plan in Lesson Study, based on case studies conducted by Project IMPULS at Tokyo Gakugei University in three Japanese schools. The paper identifies key features of the planning process in Lesson Study, including its focus on task design and the flow of the research lesson, and offers suggestions for educators seeking to improve Lesson Study outside Japan.

Keywords Lesson study · Lesson planning · Task design · Structured problem solving

1 Introduction

While the history of Lesson Study in Japan spans more than a century (Makinae, 2010), for Japanese educators, Lesson Study is like air, part of everyday school life. This situation possibly explains why Lesson Study is regarded as being under-theorised (e.g. Elliott, 2012). Educators outside Japan however, having had to learn about Lesson Study less naturally, may sometimes lose some important aspects of Lesson Study.

Lesson Study came to the attention of educators outside of Japan primarily through the publication of *The Teaching Gap* (Stigler and Hiebert, 1999), which described findings from the TIMSS video study focussing on the eighth grade mathematics lessons in USA, Germany, and Japan. Chapter seven in particular, titled "Japan's approach to the improvement of classroom teaching", which is based on Yoshida's (1999) doctoral dissertation, now available in book form (Fernandez and Yoshida, 2004), provoked enormous interest, not only in Lesson Study, but also in the typical structure of Japanese mathematics lessons. Independently, some educators such as Lewis also noticed the significance of Japanese Lesson Study (Lewis and Tsuchida, 1998).

Since then many mathematics teachers and teacher educators around the world have been involved in Lesson Study, and many books and research papers have been written on various aspects of Lesson Study (Lewis, 2002; Lewis et al., Lewis and R, Perry., & J. Hurd, 2009; Hart, Alston and Murata, 2011; Doig and Groves, 2011; Department for Children, Schools and Families, 2008; White and Lim, 2008; Ono and Ferreira, 2010). However, some aspects of Lesson Study, that may be taken for granted by Japanese teachers, seem not to be well understood outside Japan.

This paper aims to clarify the role and function of lesson planning in the Lesson Study process, based on case studies conducted in three schools in Tokyo.

2 Background

2.1 The Lesson Study process

Lesson Study is an approach to teacher professional development that differs sharply from the professional

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Table 1 Contrasting views of professional development (Liptak, cited in Lewis, 2002, p. 12)

Traditional professional development	Lesson Study
Begins with answer	Begins with question
Driven by outside "expert"	Driven by participants
Communication flow: trainer to teachers	Communication flow: among teachers
Hierarchical relations between trainer and teachers	Reciprocal relations among learners
Research informs practice	Practice is research

development practices common in other countries. Liptak (cited in Lewis, 2002, p. 12) contrasted Lesson Study with traditional professional development as practised in the United States, as shown in Table 1.

Lesson Study begins with a *question*, not with an *answer* prepared by someone else. Identifying this question, which becomes the research theme for Lesson Study, is the first step in the process (see Fig. 1).

The research theme is developed through consideration of the reality of students' current state *vis-à-vis* educational or long-term goals for their learning and development.

The second step of Lesson Study is to develop a plan to address the research theme through lessons. This means making an instructional plan for a selected unit and a detailed plan for one of the lessons in that unit in which the planning team puts forth their ideas about how to address the research theme while teaching specific academic content. That lesson is called the *research lesson.*

The third and fourth steps in Fig. 1, conducting the research lesson and having a detailed discussion about the lesson, occur in one day—a big event day for the school. Typically, it is done in a half day; one class of students stays for the research lesson while the other classes are dismissed so that every teacher can come to observe the research lesson (even the school nurse and school nutritionist usually attend). At the end of the post-lesson discussion, usually there will be final comments lasting 30 min or more by a "knowledgeable other" from outside the school, who has been invited for this purpose.

The fifth step is to reflect on the process and consolidate and carry forward the learnings from it. Teachers will usually write their reflections and publish records of Lesson Study activities in the school bulletin.

Because they are the most visible aspects of Lesson Study, some people think of the research lesson and post-lesson discussion as the most important parts of Lesson Study, or even use "Lesson Study" to refer to the research lesson alone. However, these are just two of the five components of Lesson Study.

The Lesson Study cycle, with its five steps as illustrated in Fig. 1, contrasts with similar diagrams in other publications that have four steps (e.g. Lewis, 2002; Lewis and Hurd, 2011). These five steps, while overlapping with the four steps in the other diagrams, more

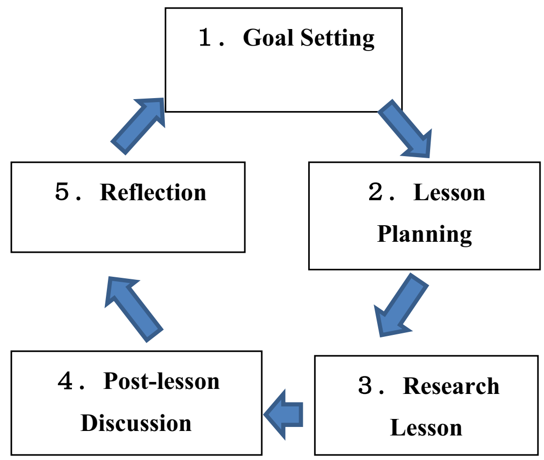

Fig. 1 The process of Lesson Study (Fujii, 2014a, p. 113)

accurately portray the reality of Japanese teachers' Lesson Study activity by having a closer correspondence between the titles of the steps and the activities undertaken by teachers.

Borrowing from Lewis' (2002) and Lewis and Hurd's (2011) descriptions, each step can be summarized as follows:

1. *Goal setting* Consider long-term goals for student learning and development. Identify gaps between these long-term goals and current reality. Formulate the research theme.
2. *Lesson planning* Collaboratively plan a "research lesson" designed to address the goals. Prepare a "lesson proposal"—a document that describes the research theme, content goals, connections between the current content and related content from former and later grades, rationale for the chosen approach, a detailed plan for the research lesson, anticipated student thinking, data collection, and more.
3. *Research lesson* One team member teaches the research lesson while the other members of the planning team, staff members from across the school, and, usually, an outside knowledgeable other observe and collect data.
4. *Post-lesson discussion* In a formal lesson colloquium, observers share data from the lesson to illuminate stu-

dent learning, disciplinary content, lesson and unit design, and broader issues in teaching and learning.
5. *Reflection* Document the cycle to consolidate and carry forward learnings, as well as new questions for the next cycle of Lesson Study. Write a report or bulletin that includes the original research lesson proposal, student data from the research lesson, and reflections on what was learned.

There are three types of Lesson Study in Japan: School-based, District-based, and National-level Lesson Study. According to Takahashi (2006), participants' motivations or interests are different in these types of Lesson Study, but the cycle itself is basically the same. The difference is in the range, or scope, of students to be considered: school-based Lesson Study is concerned with students in the school; District-based Lesson Study is concerned with students in the district; and National-level Lesson Study is concerned with the reality of students across the country, and has a research theme with a nationwide view. Lesson Study is sometimes introduced as an open lesson by a veteran teacher "jumping in" to another teacher's classroom (Takahashi, 2013, p. 84). A "jumping in" lesson is just a demonstration unless the veteran teacher has a clear goal for the lesson as in Step 1, and proposes a new idea or content to be teachable, or he or she wants to demonstrate students' potential to be greater than ordenary teachers believe, so that he, or she, plans the lesson carefully as in Step 2. This kind of Lesson Study exists in Japan and in this case the collaboration among teachers is not a critical part of Lesson Study. In any case, each step in the Lesson Study cycle is closely related to the others, with the third and fourth steps particularly related to the first and second.

In school-based Lesson Study, which is the focus of this paper, the typical Lesson Study cycle begins at the end of an academic year—i.e. in February or March in Japan—when the faculty decides upon a research theme for the next school year, which starts in April. Several research lessons are scheduled from, say, May to November. Each research lesson and its post-lesson discussion occupy only one day, but the teachers reflect on what they learned at the research lessons and usually write a booklet or long summary report by the end of school year.

While the importance of a lesson plan as a product of Lesson Study is certainly understood, compared to the research lesson, of which there are many public examples, the collaborative work of Japanese teachers in creating a lesson plan is generally mysterious, because it is difficult to observe. According to Lee and Takahashi (2011) "Lesson plans are central resources for these teachers in that they constantly refer to, problematize and act on them during the entire cycle of the [Lesson Study] procedure" (p. 210).

Japanese teachers spend a lot of energy and time crafting a lesson plan. Although the details vary from school to school and even from teacher to teacher, Lewis (2002, pp.127-130) notes that a typical template for a lesson plan for a research lesson in Japan consists of the following:

1. Name of the unit
2. Unit objectives
3. Research theme
4. Current characteristics of students
5. Learning plan for the unit, which includes connections to standards and to prior and subsequent learning, the sequence of lessons in the unit and the tasks for each lesson, and explanation of unit "flow"
6. Plan for the research lesson
7. Background information and data collection forms for observers (e.g. a seating chart)

The Japanese term for the document created for a research lesson is *gakushushido-an* (学習指導案), which is usually translated as "lesson plan". In this paper we will use that common translation, although we prefer the phrase "lesson proposal", because the document is much larger and broader in scope than what is usually meant by "lesson plan". Also the word "plan" may imply a fixed script, but in Japanese Lesson Study the teacher is expected to use his or her judgment if students respond in unanticipated ways. As Lee andTakahashi (2011) argue, researchers have taken for granted that using lesson plans, no matter how well devised, always involves judgment and interpretation, as teachers and their students face the contingencies of the lesson in the classoom. Their empirical study, in the context of Lesson Study, provided analytic descriptions of the interactive processes through which lesson plans are realized, leading to the conclusion that "classroom teachers use lesson plans as communicative resources to identify problems, specify assumptions about their teaching, and act on the evolving contingency of classroom interaction" (p. 209). However, Lee and Takahashi (2011) did not describe details of planning the lesson, including how teachers adapted or designed the task for the lesson, or how many hours they spent on planning.

In the context of Lesson Study, Lewis, Perry and Hurd (2009) focussed on one US lesson study group, of six teachers from five different schools, that conducted a research lesson in a 2-week summer workshop. This is an experimental situation, which is different from the Japanese traditional school-based Lesson Study setting. However it is worth considering in terms of the lesson planning activity. They documented that the group spent a total of six hours planning the lesson: "select research lesson, do task and share solutions, anticipate student thinking, write instructional plan using template" (Lewis et al., 2009, p.

290). However they have not offered descriptions of how they designed or adapted the task for the lesson.

On the other hand, Fernandez and Yoshida (2004) described in detail the process of planning lessons in the context of Lesson Study. This ethnographic study, focussed on a local elementary school in Hiroshima, vividly shows Japanese teachers' activities. However, the Lesson Study described there has the rather unique feature in that, following the research lesson being taught by a young inexperienced teacher, observed by the whole school and discussed by only the lower grade group of teachers and the principal, the lesson was revised by these teachers and then re-taught by a veteran teacher, with the whole school and an ouside advisor observing the lesson and taking part in the post-lesson discussion. The notion of *Re-Teaching* is extremely problematic and sensitive. In fact, the need to revise and re-teach a lesson is one of the misconceptions identified in foreign countries implementing Japanese Lesson Study (Fujii, 2014b). Whether *Re-Teaching* exists or not in the Lesson Study process affects the nature of the planning and the discussion of the lesson.

2.2 Structured problem solving

The structure of Japanese mathematics lessons is often regarded as unique by non-Japanese eyes, with researchers from outside Japan having noted patterns in Japanese mathematics lessons. For example, Becker et al. (1990) identified eight components in a typical Japanese mathematics lesson, while Stigler and Hiebert (1999) identified five components and labelled these lessons as *structured problem solving*. But their points of view are those of observers, while Japanese teachers usually do not think about the structure of their lessons in the same way. For instance, the first component of Stigler and Hiebert (1999), *reviewing the previous lesson,* is not an important activity from a Japanese teacher's point of view. Instead Japanese teachers typically consider a mathematics lesson as problem solving in terms of the four phases shown in Table 2 (see, for example, Shimizu, 1999).

This type of lesson imposes certain demands on how to interpret the lesson plan. Phase 1, *presenting the problem*, means helping students understand the context of the problem or task and what it will mean to solve the task—but it specifically excludes any exposition by the teacher about how to solve the task. Instead, students are expected to work independently on the task for 10–20 min (phase 2). Therefore teachers need to discuss the appropriateness of the task described in the lesson plan. The third phase, called *neriage* in Japanese, assumes that students will arrive at different solution methods and focusses on a comparison and discussion of those different solution methods. Therefore teachers need to discuss the plausibilty of the anticipated student solutions listed in the lesson plan. In the fourth phase, *matome*, the teacher may say something about which strategy may be the most sophisticated and why, but it should go beyond that to include comments by the teacher concerning the mathematical and educational values of the task and lesson (Fujii et al., 1998). Therefore teachers need to discuss the resonableness of the *matome* by the teacher as foreshadowed in the lesson plan. For a lesson to work in this way, the task should be understandable by the students with minimal teacher intervention; it should be solvable by at least some students (but not too quickly), and it should lend itself to multiple strategies.

Table 2 The four phases of a problem-solving lesson in mathematics

1. Presenting the problem for the day (5–10 min)
2. Problem solving by the students (10–20 min)
3. Comparing and discussing (*neriage*) (10–20 min)
4. Summing up by the teacher (*matome*) (5 min)

This paper focusses on the second, planning step in the Lesson Study cycle, and aims to illuminate the nature of the collaborative work among teachers, based on three case studies where re-teaching was not part of the Lesson Study process, with particular emphasis on planning for these four phases of the research lessons.

3 Methodology

This research took place in three local public elementary schools in Tokyo, which will be referred to as schools M, S and T. These schools were participating in the *International Math-teacher Professionalization Using Lesson Study* project (IMPULS), a recently established project funded by the Ministry of Education, Culture, Sports, Science and Technology of Japan, located at Tokyo Gakugei University, Tokyo. The purpose of this project is two-fold. First, as an international centre of Lesson Study in mathematics, Tokyo Gakugei University and its network of laboratory schools help teacher professionals learn about authentic Japanese Lesson Study, and thereby prepare them to create Lesson Study systems in their own countries for long-term, independent, educational improvement in mathematics teaching. Second, the project conducts research projects examining the mechanism of Japanese Lesson Study in order to maximize its impact on schools in Japan.

Although several research lessons were scheduled for each year, this study focusses on just one research lesson at each of these schools, and the planning meetings for those research lessons—that is, just one lesson study cycle in each school.

The author observed each lesson-planning meeting and took fieldnotes. In addition, each lesson-planning meeting was video-recorded and later transcribed; and all lesson plans and revised versions were collected and analyzed with respect to their evolution.

This paper provides a descriptive analysis of the planning process undertaken by these groups of teachers in preparation for the research lessons. In a similar vein to the research carried out by Lee and Takahashi (2011), discourse-in-interaction analysis (Sacks et al., 1974) was used to examine "the methods and procedures by which participants carry out ordinary tasks of classroom teaching and collaboration among teachers" (Lee and Takahashi, 2011, p. 215). The analysis began with unmotivated looking (Sacks, 1992) during the observations of the planning meetings in order to identify key discussions that eventually led to consensus regarding the lesson plans.

Through this overview of the lesson planning processes, the author came to realize that the discussions were based on the flow of the lesson. In particular, it seemed that teachers could imagine or visualize clearly what would happen at the research lesson through reading the lesson plan. Therefore it was clear that this study could focus on analyzing the planning of the flow of the research lesson.

Based on the flow of Japanese problem-solving lessons, thematic content analysis (see, for example, Fereday and Muir-Cochrane, 2006; Braun and Clarke, 2006) was carried out on transcripts of the lesson planning discussions. Using the framework of the four phases of problem-solving lessons (Table 2), participants' comments were coded with appropriate keywords to track their views of the lessons. These comments were examined with respect to the role of the lesson plan and planning meetings, in order to make visible an important part of Lesson Study—namely the planning process.

The following section is organized according to the main results obtained through the inductive process of examining the trajectory of revising lesson plans, transcribed records of planning meetings, research lesson, and post-lesson discussion, and field notes.

4 Results

The results of this study are presented in three sections. First, we report on the lesson planning meetings overall—e.g. the number of meetings and participants, and the duration of meetings. Second, we examine the major component of the meetings. Finally, we identify major concerns at the meetings, such as the appropreateness of the task for the lesson, anticipated student solutions, and how to organize the comparison and discussion phase in the lesson.

4.1 The lesson planning process overall

The dates of the research lessons held at school M, S and T, together with the dates of the planning meetings are shown in Table 3. The planning meetings began between 4 and 6 weeks before the research lessons. Two schools, M and S, had four planning meetings and school T had just two meetings.

Table 3 Dates of research lessons and planning meetings

	Meeting 1	Meeting 2	Meeting 3	Meeting 4	Research Lesson
School M	15 May	22 May	13 June	21 June	1 July
School S	30 May	6 June	11 June	19 June	3 July
School T	28 May	4 June			26 June

Table 4 Number of participants at the planning meetings

	Meeting 1	Meeting 2	Meeting 3	Meeting 4
School M	7	8	8	8
School S	5	6	7	4
School T	8	8		

It should be noted that there was no rehearsal or trial implementation of a tentative lesson plan between planning meetings. It should be noted also that this schedule fails to reveal the amount of time that the teachers may have spent thinking about their research lesson beforehand, since the grade, unit, and lesson may have been selected at the end of the previous academic year in March.

Table 4 shows the number of participants at each of the planning meetings.

In the case of school M, the regular members of planning meetings were: the leader of the research steering committee, who also chaired the meeting and was the lead teacher for mathematics in the school; three Grade 3 teachers, one of whom taught the research lesson; and four Grade 4 teachers—a total of eight participants. The first planning meeting, held in the principal's office, was rather informal, since the knowledgeable other, who had given a talk at a research lesson that day, joined the meeting, together with the principal of the school. Beside these two participants, three Grade 3 teachers and two Grade 4 teachers attended. But at later meetings, in the school conference room, the only participants were the eight regular members.

At school S, which is a small school with only one class at each grade, the first meeting included five regular members: two classroom teachers for Grades 5 and 6, the music teacher, the art teacher, and the teacher for mathematics. The Grade 6 teacher was the leader of the school research steering committee and taught the research lesson. In Tokyo, in the case of mathematics only, if a school wants to divide classes into two or three groups for teaching mathematics, in order to help cater for individual differences,

Table 5 Duration of planning meetings (min)

	Meeting 1	Meeting 2	Meeting 3	Meeting 4	Total time
School M	30	128	114	81	353
School S	60	60	30	54	204
School T	78	87			165

the school gets an extra teacher—in this case this teacher. The music teacher and the art teacher were teaching Grade 5 and 6 students, therefor the regular members were the upper year level team. At the second meeting, the principal joined them; at the third meeting, the knowledgeable other also joined; but the fourth meeting included only the Grade 1 teacher and the Grade 6 teacher, the music teacher, and a special needs teacher—these four constituted the school research steering committee. The venue was always a meeting room in the school.

At school T, regular members were the leader of the research steering committee, three Grade 3 teachers and three Grade 4 teachers, and the principal of the school, who attended the planning meetings—so the total number was 8. One of the Grade 4 teachers taught the research lesson. There were only two meetings, both of which were held in the principal's office.

School M, S, and T each organized a research steering committee. According to Takahashi and McDougal (2014), a research steering committee in Japan consists of representatives of each grade level and, in the case of the Lesson Study focussing on mathematics, the lead teacher for mathematics. In addition, representatives of special subject teams, such as music, science and home economics may join. The research steering committee leads the school's efforts and maintains the cohesion of ideas across the grades. Takahashi and McDougal (2014, p. 16) list roles and functions of research steering committees as follows (parenthesis added by author):

- Developing a master plan for the school research;
- Scheduling and leading monthly meetings to find strategies to address the school's research theme based on the ideas of the teachers;
- Publishing a monthly (not always the case) internal newsletter to record the findings from each research lesson;
- Planning, editing, and publishing the school research reports, including those for the research open house; and
- Arranging for knowledgeable others to present lectures, teach demonstration lessons (not always the case), and give final comments at research lessons.

As shown in Table 5, the duration of the planning meetings ranged from a minimum of 30 min to a maximum of 128 min.

The chairperson of the school research steering committee led most of the meetings at schools M, S, and T. As these schools were conducting Lesson Study focussing on mathematics, the lead teacher for mathematics tended to also be in charge of the school research steering committee. Besides regular members from the school, the knowledgeable other, who had given comments on a research lesson that day, attended the first meeting at school M and the third meeting at school S. Involving a knowledgeable other in this way is common; after a research lesson and discussion ends, the team responsible for the next research lesson will meet with the knowledgeable other for further discussion and to get advice for their lesson.

As both of the 30-min meetings were with the knowledgeable other, these could be regarded as atypical. The average duration was 72 min, with the average duration excluding the 30-min meetings being 83 min.

One reason that may account for the differences in the duration of planning meetings between schools could be that the principals of schools S and T attended and participated actively in these meetings, with teachers in both schools appearing to have great confidence in them. When teachers asked, these principals gave suggestions to help break deadlocks. As a result, the duration could become shorter. In the case of school M, some of the regular members of planning meetings were young and inexperienced. Therefore, the leader of the research steering committee, who was also the lead teacher for mathematics, sometimes needed to explain the position of the lesson in the scope and sequence of the Japanese course of study, and the mathematical value of the task for use in the lesson. These factors may have had an effect on the longer duration of the meetings.

4.2 Major components and structure of the planning meetings

The first meetings held at school M and S were unusual in that the teachers discussed ideas about the research lesson in depth without a written lesson plan. At all other meetings, the discussion was based on a draft lesson plan, which had been written, either with or without the support of colleagues, by the teacher who would be teaching the lesson. Furthermore, the flow of the planning meetings followed the flow of the lesson plan. Other issues, such as the logistics of the research lesson or post-lesson discussion, were not discussed.

The format of the first draft of the lesson plan for schools M, S, and T was basically the same as Lewis' (2002) template as described earlier in this paper. In the case of school M, component 5 in Lewis' (2002) template, *Learning plan for the unit*, was missing at the beginning, but was added later.

Among the seven components in Lewis' (2002) template, component 6, *Plan for the research lesson*—which we will refer to here as *Planning the flow of the research lesson* in order to distinguish it from the overall lesson plan—is the most prominent in terms of both quantity and quality. At school T, the draft lesson plan had already been prepared for the first meeting, written by the teacher who was to teach the research lesson. The items discussed at the first meeting were as follows:

1. The research theme of the school (8 min).
2. The goal of the unit; evaluation points for learning (i. Interest, Eagerness, and Attitude; ii. Mathematical Way of Thinking; iii. Mathematical Skills; and iv. Knowledge and Understanding); the relationship between this unit and the research theme; other units related to this unit; students' reality; and teachers' vision of ideal students (6 min).
3. What ideal students would look like (11 min).
4. Unit and lesson plans (2 min).
5. Planning the flow of the research lesson (51 min)

These items were exactly the items written in the draft lesson plan.

In both meetings at school T, discussion relating to planning the flow of the research lesson occupied the majority of the time: 51 min (65 %) of the first meeting as shown above, and 87 min (78 %) of the second meeting.

At school S, the first meeting was held without a written lesson plan. At this stage, teachers had not yet decided exactly which unit or content to teach for the research lesson and how. From the second meeting onwards, the teachers' discussions were based on the lesson plan drafted by the teacher who was to teach the research lesson. The knowledgeable other attended the third meeting. Excluding the first and third meetings, the proportion of time spent on planning the flow of the research lesson was 74 %, while when all four meetings are included, 52 % of the time was spent on planning the flow of the research lesson.

At school M, the first meeting was also held without the written lesson plan. From the second meeting onwards, the discussion was based on the draft lesson plan which had been written mainly by the teacher who was to teach the research lesson, but as a team, with support from the third grade teachers. In the second, third and fourth meetings, the proportion of time spent planning the flow of the research lesson was 74 %, while if the first meeting is included the proportion was 66 %. Across the three schools, omitting meetings without the lesson plan, the average proportion of time spent on planning the flow of the research lesson was 72 %; while if all meetings are included the proportion was 63 %.

Thus we have two findings: one, that the planning meetings followed the structure of the lesson plan; and two, that the discussion among teachers was particularly focussed on planning the flow of the research lesson.

The discussions specific to the flow of the research lesson during the planning meetings at the three schools could be aligned with the four phases of a problem-solving lesson (see Table 2). For example, at the second meeting at school S, a discussion on how students might grasp the given task (15 min) was related to phase 1, *Presenting the problem for the day*; discussion about likely student responses (14 min) was related to phase 2, *Problem solving by the students*; discussion about how to organize the comparison and discussion period (15 min) was obviously related to phase 3, *Comparing and discussing*; and discussion about how to conclude the lesson (5 min) was related to phase 4, *Summing up by the teacher*. Of the 49 min focussed on the flow of the research lesson, the proportions of time related to these four phases was approximately 31, 29, 31 and 10 %. The other two schools showed a similar pattern.

In the next section we will present, in more detail, what the teachers talked about regarding each phase of their lessons.

4.3 Major concerns when planning the flow of the research lesson

Discussions by the teachers, while planning the flow of the research lesson, were classified into three key categories: Appropriateness of the task, Plausibility of the anticipated student solutions, and Quality of the comparison and discussion (*neriage*) phase.

4.3.1 Appropriateness of the task

Discussions about the task for the research lesson can be classified into two types. One is discussion about the task and unit from an advanced mathematical perspective, where teachers clarify the scope and sequence of relevant topics, or relationships within and expansion of the content. The second is to discuss the appropriateness of the task to the goal of the lesson, including detailed consideration of the numbers in the task, the context of the task, and so on.

When teachers talked about the position of the unit within the curriculum, they carefully referred to the National Course of Study (2008) published by the Ministry of Education, Culture, Sports, Science and Technology. According to Lewis' (2002) typical lesson plan template, this discussion is related to "connections to standards and prior and subsequent learning", which is included in the fifth componentof the template, *Learning plan for the unit*, where related units in former and later grades are explained and shown by using a diagram. In fact, teachers at school

M used their own diagram as they discussed why the unit was important and as they traced the students' learning path leading to the unit. In the case of school S, at the second meeting where teachers talked about sequence of units, they recalled an old version of the National Course of Study (1998) in which "speed" was placed in fifth grade. "Speed" was now in sixth grade in the National Course of Study (2008). In fact, one teacher said "At fourth grade we teach multiplication and division of decimal numbers, and in fifth grade we teach the size of per-unit quantities.[1] The closest content to speed is size of per-unit quantities.… We used to teach speed in fifth grade, together with the size of per-unit quantities".

Teachers also talked a lot about the task itself. The tasks in all three cases were not directly from textbooks; they were newly created, or modified from tasks in the textbook. Teachers discussed why they selected the particular tasks; what roles the tasks were expected to play in the unit; what benefits students might gain from solving the tasks: whether it helped to develop a new concept, a new way of thinking, or some important procedure.

The discussion of the curriculum was closely related to the solution of the task, because related content in the curriculum was expected to be a resource for students to solve the task. For example, in the second meeting in School S, there was the following exchange:

Teacher A:	Students learned how to arrange to get the same numbers for time or distance, didn't they?
Teacher B:	Yes, I suppose. However, the idea of a common multiple was learned a long time ago from the students' point of view.
Teacher C:	Probably they forgot the procedure to find the common multiple.
Teacher B:	When they learned division of decimal numbers, they learned the idea of per-unit. It's the same thing here. However, the idea of per-unit was not learned in the context of comparing things.
Principal:	The idea of per-unit quantity was applicable for comparing crowdedness. That is a mathematical way of thinking that could be applicable for Speed.

[1] A per-unit quantity is a ratio of two quantities from different measure spaces. As a ratio, it is expressed as the amount of one measured quantity for one unit of the other measured quantity. For example, population density is typically expressed as the number of people per unit area, or speed as the distance travelled per unit time.

This kind of detailed and concrete consideration of previously-learned content was observed in all three schools.

Teachers also engaged in detailed discussions about the task itself, including which numbers to use and why. This aspect of Lesson Study was noted by Stigler and Hiebert (1999), who reported that teachers would talk about the "problem with which the lesson would begin, including such details as the exact wording and numbers to be used" (p. 117). However, the selection of numbers is not always from a purely mathematical point of view.

For example, in the case of School S, teachers thought about numbers both in terms of their students' reality and also from a procedural or calculation point of view. The teacher who would teach the research lesson said:

> Child A in the problem can run 40 metres in only 6 s. In my class there is no such fast runner. However I decided to use these numbers, because these numbers are easier for children to calculate.

Time and distance data for the first three people in the problem (A, B, C) were not changed, but data for two people (E, F) were changed from E (42 metres in 6.7 s), F (28 metres in 4.9 s) to E (45 metres in 6.5 s), F (50 metres in 8 s), in order to provide some faster speeds. Numbers for D, E, and F were considered hard for students to calculate and the teachers also worried about having decimal fractions as the result of calculations. However, they decided to keep the numbers and let students use calculators if they wanted.

In the case of school M, the task was to contrast partitive and quotitive division problems obtained from one mathematical sentence. The teachers chose to use $8 \div 2$ after also discussing $12 \div 3$, $18 \div 6$, $6 \div 2$, and $10 \div 2$ as possible candidates. They considered the numbers 8, 2, and 4 as the most easily distinguishable for students, so that students would not confuse them in using, or explaining, their ideas.

In the case of school T, the research lesson was on learning about quadrilaterals and the task was to classify quadrilaterals. The teachers changed the plan from *asking students to draw* figures freely on dot paper to *giving students* figures already drawn by the teacher. The teacher worried that students might not construct certain figures that the teacher particularly wanted to discuss in the lesson. The teachers also discussed what would be a suitable number and what types of quadrilaterals to give. If the number of figures was too small, students would not be interested in classifying them, or they would not feel any necessity to make groups. Eventually the teachers decided on nine figures: a square, a rectangle, two parallelograms, two rhombi, an isosceles trapezium, a general trapezium, and a general quadrilateral. The team decided not to include a trapezium with a right angle. As part of their discussion, teachers simulated individual students solving the problem to get an idea of the time required. Further, they considered the quality of the problem-solving activity in terms of the appropriateness of the task and the goal of the lesson.

At all three schools, the teachers discussed the unit in reference to the curriculum, as well as discussing the main task in terms of its appropriateness within the unit, its value for clarifying mathematical ideas, and its appropriateness for accomplishing the goal of the lesson. In terms of the appropriateness of the task for the goals of the lesson, teachers considered what solutions or ideas the students would be likely to bring up. This is the topic of the next section.

4.3.2 Anticipated student solutions

In all three schools, teachers spent time discussing likely student responses to the main task in the research lesson. These discussions usually began by considering what was most likely from the class as a whole. They then went on to consider likely responses from students who were rather slow learners and from students who were fast learners.

In the case of school S, teachers pretended to be students in order to solve the speed task, *Who is faster?* (see Table 6), from the students' point of view. Through this activity, teachers confirmed the plausibility of the four anticipated solutions already written in the lesson plan: (1) finding a common multiple of distance to compare; (2) finding a common multiple of time to compare; (3) finding the amount of time per metre to compare; and (4) finding the distance per second to compare.

In the case of school T, one teacher was asked to pretend to be a student to solve the task, and the other teachers watched his activity. In the case of school M, teachers wondered whether students would be able to create two kinds of division stories or just one story. The team leader asked the other teachers if they felt uneasy partly because of their own experiences. Teachers made explicit reference to their own experiences as they tried to anticipate students' responses to the task.

In all three schools, teachers considered how to deal with slow learners. In the case of school S, the teacher had already decided to provide hints to students who wanted them during the individual problem-solving period. The team discussed specifically what should be on these hint cards. While a hint card suggesting using common multiples was reasonable from the teachers' initial point of view, they no longer thought this might be the case when they imagined, or visualized, the lesson. They thought this strategy would eventually be rejected in favour of a better strategy: finding the distance per second. One teacher said, "Students might ask the teacher, '*Why did you not give me the best hint, if you knew*?'" The other teachers agreed that was likely to happen. So they discussed how to let students notice the per second strategy. Finally teachers thought of using 30 metres and 5 s as the data. "It divides beautifully". "If the teacher asks a question such as, '*Five seconds to go (30 m), so if it were one second how far could you go?*, students may be able to notice the idea of *per second*'". "It will work," one teacher said, "it looks fine". Eventually the teacher decided to suggest using the "per second method" to solve the task using the data of 30 metres and 5 s.

Table 6 The task given: Who is faster? Let's think about the order of speed of these 3 children: A, B and C

Children	Distance (m)	Time (s)
A	40	6
B	30	6
C	30	5

In all three schools, teachers also considered how to deal with fast learners in the lesson. For instance, at school M, a teacher said, "Students who have finished solving the task, I would ask them to write mathematical sentences, possibly like $4 \times 2 = 8$ or $2 \times 4 = 8$, showing the process to get the answer".

4.3.3 The comparison and discussion (neriage) phase

The comparision and discussion (*neriage*) phase follows the problem solving by the students. This phase in the structured problem-solving lesson is the most difficult for teachers to deal with. Each correct solution has equal value in terms of getting an answer. However, the ideas involved may not have equal value. The *neriage* phase is when the teacher elicits these ideas and discusses the value of each solution. The teacher at school S clearly stated, "Although each strategy is sure to get the correct answer, we should not end there … I want the students to know that getting the answer is not the final goal".

In the case of school M, teachers wanted students to compare two word problems, for partitive and quotitive division, through the use of multiplication sentences to model situations. (See the "Appendix" for the actual task.) The lead teacher of the research steering committee posed the question, "What should we ask to elicit a multiplication maths sentence?" For the next 17 min the teachers discussed what the question should be, including its exact wording.

At school T, teachers talked about which point or theme for discussion would be best: the number of groups of quadrilaterals, where the teacher might say "this student made two groups and the other student made three groups, what made these difference? What were the thoughts behind these categorizations?" or how to characterize each group, for example "This student made two groups. Can you see the common characteristics of the quadrilaterals in each group?" One teacher asked, "Which is the higher level of thinking?" to which another teacher responded, "Probably the number of groups is higher. This point is proposed in the lesson plan". So they decided to ask students to discuss how many groups there were and reasons behind this in the *neriage* phase.

The teams at all three schools discussed how to elevate students' mathematical thinking by comparing individual students' solutions.

5 Discussion

It is well known that Japanese teachers get together before a research lesson to discuss the lesson. What do teachers discuss? This study reveals that their discussions followed the lesson plan, which had been drafted or created before meetings, and they devoted approximately two thirds of the time to discussing the flow of the research lesson. Within that time, teachers focussed on the appropriateness of the task, anticipated student solutions, and the plan for comparing and discussing those student solutions. The teachers also referred to the Japanese National Course of Study and its guide for teachers.

5.1 The role of the Japanese National Course of Study in designing and adapting the task for the research lesson

At planning meetings, teachers frequently referred to the National Course of Study when they needed to confirm the role of the unit, or focus lesson, within the entire curriculum. Teachers at school S talked about the placement of speed in the previous National Course of Study. This is a more difficult conversation to have in countries lacking a clear curriculum. Lewis and Tsuchida (1998) argued that having a frugal, shared curriculum was necessary for implementing Lesson Study. With a clear curriculum sequence, teachers could identify the value of the research lesson and the unit within the curriculum: by identifying closely related content in former and later grades, teachers can understand why the research lesson is important for later learning. And, identifying similar units or content in earlier grades helps teachers infer what students might do to solve the task, based on their previous learning. All three teams of teachers identified the position of the research lesson in the curriculum in order to clarify students' learning trajectory.

5.2 The value of discussing anticipated solutions

Data from the three schools revealed that teachers tried hard to anticipate student solutions in detail; and what they anticipated influenced the design of the lesson. For example, it influenced the design of the task, such as in the case of school T where the decision whether to include a trapezium with a right angle was made through considering students' anticipated solutions. Anticipating student responses also influenced how teachers decided to pose the problem. For instance, teachers at school S considered how students would react to the question of which person is faster when only times were given. Also teachers tried to predict students' difficulties, and discussed how to reduce students' confusion in comparing three speeds. They eventually decided to erase the slowest person's data in order to focus on only two people.

Based on their experience, Japanese teachers know that the conditions, or characteristics, of the task influence students' thinking processes and solution methods. In the case of school T, the teachers thought that the right angle might cause students to go in a direction that was not consistent with the goal of the lesson. Anticipating student solutions at planning meetings is therefore important in Lesson Study, and this unique activity is a characteristic of task design in Lesson Study (Fujii, 2015).

Teachers also think carefully about the numbers used in a task because this can strongly influence students' ways of solving the task. In the case of school S, teachers deliberately chose awkward numbers for the additional speed data, of persons D, E, F. The teacher explained, "I want students to say that it is awkward to calculate common multiples among them". She deliberately chose numbers that would push students to calculate distance divided by time. On the other hand, the numbers for B and C were (30, 6) and (30, 5) respectively, with these chosen because the numbers "divide beautifully". The teacher clearly anticipated that some students would calculate $30 \div 6$ and $30 \div 5$ to get the distance per second.

Close attention to the specific numbers does not mean that teachers are sticking to a concrete level of thinking or encouraging students to think concretely. On the contrary, teachers consider the general aspect of the numbers—their quasi-variable aspects. A quasi-variable is a number deliberately used in a general way, so that it serves as a representative of many numbers, just as a variable would (Fujii and Stephens, 2001, 2008; Fujii, 2008, 2010). Numbers are often chosen based on their quasi-variable power, or how well they can demonstrate a general truth—a general truth that is brought out during whole class discussion.

A structured problem-solving lesson includes a *neriage*—comparison and discussion—phase for students to compare or experience their friends' methods and discuss similarities and differences between strategies as a whole class. When designing the task, there needs to be consideration of whether the task will elicit the alternative approaches needed for an effective *neriage*. Therefore teachers carefully discuss and choose appropriate numbers for the task.

Discussing students' anticipated solutions while considering the specific numbers in the task clarifies the mathematical value of the task. In their book *The Teaching Gap* Stigler and Hiebert (1999, p. 118) have another example: teachers discuss appropriate number sentences to use in the context of teaching subtraction across 10. Subtraction across 10 can be solved by subtraction-addition (e.g. 12–9 = 10–9 + 2), subtraction–subtraction (e.g. 12–9 = 12–2–7), counting down, and counting up. In this example, the teachers believed that the subtraction-addition strategy was the most valuable for students to learn, so they examined the potential of different choices of numbers to lead to that strategy. For the same reason, almost all textbooks in Japan choose 13–9 or 12–9 to elicit the subtraction-addition strategy (Doig, Groves, and Fujii, 2011). In the case of school S, numbers were chosen to lead students to calculate distance divided by time. In the case of school T, teachers chose geometrical figures which could lead students to classify them in terms of characteristics related to their parallel or perpendicular sides. Anticipating student solutions in Lesson Study helps clarify the mathematical value of the task, and helps teachers make sure that the goal of the lesson is reached.

5.3 The value of designing the *neriage* phase of the lesson

The comparison and discussion of multiple student solutions needs to be more than "show and tell" (Takahashi, 2008). This *neriage* phase of a lesson should be an actualization of Vygotsky's zone of proximal development (Ohtani, 2014), and the role of the teacher is critical. Teachers at the three schools, M, S, and T, discussed at length how to deepen students' ideas in the *neriage* phase. A teacher at school S said, "Although each strategy is sure to get the correct answer, we should not end there". This comment shows teachers' deliberate efforts to elevate all students' ways of thinking.

During the planning meetings, the focus of designing the *neriage* phase of the lesson was on deepening students' understanding and ways of thinking. From the point of view of mathematical value, the lesson should clarify the relative value of the different solutions, generally by contrasting these. The lesson is less likely, obviously, to do this without sufficiently rich and diverse solutions to compare. Therefore, teachers carefully examine anticipated student solutions in detail in order to make sure valuable solutions are likely to appear in the comparison and discussion phase. The value of designing the *neriage* phase of the lesson lies in its potential to elucidate or expose ways to highlight different solutions, and how to compare them in order to reach the goal of the lesson.

5.4 Designing and adapting tasks in lesson planning goes with lesson evaluation

As we have seen, teachers give much thought to the selection and design of the task during the planning phase of Lesson Study. The task is later evaluated during the post-lesson discussion. This is another distinguishing aspect of Lesson Study. The task is not judged based on some abstract determination of whether it is good for teaching a certain skill or concept, but based on concrete evidence from the research lesson of how the students responded to it. In the case of school S, three pairs of data points were added for students to compare, but at the post-lesson discussion teachers argued about whether these additional data were useful or not. The arguments were based on how students actually responded to the task in the lesson. Similar arguments occurred at the other two schools.

In the case of school S, the arguments progressed from evaluating the task to modifying the task. In fact, the final commentator, the knowledgeable other, suggested more direct ways to manipulate numbers to identify faster speed without calculating six pairs of numbers. He gave the example shown in Table 7 of two pairs of numbers in the context of population density:

The final commentator suggested using these numbers instead the six pairs of numbers that were used in the research lesson, as some students struggled to carry out the calculations in the time available, and then missed the educational value of the task, and the whole-class discussion. The post-lesson discussion provided a context for revising the task used at the research lesson, since points missed in planning meetings were revealed in the post-lesson discussion. This shows that the planning meetings of the Lesson Study cycle are closely related to the research lesson itself, and to the post-lesson discussion.

The post-lesson discussion provided a context for revising the task used at the research lesson. However, this does not imply that re-teaching is necssarily part of Japanese Lesson Study. Based on their experience, Japanese teachers know that if students are different then their reactions will be different. They understand that a lesson is itself an organic system, it is not like a machine. A non-organic system, such as a car, is composed of parts that may be easily replaced. However, in organic systems, like a lesson, each part supports the whole ecology. In the case of school S,

Table 7 An example of two pairs of numbers used in the context of population density

Pool	Area (m^2)	Number of people
A	200	15
B	400	45

important ideas missed in planning meetings were revealed in the post-lesson discussion. Teachers then regretted that their *kyozai-kenkyu* (study or research on teaching materials— see, for example, Watanabe, Takahashi, and Yoshida, 2008) was not profound enough and broad enough to cover the idea. In other words, Japanese teachers' attitude towards research lessons and lesson plans is that their best lesson plan should be implemented at a research lesson, and that a research lesson is the proving ground for teachers (c.f. Lewis and Tsuchida, 1998).

6 Conclusion

It is widely understood that a lesson plan is an important product of Lesson Study, but despite much research into Lesson Study, the process of creating a lesson plan, as a collaborative effort by teachers, is largely invisible to non-Japanese adopters of Lesson Study. This paper tries to clarify the process of lesson planning and the role and function of the lesson plan, based on case studies of Lesson Study in three Japanese schools.

In each of these case studies, we see that the planning meetings began with a lesson plan already written by the teachers and most of the time was spent discussing the flow of the research lesson. While discussing the flow of the research lesson, teachers spent time designing and adapting the task for the lesson, during which time they typically did the following: consulted the National Course of Study to clarify the position of the task in curriculum, as well as for guidelines in designing and adapting tasks; verified the mathematical value of the task by anticipating student solutions; carefully designed the comparison and discussion (*neriage)* phase of the lesson to ensure that the goal of the lesson was reached.

In addition, teachers evaluated the task during the post-lesson discussion in light of the actual student responses in the research lesson, and they also explored how the task might be revised based on this discussion.

Some potentially interesting aspects of lesson planning were not addressed in this paper: the author did not consider the relationship between the quality of the lesson planning and the quality of the research lesson. This paper did not look at the impact of lesson planning on teachers' mathematical and pedagogical knowledge (Lee and Takahashi 2011, Lewis 2009). And the paper did not look at how the lesson planning process exposes teachers' beliefs. The author hopes, however, that by making aspects of the planning phase of Lesson Study visible, this paper will contribute to helping educators outside Japan appreciate the full richness of Lesson Study, and better understand how it can improve teaching and learning.

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Appendix

The task given by the teacher was: "let's write word problems that can be solved by $8 \div 2$. Draw a picture or diagram for the problem situation. Also, write an equation and the answer, too."

A: Division to find the group size (partitive division)
2 people are sharing 8 strawberries. How many strawberries does each person get?
Equation: $8 \div 2 = 4$ Answer: 4 strawberries

B: Division to find the number of groups (quotative division)
We are going to give 2 strawberries to each person. If there are 8 strawberries, how many people will get strawberries?
Equation: $8 \div 2 = 4$ Answer: 4 people.

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Implementing a New National Curriculum: A Japanese Public School's Two-Year Lesson-Study Project

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The Common Core State Standards for Mathematics (National Governors Association Center for Best Practices [NGA Center] and Council of Chief State School Officers [CCSSO] 2010) present significant implementation challenges. In particular, the Standards for Mathematical Practice will require substantial changes in how most teachers teach. In Japan, where the national curriculum is revised approximately every eight years, teachers use lesson study to understand and implement the changing standards (Lewis 2010; Takahashi 2011), as well as to implement ideas from the latest research. Because Japanese educators have been successful in the past at implementing significant changes in teaching based on shifts in the standards (Lewis and Tsuchida 1997; Watanabe in press; Yoshida 1999b), it may be useful for U.S. educators to consider what Japanese schools do to support such changes. This article describes a two-year research program undertaken by the faculty and staff of a public elementary school in Tokyo. They used lesson study to implement recent revisions of the national curriculum and to investigate ways to improve student learning through the process.

■ The Role of Lesson Study in Implementing New Ideas and Curriculum

To support teachers in improving teaching and learning, the Japanese school system uses lesson study as the primary mechanism of professional development (Lewis 2000; Lewis and Tsuchida 1998; Murata and Takahashi 2002; Takahashi 2000; Takahashi and Yoshida 2004; Yoshida 1999a). In lesson study, teachers study the standards, read relevant research articles, examine available curricula and other materials, and work together to design a lesson focused on a problematic topic while also addressing a broader research theme related to teaching and learning. That lesson, called a *research lesson* (*kenkyu jugyou*), is taught by one teacher from the planning team while others observe. The planning team and observers then conduct a postlesson discussion focusing on how students responded to the lesson so the teachers can gain insight into the teaching-learning process.

Lesson study affords teachers the opportunity to look closely at teaching practices and judge, based on student learning, whether those practices properly support students in learning mathematics. Researchers credit Japanese lesson study with enabling a national shift from didactic, teacher-centered mathematics instruction to a student-centered approach based on problem solving (Lewis, 2002; Lewis and Tsuchida 1998; Stigler and Hiebert 1999; Yoshida 1999b).

Although lesson study is commonly used as a medium of professional development that focuses on teachers and schools improving their teaching and learning, it can also be used to implement new curricula and research findings (Murata and Takahashi 2002).

The Case

The school we examine in this article is a public elementary school in Tokyo with about 760 students in grades 1 through 6 and sixty-four teachers and staff members. In 2008, the Japanese Ministry of Education released a revision to the national standards, known as the Course of Study. The teachers at the school decided to focus their lesson-study work over the next two years on the new standards and their effective implementation. Schools in Japan often do this, especially when the new standards include unfamiliar material. Sometimes, schools apply for a small grant from their district to support their implementation work, especially to pay for outside experts and publication of their findings. The school gives back to the district through a written report and an open house to share the learning with other schools. The school in this case study applied for and received such a grant.

The *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics,* a document published by the Ministry of Education as a companion to the 2008 Course of Study (Asia-Pacific Math and Science Education Collaborative 2008), explicitly linked student thinking with expression:

> In this revision, the phrase "to express" was added. The ability to think and the ability to express are considered to be complementary. In the process of expressing their thoughts, students may realize their own good points or errors in their ideas. By expressing thoughts, they become better able to organize logical steps and produce better ideas. In class, they can express various ideas and they can learn from each other. For this reason, thinking and expressing are mentioned in parallel. (p. 7)

Inspired by this text, the teachers at the school decided to spend two years seeking ways to encourage all students to think mathematically and to communicate their thinking with each other.

The School Research Organization and the Research Steering Committee

During the two years of the school research program, all full-time teachers at the school worked within a structure based on existing grade-level groups (see fig. 2.1). Grade-level groups in Japanese elementary schools typically facilitate sharing responsibilities for running school events and for academic activities. Most public schools have time for grade meetings in their weekly schedule, typically about one hour, and teachers have desks in a common work area so they can collaborate regularly. For this research project, each grade-level group was made responsible for crafting a plan for a research lesson, conducting their research lesson in front of the rest of the faculty, serving as panelists during the postlesson discussion, and supporting the other teams' research lessons.

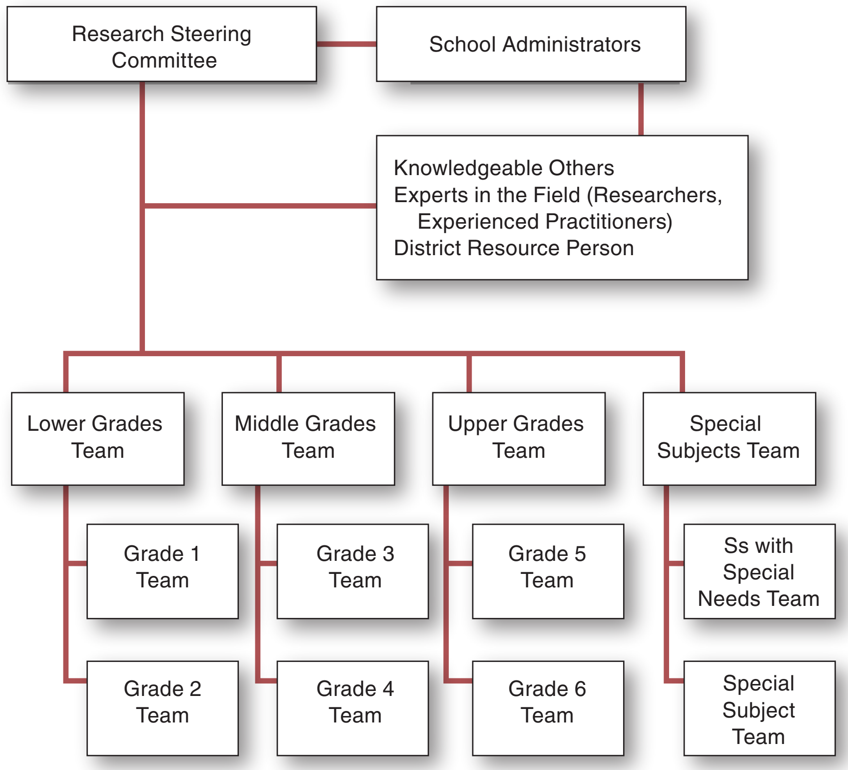

Fig. 2.1. Structure of the school research program

The school also had grade-band teams, which consisted of all the teachers from adjacent grades, such as grade 1 and grade 2. Although the responsibility for lesson planning belonged to each grade group, most of the lesson planning was done in grade-band meetings, since the teachers felt that the single grade-level groups, comprising only three or four teachers, were too small on their own to generate enough variety of ideas to lead to high-quality research lessons. Also, the grade-band meetings helped the teachers develop a shared view not only of their students but also of the scope and sequence of the curriculum in adjacent grades. This is important since Japanese elementary school teachers typically teach the same students for two consecutive years. Finally, the grade-band meetings allowed additional opportunities to participate in research lesson planning, a valuable experience, especially for novice teachers. The grade-band meetings allow these new teachers not only to learn how to design lessons but also to deepen their understanding of the topics they teach (Takahashi et al. 2005; Takahashi and Yoshida 2004; Watanabe, Takahashi, and Yoshida 2008).

Following common practice, the school organized a research steering committee, which consisted of representatives of each grade level and the lead teacher for mathematics, who was appointed chairperson of the committee by the principal on the basis of his leadership ability and knowledge of mathematics teaching and learning. (Typically, the lead teacher has his or her own self-contained class but also has responsibility for providing support for the upper-grade teachers and for preparing curriculum materials for the school. The lead teacher at this school taught mathematics to upper-grade students.) The committee led the school's efforts and maintained the cohesiveness of ideas across the grades. Among other things, the research steering committee was responsible for the following:

- Developing a master plan for the school research
- Scheduling and leading monthly meetings to find strategies to address the school's research theme based on the ideas of the teachers
- Publishing a monthly internal newsletter to record the findings from each research lesson
- Planning, editing, and publishing the school research reports, including those for the research open house
- Arranging for knowledgeable others to present lectures, teach demonstration lessons, and give final comments at research lessons

The first task of the research steering committee was to propose a focus for the school's research. That proposal was discussed by the full faculty at their first faculty meeting of the 2010 school year and the faculty approved the following research theme and focus of study:

- Research theme: The development of individual thinking and the expression of these thoughts
- Focus of study: Seeking effective ways to support students' individual problem-solving skills and better facilitation of whole-class discussion in teaching through problem solving

The research theme articulated a goal for students, inspired by the *Teaching Guide,* while the focus of study expressed the faculty's idea about a path to accomplish the goal.

During year one, each grade-level team developed a lesson plan for a research lesson and conducted the research lesson and postlesson discussion to address the theme. All full-time teachers observed the lessons and participated in the discussions, so each full-time teacher had the opportunity to be a part of eight research lessons during the school year. The school also invited two distinguished mathematics educators to give lectures, one in the first month of the school year (April) and another during the summer break, about the issues and trends in mathematics education and implementing the new Course of Study.

The teachers at the school shared many responsibilities for making the research lessons and discussions go smoothly. For example, for the research lesson held in June, the grade 1 team planned and taught the lesson. During the postlesson discussion, the grade 4 team facilitated the discussion while the team of teachers of students with special needs took notes for the school's official record.

At the first faculty meeting of year two, the research steering committee proposed a change in the research theme based on their reflections on the first year's activities. The first year's theme had emphasized developing individual students' ability to think and express their thoughts. Having made some gains with respect to this theme—for example, teachers were observing that students were more often able to solve problems independently—the teachers were now concerned that students were not appreciating the benefits of learning from others' ideas and developing better ideas by exchanging and combining ideas. This led to the following new research theme:

- Research theme: Mathematics teaching that helps students explain their ideas to each other and learn from each other—learning through problem solving.

The faculty also approved the schedule of activities for year two, which included a public open house near the end of the school year in December. To meet the deadline for this event, the teachers in the school had to complete all research lessons by the middle of the fall and compile their findings prior to the open house. Six research lessons and two invited experts' lectures on the theme topic occurred during the second year.

Throughout the two years of the project, the research steering committee met between the research lessons to summarize the ideas that had been proposed by each lesson planning team and addressed during the postlesson discussion. They published their summaries as a school research newsletter each month. These newsletters documented the process of this long-term collaborative effort, and more important, they allowed the teachers to share what was discussed and helped other teams build on the results of previous research lessons.

Lesson Plans and Their Development

In each stage of lesson plan development, members of the research steering committee reviewed the lesson plan and provided feedback to the team. Through this process, committee members tried to ensure that all the lesson plans developed by the school were of sufficient quality to merit discussion by the entire faculty and contributed to the school's effective implementation of the revised standards. But the steering committee and the school administrators found that the quality of the research lesson plans in year one was not satisfactory. So the committee distributed to each teacher at the beginning of year two a list of questions to guide them toward higher-quality lesson plans:

- Does the lesson plan provide sufficient information for the teacher to understand the task and the flow of the lesson?
- Does the lesson plan provide sufficient information about how the planning team decided to teach the lesson as described by the plan?
- Do the objectives of the lesson plan clearly address the Course of Study?
- Are the tasks appropriate for the students given the date of the lesson?
- Are the key questions clear? Will they encourage students to think mathematically and help them complete the task independently?
- Does the lesson plan include reasonable anticipated student responses and indicate how the teacher will help students overcome any misunderstandings?
- Does the lesson plan include a plan for formative assessment and a plan to accommodate individual student differences during the lesson?

The list seemed to be helpful; according to the principal, the research lesson plans in year two reflected much deeper thought compared to those from the previous year.

Disseminating the Results of the School Research

Toward the end of year two, the school faculty and staff hosted a half-day public open house to share their findings. All the district content specialists and principals of other area schools were invited, and many other schools sent their teachers. In all, a total of 612 participants attended, including teachers, administrators, educators, and parents.

The public open house consisted of three major parts: public research lessons, research presentations by the school's research steering committee, and a panel discussion by experts in the field of mathematics education who had been involved with the school's research project. Twenty-eight mathematics lessons, based on twenty-five different lesson plans, were conducted simultaneously for the participants to observe at the beginning of the open house. All twenty-five lesson plans were in a booklet given to each participant on arrival at the school. The participants were thus able to witness strategies for the effective implementation of the Course of Study in live lessons and were able bring these ideas back to their school as a set of lesson plans. The presentation given by the members of the steering committee informed participants about the philosophy and the rationale behind the strategies being used at the school. The presentation also provided educators from other schools an opportunity to learn how the school conducted its research using lesson study and what the faculty at the school had learned.

Two sets of research reports, from year one and of year two, were also made available for teachers and administrators of other schools as summaries of the school research effort. Since the school used a district grant to produce them, all the research reports were made available free. In the second year, the school compiled a report covering the entire two-year study. The report was produced as four booklets: three of them were distributed at the public open house and the last was sent to all the schools in the district at the end of the school year. An English translation of one of these booklets is available at http://www.impuls-tgu.org/en/resource/readings/page-26.html.

Discussion

The Japanese national standards released in 2008 contained a new emphasis on having students learn to express their ideas and learn from each other as a way to help students with their own thinking. The teachers at this school chose to spend two years working through lesson study to research changes in practice that would address this new emphasis. Some of what they learned—and what they put into practice—is evident in the booklet they published for the open house. Here are a few points from that booklet:

- Students were able to express their ideas by using not only words but also mathematical expressions and diagrams. Because of the cohesive use of diagrams, such as tape diagrams, area diagrams, and number line diagrams, and of expressions and equations throughout the grades, whole class discussions became deeper and productive. Moreover students were able to express their ideas in similar ways regardless of who was teaching the lessons.
- By crystallizing what was expected of students in each stage of problem solving (e.g., understanding the problem, solving the problem, reflecting upon the solution) and at the major points of teacher instruction, students were able to learn independently.
- By preparing effective key questions for each stage of problem solving, students were able to express their ideas in various ways and talk to each other clearly focusing on what should be discussed.

- By planning blackboard writing, the flow of the lessons became more coherent. Students became able to look back at what they learned by looking at the board. Then they could use it to put the various ideas together in integrated and expanded ways, and to evaluate their learning during the lessons by themselves (Matsuzawa Elementary School 2012, p. 19).

Although the school made use of outside experts—either to teach demonstration lessons, give lectures, or provide final comments at the research lessons—it is through lesson study that teachers made the changes described above.

Each teacher was deeply involved in planning only one research lesson per year, which may not seem like enough to support such profound growth. But the school's work over the two years was carefully organized to support teacher learning in various ways. Each teacher at the school had at least two opportunities to critique lesson plans from another team during the planning process through the grade-band meetings. Teachers observed and discussed the lessons of all the other grades at the school. And the newsletters published by the research steering committee helped each successive team build on what was learned before.

Conclusion

Implementing new standards and implementing findings from research share a common challenge: teachers must determine what the necessary changes will look like in their own classrooms, with their own students. To figure out these changes, teachers need to conduct their own research, and lesson study provides an organized way to do so. Because lesson study is tied to teachers' practice, there is no gap between research and practice.

In the United States, many lesson-study projects have been conducted by a few volunteers within a school with support from outside the school. Individual teachers can certainly improve their own teaching by participating in such volunteer groups. But in Japan, as this case study illustrates, improving teaching is the responsibility of all teachers at a school, to be worked on together.

Meeting the challenges of implementing the Common Core State Standards, especially the challenges of the Standards for Mathematical Practice, requires fundamental changes in teaching, although the exact nature of those changes is not clear. Lesson study is a way for teachers to simultaneously investigate and implement changes in curriculum and practice, if a school faculty can work together in a coordinated way. Based on this case study, the following features of whole-school lesson study appear to be important:

- A clear research focus (the research theme and focus of study)
- A structure to support collaboration (grade-level teams and grade-band teams)
- Distributed leadership in the form of a research steering committee that comprises teachers from multiple grades
- Guidelines to help teams create high-quality lesson plans
- A conscious effort to extend what is learned in the research lessons to later research lessons (through the monthly newsletter published by the research steering committee)
- Support from knowledgeable others outside the school

None of these components is commonplace in U.S. schools. Implementing school-based lesson study in the United States would require a significant change in how school leaders view professional development—from viewing it as something that is done *to* teachers by experts from outside, to viewing it as something that is done *by* teachers. Such a change in thinking about professional development is likely to be important to successfully implement the Common Core standards.

This chapter summarizes key findings from a study supported by Project IMPULS at Tokyo Gakugei University. The full report from that study has been published as "Supporting the Effective Implementation of a New Mathematics Curriculum: A Case Study of School-Based Lesson Study at a Japanese Public Elementary School" (Takahashi 2014).

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Section 4
Program Overview

IMPULS Lesson Study Immersion Program

(Version _ April 14)

Tuesday, June 20, 2017 to Thursday, June 29, 2017
Tokyo, Japan

The lesson study immersion program is designed to give mathematics education researchers and practitioners from outside Japan an opportunity to examine authentic Japanese Lesson Study in mathematics classrooms.

The major purpose of this program is for us to receive feedback on the strengths and weaknesses of Japanese Lesson Study and to discuss how to improve mathematics teacher professional development programs. To accomplish this, we are inviting leaders of mathematics education to immerse themselves in authentic Japanese lesson study and to observe mathematics research lessons in elementary and lower secondary grades.

Tentative Schedule in Japan

All the research lessons and the post-lesson discussions will have simultaneous English translations by experts. Latest Schedule will update on Basecamp.

Date	AM	PM	Stay
19, June MON	Arrival Day 15:00 ~ Check-in at Hotel Mets Kokubunji		Hotel Mets Kokubunji
20, June TUE	Opening Session Mathematics teaching and learning in Japan,Lesson Study in Japan,Teaching through problem solving and Kyouzai-Kenkyu	Workshop: Japanese mathematics lessons and lesson study *Welcome dinner	Hotel Mets Kokubunji
21, June WED	Preparation for the research lesson observation	<School Visit 1> Heisei Elementary School (School-based LS)	Hotel Mets Kokubunji
22, June THU	Preparation for the research lesson observation	<School Visit 2> TGU Koganei Junior High School (Specially-appointed LS)	Hotel Mets Kokubunji
23, June FRI	Move to Yamanashi using a charter bus	<School Visit 3> Public Elementary School (School-based LS)	Tokiwa Hotel
24, June SAT	<School visit 4> University of Yamanashi Attached Elementary School (Cross-district LS)	Sightseeing Move back to Tokyo	Hotel Mets Kokubunji
25, June SUN	Free		Hotel Mets Kokubunji
26, June MON	Preparation for the research lesson observation	<School Visit 5> Junior High School (School-based LS)	Hotel Mets Kokubunji
27, June TUE	Preparation for the research lesson observation	<School Visit 6> Sarugaku Elementary School (School-based LS)	Hotel Mets Kokubunji
28, June WED	Preparation for the research lesson observation	<School Visit 7> Elementary School in Ohta ward (District-wide LS)	Hotel Mets Kokubunji
29, June THU	Discussion to wrap up the Lesson Study Immersion Program	Closing Session *Farewell dinner	Hotel Mets Kokubunji
30, June FRI	Departure Day		

(3) The hotel information:

<u>In Tokyo (June 19 - 23, June 24 - June 30)</u>

Hotel Mets Kokubunji

Address: 3-20-3 Minami-cho, Kokubunji, Tokyo 185-0021 JAPAN

Tel: + 81-42-328-6111

Website: http://www.jrhotelgroup.com/eng/code/codeeng123.htm

*Facilities Include: TV, Internet (Wi-Fi, Free), radio, clock, internal telephone, hangers, a clothes brush, shoehorn, slippers, mirror, safe, water boiler, tea cup, refrigerator, hair dryers, toilet.

*Amenities Include: Face towels, bath towels, shampoo with conditioner, body soap, and soap, toothbrushes, toothpaste, razor-sharp, night wear (pajamas).

*There are laundromats at 10th Floor.

<u>In Yamanashi (June 23 -24)</u>

TOKIWA HOTEL

Address: 2-5-21 Yumura, Kofu-shi, Yamanashi, JAPAN〒400-0073

Tel :+81-055-254-3111

Website: http://www.tokiwa-hotel.co.jp/e_index.html

*Facilities Include: Air conditioning, bathtub/shower, toilet, refrigerator, Japanese style wear (Yukata), TVs, Internet (Free Wi-Fi is available at lobby and room), and safe, hair dryers.

*Amenities Include: Bath towels, face towels, tooth brushes, shampoo, conditioner, soap.

*Project IMPULS will arrange for a <u>Japanese style room (tatami room) for 4 - 5 persons in one room</u>. If you have any request, please contact us.

IMPULS Immersion Program 2017 Schedule

Ver June19

Date	Time	detailed contents
June 19 (Mon)		Arrival day
	15:00	Check in at Hotel Mets Kokubunji *Check in time is 3pm. When you arrived before 3pm, you can leave your bags at the frontdesk.
June 20 (Tue)	9:00	Meet at lobby *IMPULS staff will guide to TGU.
	9:30	General Orientation
	10:00	Mathematics teaching and learning in Japan,Lesson Study in Japan,Teaching through problem solving and Kyouzai-Kenkyu
	11:30	Lunch on your own
	13:00	Workshop: Japanese mathematics lessons and lesson study
	15:00	Tea break
	16:30	About Research Lesson Report
	17:30	Welcome Reception
	19:30	Move back to the hotel
June 21 (Wed)	10:45	Meet at Kokubunji station
	10:54	Move to Heisei Elementary School by public transportation (train and bus)
	12:00	Lunch on your own
	13:00	Preparation for the research lesson observation
	14:00	<Research lesson 1>Heisei Elementary School (Schol-based LS) (Let's think about division of decimal numbers , Grade 5 , Ms KONOHARA, Nami)
	15:00	Post Lesson Discussion
	16:30	End of the program
	17:00	Dinner with Japanese teachers (optional)
June 22 (Thu)	9:30	Meet the hotel at lobby
	10:00	Preparation for the research lesson observation
	11:30	Lunch on your own
	13:00	Preparation for the research lesson observation
	13:50	Move to the school
	14:20	<Research Lesson 2> TGU Koganei Junior High School (Specially Appointed LS for Fuzoku teachers)(Mr. Sho Shibata, Grade 8)
	15:30	Post Lesson Discussion
	17:00	End of the program

		Dinner on your own
June 23 (Fri)	7:45	Meet at lobby, Check out
	8:00	Move to Yamanashi by a charter bus
	11:00	Arrival at Oshihara Elementary School
	11:15	Introduction and tour of the school
	11:40	Lesson Observation
	12:40	School lunch with pupil
	13:55	<Research Lesson 3> Oshihara Elementary School (School-based LS)(Grade 4 , Mr Ohma)
	15:15	Post Lesson Discussion
	17:00	Move to Hotel by a charter bus
	19:00	Dinner at the Tokiwa Hotel (Japanese style)
June 24 (Sat)	6:40	Breakfast
	8:15	Meet at lobby, Check out
		Move to University of Yamanashi Attached Elementar School by a charter bus
	8:30	Arrive at elementary school attached to Yamanashi University
	9:00	<Research Lesson 4> University of Yamanashi Attached Elementary School, Cross-district LS(Grade 3 ,)
	10:00	<Research Lesson 5> University of Yamanashi Attached Elementary School, Cross-district LS(Grade ,6)
	11:00	Post Lesson Discussion
	12:30	Lunch(bento box)
	13:30	Move back to Tokyo *Sightseeing at Takeda shrine and Mt Fuji
	17:30	Arrive at Mets Kokubunji, check in
		Dinner on your own
June 25 (Sun)		Free
June 26 (Mon)	9:00	Meet at lobby
	9:30	Preparation for the research lesson observation
	11:45	Lunch on your own
	12:42	Move to the school by public transportation

	14:30	<Research Lesson 6> Kiyose Daiyon Junior High School *ordinal lesson
	15:30	Students will demonstrate japanese musical instrument "koto" and english short drama
	16:30	End of the program
June 27 (Tue)	9:50	Meet at lobby
	10:12	Move to the school by public transportation
	11:00	Arrive at the school. Preparation for the research lesson observation
	12:30	Lunch on your own
	13:45	<Research lesson 7> Sarugaku Elementary School (Schol-based LS) (, Grade , M)
	15:00	Post Lesson Discussion
	15:30	Final Comment
	16:30	End of the program
June 28 (Wed)	10:00	Meet at lobby
	10:28	Move to the school by public transportation
	12:15	Arrive at the school, Preparation for the research lesson observation, lunch on your own
	13:35	<Research Lesson 8> Kojiya Elementary School (District-wide LS)(Grade ,)
	14:30	Post Lesson Discussion
	16:30	End of the program
June 29 (Thu)	9:30	Meet at lobby
	10:00	Discussion to wrap up the Lesson Study Immersion Program
	11:30	Lunch on your own
	13:00	Discussion to wrap up the Lesson Study Immersion Program
	16:00	Colosing session
	17:00	Farewell Party at Umenohana
June 30 (Fri)	10:00	Check out
		Departure Day

2017 Immersion Program Schedule
Thursday June 29th

Time	Activity
10.00am - 10.20am	**Reflection on Grade 6 Lesson (Ohta Ward Kojiya Elementary School)**
10.20am - 11.10am	**Q&A** Norm Setting Post questions: 1. Japanese Mathematics teaching and learning 2. Lesson Study in Japan Sort questions Discussion of questions
11.10am - 11.30am	**Prepare Lesson Report in teams**
11.30am - 1.00pm	**Lunch** (optional to continue to work on lesson report)
1.00pm - 2.30pm	**Individual Learning Journey** **Part 1:** ● Review your daily reflections, notes and lesson plans and write down your thoughts about the following: (15 minutes quiet time) 1. What opportunities for teacher learning does Lesson Study afford? 2. How did your views of lesson study change from your participation in the IMPULS program? 3. How do you view teaching and learning now? ● Find a partner and share your thinking (3 minutes per question for both partners to share) ● Each person shares out to the whole group 2-3 important take-aways (monitor your own air time) **Part 2:** ● How will you take this back to your own context? (open discussion)
2.30pm - 3.20pm	**Next Steps: What would you suggest to your colleagues?** ● In small groups* write out recommendations you would make for: ○ Your own classrooms (instruction, pedagogy) ○ Lesson Study Teams at your site (or in your context) ○ Your School (for your admin, ILT, or other decision-makers regarding instruction, pedagogy, or PD) ○ Central Office if applicable (may include math department,

	lesson study supports through central office, etc.) ○ Networking across sites, districts, external partners ** Groups may choose to do this through a visual, a timeline of actionable next steps, a chart, or any other format conducive to generating a concrete plan*
3.20pm - 4.00pm	**Each group shares out ideas for next steps** (3 minutes per group)

5pm - Final party at Umenohana

***Groups**

1. AWE
2. Argonne
3. Hillcrest
4. Lawton
5. Middle Schools (David/Stephanie)
6. Muir
7. Prieto
8. SFC
9. Ed, Millie (University based)
10. Belle, Karen, Ruth (consultants)
11. Meghan, Nakachi, Rory, Shelby (Starting lesson study)

Commitment to complete by July 31st, 2017

- Lesson report (submit to Shelley Friedkin)
- Daily reflections
- Final reflections (Day 9)
- Post program survey (to be sent out to participants July 5th**)**

Section 5

Program Orientation

Project IMPULS
Lesson Study Immersion Program
2017

Day 1

Lesson Study in Japan

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Outside Japan, lesson study has been introduced as teacher-led professional development.
In the United States, many lesson-study projects have been conducted by a few volunteers within a school with support from outside the school.
Individual teachers can certainly improve their own teaching by participating in such volunteer groups. But ……..

Why Lesson Study?

Stigler and Hiebert argue that Japanese mathematics lessons better exemplify current U.S. reform ideas than do typical U.S. mathematics lessons (1999).

> When we watched a Japanese Lesson, for example, we noticed that the teacher presents a problem to the students without first demonstrating how to solve the problem. We realized that U.S. teachers almost never do this. U.S. teacher almost always demonstrates a procedure for solving problems before assigned them to students.

Lesson Study was introduced a form of professional development to improve mathematics teaching and learning.

What we learned from the TIMSS video study

The Teaching Gap (J. Stigler & J. Hiebert, 1999)

The study samples included 231 eighth-grade mathematics classrooms:

81 in the US,

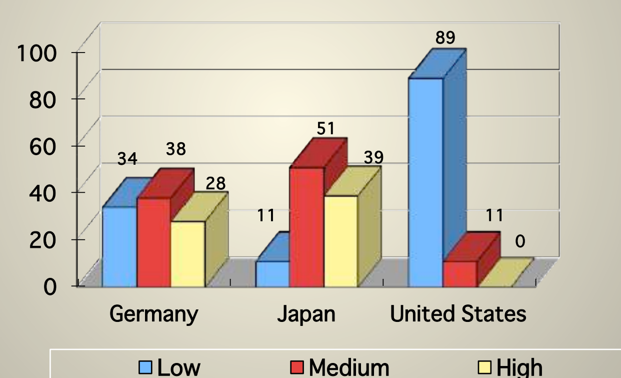
100 in Germany, and

50 in Japan.

These samples were designed as a nationally representative sample of eighth-grade students in the three countries.

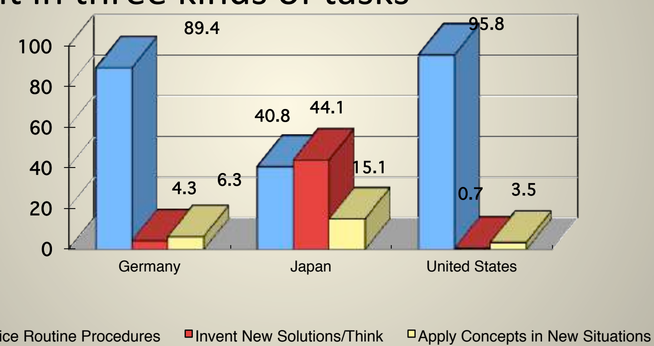
Focused on important mathematics

- Percentage of lessons rated as having low, medium, and high quality of mathematical content



Learning mathematics with understanding

Average percentage of seat working time spent in three kinds of tasks



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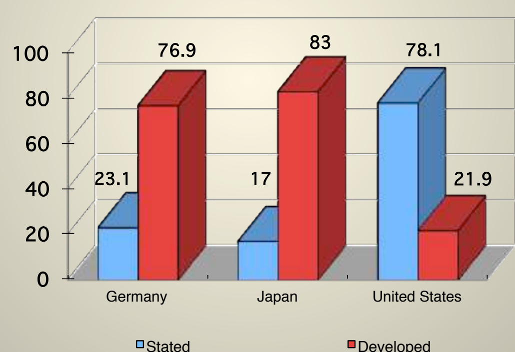
New knowledge from experience and prior knowledge

Average percentage of topics in eight-grade mathematics lessons that contained concepts that were DEVELOPED or STATED.



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Problem Solving

(Standards and Focal Points, NCTM)

- An Agenda For Action (NCTM 1980)
  Recommendation 1
  Problem Solving must be the Focus of School Mathematics in the 1980s
- Problem solving means engaging in a task for which the solution is not known in advance.
- Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts.
- Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases.

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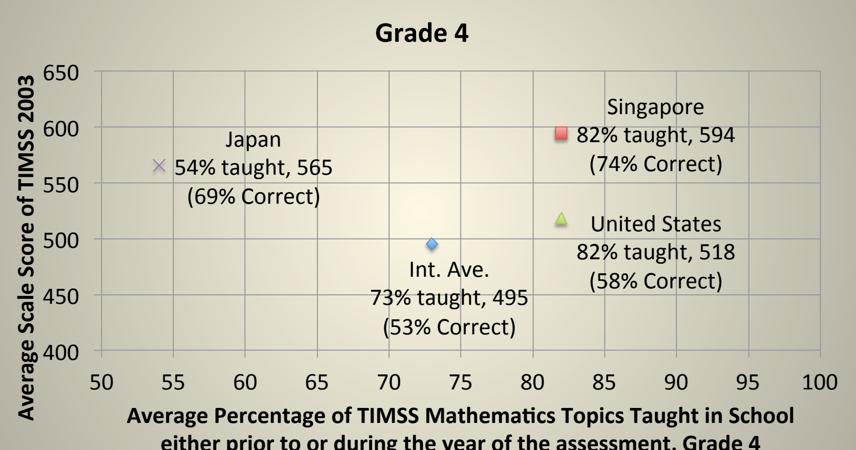
Average Percentage of TIMSS Mathematics Topics Taught in School and the Achievement (Average Scale Score) of the TIMSS 2003



Source TIMSS 2003 International Mathematics Report
Grade 8: Exhibit 5.7 (p.192), Exhibit C. 1 (p.400)
Grade 4: Exhibit 5.7 (p.193), Exhibit C. 1 (p.402)

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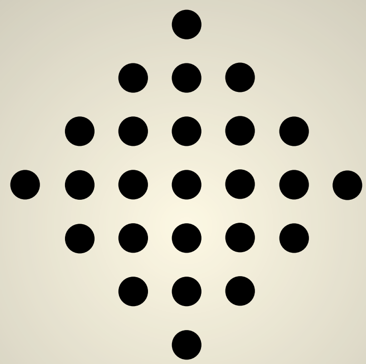
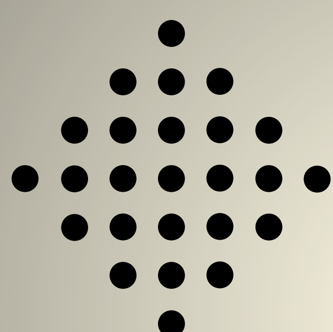
How many marbles are there in the picture below?



Find the answer in as many different ways as you can. Write your ways of finding the answer and write your answer

Nagasaki (1990) in Fujii, T. (1993). Japanese Students' Understanding of School Mathematics; Focusing on Elementary Algebra. In G. Bell (Ed.), Asian Perspectives on Mathematics Education (pp. 70-89). Lismore, Australia: The Northern Rivers Mathematical Association.



- Category 1 (words/verbal explanation) Counting one by one, count by lining, count from the top
- Category 2 (words/verbal explanation) addition: sum up, by adding the marbles
- Category 3 (verbal explanation) multiplication: taking them into groups, count by fives
- Category 4 (explanation with mathematical expression) addition: 1+3+5+7+5+3+1, 4+3+4+3+4+3+4
- Category 5 (explanation with mathematical expression) multiplication: 5x5, 4x4+3x3, 3x8=24 24+1

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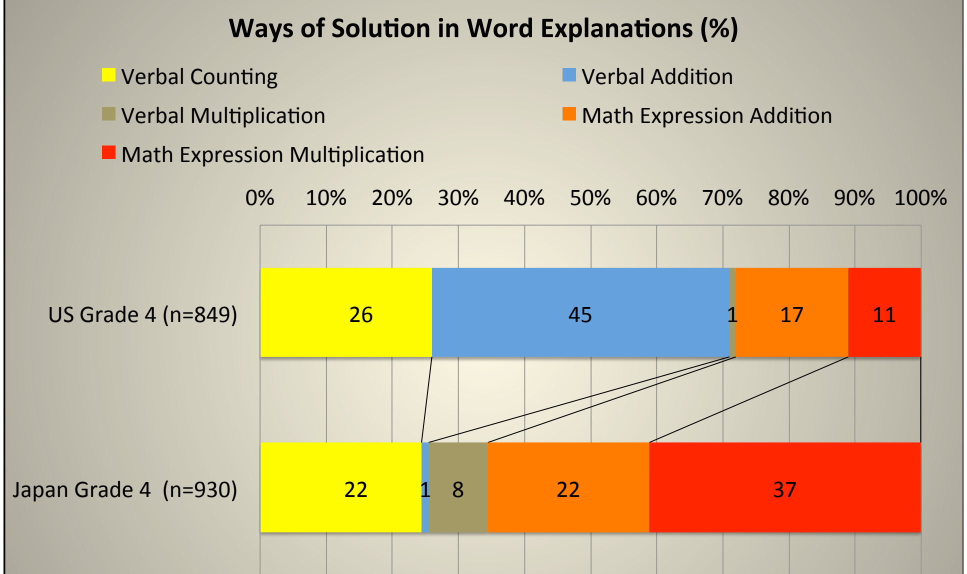

Nagasaki (1990) in Fujii, T. (1993). Japanese Students' Understanding of School Mathematics; Focusing on Elementary Algebra. In G. Bell (Ed.), Asian Perspectives on Mathematics Education (pp. 70-89). Lismore, Australia: The Northern Rivers Mathematical Association.

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Three Levels of Teaching

Japanese mathematics educators and teachers identify three levels of expertise of mathematics teaching:

- Level 1: The teacher can tell students the important basic ideas of mathematics such as facts, concepts, and procedures.
- Level 2: The teacher can explain the meanings and reasons of the important basic ideas of mathematics in order for students to understand them.
- Level 3: The teacher can provide students with opportunities to understand these basic ideas, and support their learning so that the students become independent learners.

(Sugiyama, Y. 2008, Trans. Takahashi, A., 2011a)

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Two Major Types of Professional Development

- Phase 1 professional development focuses on developing the knowledge for teaching mathematics,
  - through reading books and resources, listening to lectures, and watching visual resources such and video and demonstration lessons.
- Phase 2 professional development focuses on developing expertise for teaching mathematics
  - teachers should plan the lesson carefully, teach the lesson based on the lesson plan, and reflect upon the teaching and learning based on the careful observation. Japanese teachers and educators usually go through this process using Lesson Study

Lesson Study is not an end in itself, but a process for accomplishing specific teaching-learning goals.

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Can Lesson Study Make An Impact on Student and Teacher Learning?

A recent randomized controlled trial **demonstrated a significant impact of lesson study** supported by mathematical resource kits on both U.S. teachers' and students' mathematical knowledge

Teachers randomly assigned to lesson study conditions reported their professional learning to be of significantly higher quality than did educators randomly assigned to self-chosen professional learning, on indicators such as "Encouraged my active participation," "Valued my opinion, experience, and contributions" and "Included intellectual rigor, constructive criticism, and challenging of ideas" (Lewis & Perry, under review-b).

- **This is one of only two mathematics professional learning interventions (of 643 studied) identified by the What Works Clearinghouse to meet scientific criteria and demonstrate impact on students' mathematical proficiency (Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014).**

However…..

- Although many schools and teachers have tried to use ideas from Lesson Study in various ways, only a few cases have been documented in which there was strong evidence of impact on teaching and learning (e.g., Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014; Lewis, Perry, Hurd, & O'Connell, 2006).
- Why Lesson Study has been less consistently impactful outside of Japan?
- There are important aspects of *jugyou kenkyuu* that are getting "lost in translation" and can be fixed.

Interesting Cases (1)

Some school districts decided to try Lesson Study in one day. In the morning, a team of teachers came together to spend 30 minutes planning a lesson. They taught the lesson with students and reported what they observed. That afternoon, they modified the lesson plan in 30 minutes and taught the revised lesson.

- Although this one-day process seems to include all the components of Lesson Study that are described in journal articles and resources, this is far from *jugyou kenkyuu*. The typical duration of one *jugyou kenkyuu* cycle in a Japanese elementary school is more than 5 weeks (Murata & Takahashi, 2002) – it is certainly never done in just one day. Moreover, re-teaching a research lesson is not a common practice in *jugyou kenkyuu* (Fujii, 2014).

Interesting Cases (2)

After a public research lesson and post-lesson discussion, the team members commented that "we did not learn much from the research lesson and the discussion because we have already done six lesson study on this lesson."

- This team thought that the purpose of Lesson Study was to reteach a lesson until perfecting the lesson. The true purpose of *jugyou kenkyuu*, however, is to establish shared knowledge for teaching and learning among professionals, and not perfecting a lesson plan. In fact, Japanese teachers share the belief that there is no perfect lesson plan available. For Japanese teachers, the lesson plan is a plan for contingency and not a script for teaching the topic (Lee & Takahashi, 2011).

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Interesting Cases (3)

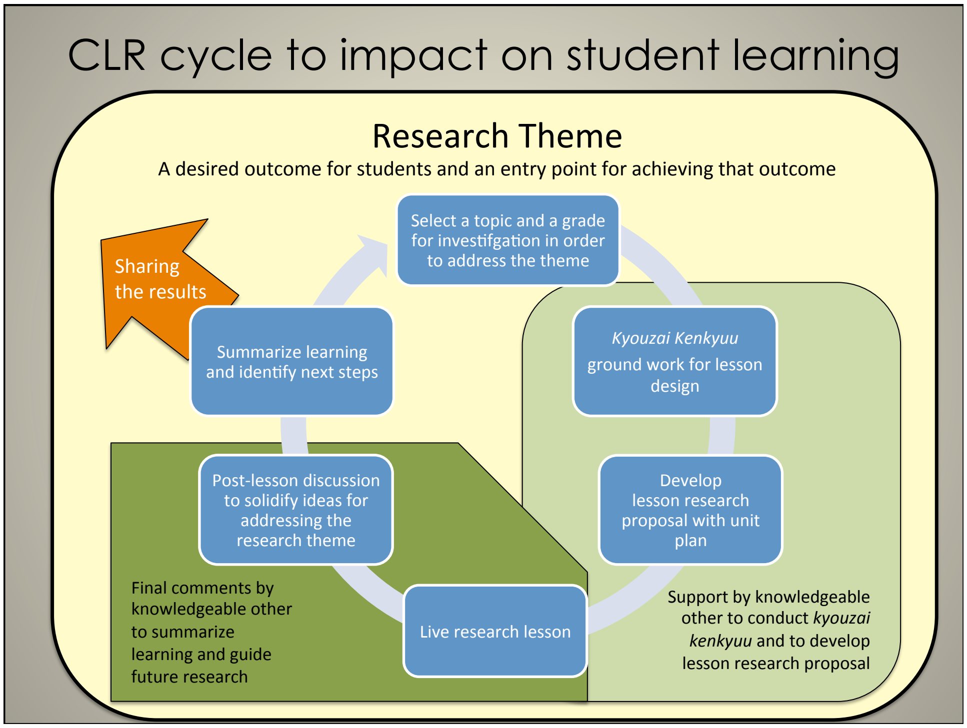
Lesson planning team chose their favorite task/activity for the students to demonstrate the teacher's ability to teach mathematics as a research lesson.

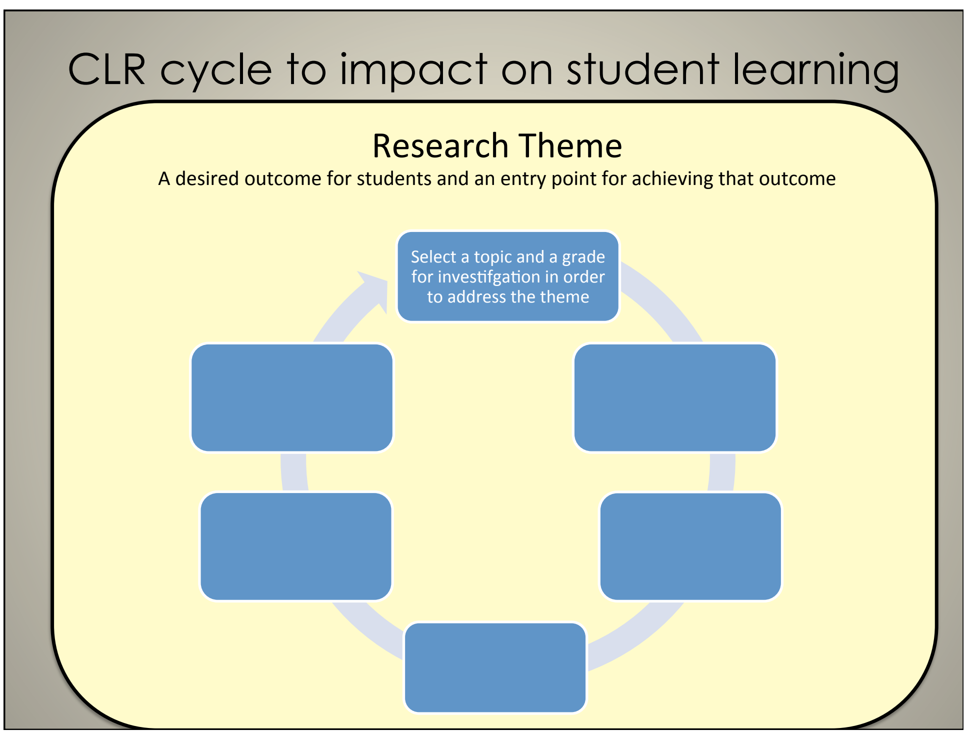
- Observing a demonstration lesson may be a powerful opportunity of teacher professional development but may not be a Phase 2 PD.

Essential elements for effective Lesson Study

We define Collaborative Lesson Research (CLR) has having the following components:

1. A clear research purpose
2. Kyouzai kenkyuu
3. A written research proposal
4. A live research lesson and discussion
5. Knowledgeable others
6. Sharing of results


CLR cycle to impact on student learning
Research Theme
A desired outcome for students and an entry point for achieving that outcome
Select a topic and a grade for investifgation in order to address the theme
Sharing the results
Summarize learning and identify next steps
Kyouzai Kenkyuu
ground work for lesson design
Post-lesson discussion to solidify ideas for addressing the research theme
Develop lesson research proposal with unit plan
Final comments by knowledgeable other to summarize learning and guide future research
Live research lesson
Support by knowledgeable other to conduct kyouzai kenkyuu and to develop lesson research proposal


CLR cycle to impact on student learning
Research Theme
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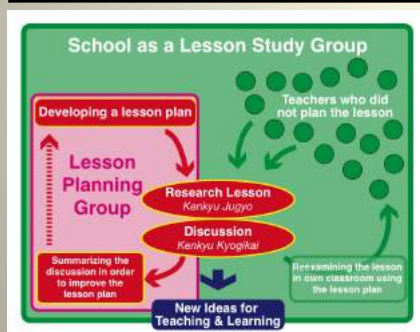
School-based lesson study

Description	Main Purpose
Usually all teachers from a school participate Establish a school lesson study goal •Form several sub-groups that engage in a lesson study cycle	Achieving systematic and consistent instructional and learning improvement in the school as a whole Develop a common vision of education at the school through teacher collaboration



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District-wide Lesson Study

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District-wide Lesson Study

Description	Main Purpose
Organized as an intra-school lesson study group Usually subject oriented groups (e.g., math teachers from each school in the district gather to conduct lesson study) Meet once or twice a month	Developing communication among the schools in the district. Exchanging ideas between the schools. Improving instruction and learning in the district as a whole

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Cross-district Lesson Study

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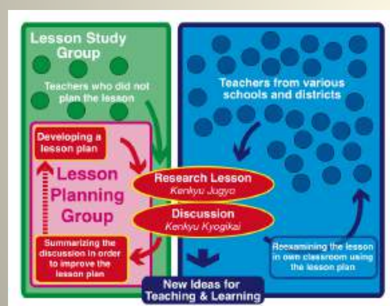
Cross-district Lesson Study

Description	Main Purpose
Usually a voluntarily organized group Group of enthusiastic practitioners with purpose of improving teaching and learning or curriculum in a certain subject Meet once or twice after school on off-school days	Developing new ideas for teaching chosen topics. Investigating curriculum sequences and contents. Developing curriculum



Essential elements for effective Lesson Study

We define Collaborative Lesson Research (CLR) has having the following components:

1. A clear research purpose
2. Kyouzai kenkyuu
3. A written research proposal
4. A live research lesson and discussion
5. Knowledgeable others
6. Sharing of results

Section 6

School Research Reports, Lesson Plans & Participant Lesson Reports

Section 6 Content

Grade 5 Mathematics Lesson Plan

Date: June 21, 2017
Heisei Elementary School, Taitoh Ward, Tokyo
Grade 5 Classroom 1 (27 students)
Gungun (standard pace) course: KONOHARA, Nami
Jikkuri (steady pace) course: NAGASHIMA, Keita

Research Theme
Nurturing students who can identify and solve mathematical questions and express their ideas on their own
~ Through problem solving in mathematics ~

1 Name of the Unit: Let's think about division of decimal numbers (14 lessons)

2 Goals of the Unit

Students will understand the meaning and ways of calculating quotients when the divisor is decimal numbers.

3 Goals and standards for each assessment domain

Domain	◆ Goals • Assessment standard
Interest, Eagerness, and Attitude	◆ Students try to think about the meaning of division by decimal numbers by generalizing their understanding of division of whole numbers. The think about ways to calculate the quotients based on the properties of the base-10 numeration system.
	• Students try to connect ways to calculate division with decimal numbers with the ways of division with whole numbers. • Students recognize the merit that Decimals ÷ Decimals can be calculated in a similar way with division of whole numbers.
Mathematical Way of Thinking	◆ Students can think about ways to calculate the quotients when the divisors are decimal numbers by using number lines and properties of operations, and they can explain their ideas concisely.
	• Students think about meaning of calculating division of decimal numbers by making use of their prior knowledge of calculations and number lines. • Students are thinking about ways to calculate division of decimal numbers. • Students think about and represent Decimal ÷ Decimal by making use of their knowledge of division properties and calculations with whole numbers. • Students think about the size of the remainders. • Students think about the ways to calculate the quotients when the quotients must be rounded. • Students think about represent the way to set up the appropriate division calculation by making use of number lines. • Students think about ways to determine how many times as much or the base number using decimal number division just as they did with whole numbers.

Skills with Numbers, Quantities, and Figures	◆Students can calculate the quotients even when the divisors are decimal numbers.
	• Students can calculate Decimal ÷ Decimal using the standard division algorithm. • Students can find the quotients to the specified number of decimal places. • Students can divide by decimal numbers accurately.
Knowledge and Understanding about Numbers, Quantities, and Figures	◆Students understand the meaning and ways to calculate the quotients when the divisors are decimal numbers.
	• Students understand the size relationship between the dividend and the quotient based on the size of the divisor (greater or less than 1). • Students understand the size of the remainder (where the decimal point must be placed). • Students understand that we can use the idea of times as much to compare quantities. • Students understand the meaning of division of decimal numbers.

4 About the Unit

(1) With respect to the National Course of Study

The content of this unit is described in the National Course of Study as follows.

(3) Students will deepen their understanding of multiplication and division of decimal numbers, and be able to use them appropriately.

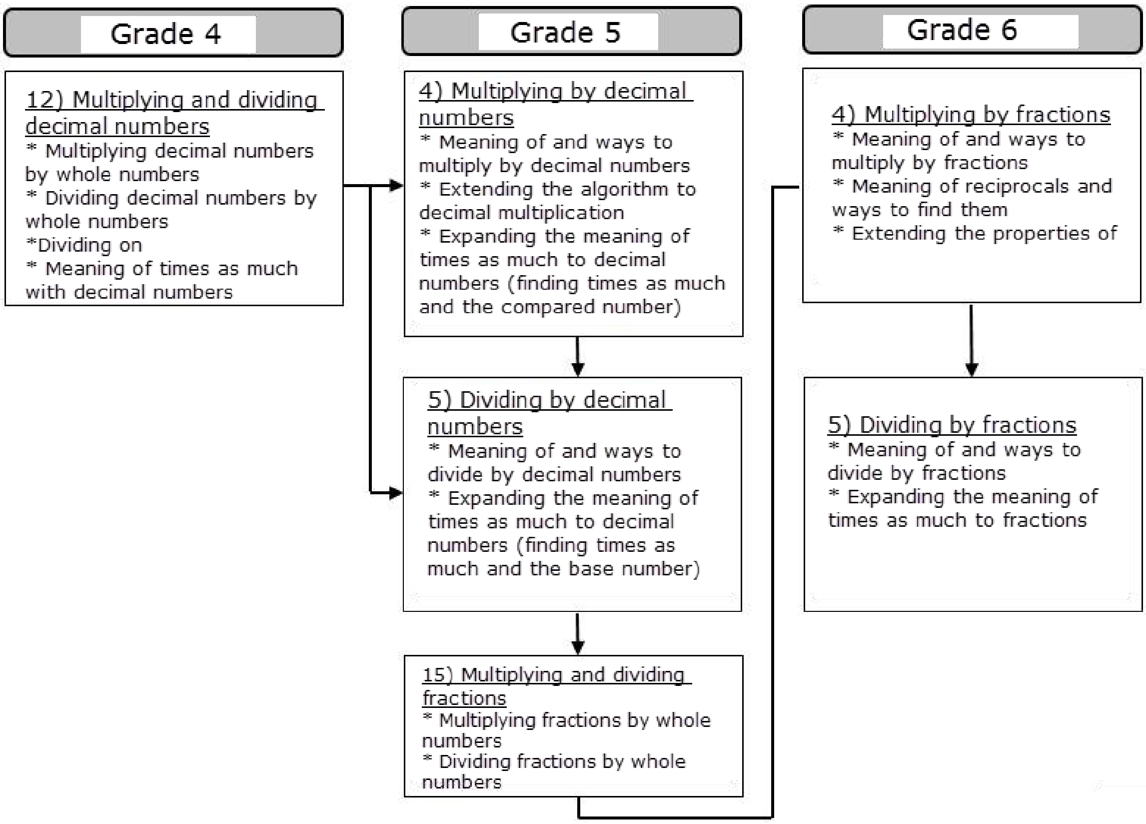
a. To understand the meaning of multiplying or dividing by decimal numbers based on the calculation ideas for multiplying and dividing decimal numbers by whole numbers.
b. To explore way to multiply and divide by decimal numbers, and be able to calculate accurately; to understand the size of numbers.
c. To understand that the same properties of multiplication and division for whole numbers will apply to decimal multiplication and division.

[Mathematical Activities]

Investigate and explain the meaning and the ways of calculating with decimal numbers, using words, numbers, mathematical expressions, diagrams, and number lines.

In this unit, students will expand the meaning of division by studying the meaning of division when the divisors are decimal numbers. Students cannot explain division by decimal numbers using the “fair sharing” idea that they have been using. Therefore, by using number lines and equations with words, students will understand the meaning of division as “the calculation to find the amount per unit.” They will also understand ways to carry out the calculation and develop their ability to apply their learning. They will also investigate times as much relationships with decimal numbers as the foundations for their future study of ratios and rates.

(2) Scope and Sequence



5 About the students

(1) Current state of students in mathematics

Students generally approach everything with eagerness. Many are mild-mannered but willing to share their ideas during lessons. However, even though they can share their ideas orally, some students find it difficult to express their ideas in writing. Some students often do not know what to write in their own notebooks. During independent problem solving time, they often require hints from the classroom teacher.

In addition, there is a wide range of mathematical achievement levels within the class. Therefore, instead of teaching mathematics with the whole-class instruction, we have been splitting the class into 2 smaller groups. To create the 2 groups, we administer a readiness test before a unit. The homeroom teacher and the partner teacher who has been assigned specifically to support small group instruction analyze the results of the test and create the 2 groups. In the area of numbers and calculations, the achievement levels vary significantly, and some students still struggle with whole number division. In this unit, we have created 2 groups, "*gungun* course" and "*jikkuri* course" considering the differences in their prior achievements.

<About the 2 Courses>

Gungun course --- According to the readiness test results, these students have mastered the prior contents fairly well. The instruction will proceed with the suggested pacing by the textbook.

Jikkuri course --- According to the readiness test results, these students have yet to mater the prior contents at the satisfactory level. More concrete and semi-concrete materials will be incorporated in instruction so that students can engage in more hands-on activities.

(2) Results and analysis of survey on students' attitudes toward mathematics
Administered to all 27 students on May 12

(a) Students placed in *gungun* course (16 students)				
	Definitely agree	Somewhat agree	Somewhat disagree	Definitely disagree
I like studying mathematics.	12 (75 %)	3 (19 %)	0 (0 %)	1 (6 %)
Studying mathematics is important.	15 (94 %)	0 (0 %)	1 (6 %)	0 (0 %)
I understand mathematics lessons well.	12 (75 %)	3 (19 %)	1 (6 %)	0 (0 %)
When I see a novel problem in a math lesson, it makes me want to solve it.	11 (69 %)	4 (25 %)	0 (0 %)	1 (6 %)
When I can't figure out a problem in a math lesson, I keep trying instead of giving up.	9 (56 %)	4 (25 %)	3 (19 %)	0 (0 %)
I think about ways to use what I learn in math lessons in my daily life.	4 (25 %)	10 (63 %)	1 (6 %)	1 (6 %)
I think what I have learned in math lessons has been useful in my daily life.	10 (63 %)	5 (31 %)	1 (6 %)	0 (0 %)
When I am solving a math problem, I think about easier and simpler ways of solving it.	10 (63 %)	4 (25 %)	2 (12 %)	0 (0 %)
When we learn a formula or a rule in math lessons, I try to understand why it works.	7 (43 %)	6 (38 %)	3 (19 %)	0 (0 %)
I try to write ways to solve or think about a problem in my notebook.	9 (56 %)	6 (38 %)	1 (6 %)	0 (0 %)

(b) Students placed in *jikkuri* course (11 students)				
	Definitely agree	Somewhat agree	Somewhat disagree	Definitely disagree
I like studying mathematics.	3 (27 %)	5 (46 %)	1 (9 %)	2 (18 %)
Studying mathematics is important.	8 (73 %)	3 (27 %)	0 (0 %)	0 (0 %)
I understand mathematics lessons well.	3 (27 %)	7 (64 %)	1 (9 %)	0 (0 %)
When I see a novel problem in a math lesson, it makes me want to solve it.	6 (55 %)	3 (27 %)	2 (18 %)	0 (0 %)
When I can't figure out a problem in a math lesson, I keep trying instead of giving up.	6 (55 %)	3 (27 %)	2 (18 %)	0 (0 %)
I think about ways to use what I learn in math lessons in my daily life.	4 (36 %)	5 (46 %)	1 (9 %)	1 (9 %)
I think what I have learned in math lessons has been useful in my daily life.	7 (64 %)	4 (36 %)	0 (0 %)	0 (0 %)
When I am solving a math problem, I think about easier and simpler ways of solving it.	5 (45.5 %)	5 (45.5 %)	1 (9 %)	0 (0 %)
When we learn a formula or a rule in math lessons, I try to understand why it works.	6 (55 %)	3 (27 %)	1 (9 %)	1 (9 %)
I try to write ways to solve or think about a problem in my notebook.	6 (55 %)	3 (27 %)	2 (18 %)	0 (0 %)

Students in both groups generally agree that "studying mathematics is important" regardless of whether or not they like mathematics. However, there are obvious differences in the response patterns in the 2 groups. In *gungun* course, about 70 % of students strongly agree that "when I see a novel problem in a math lesson, it makes me want to solve it," but the students in *jikkuri* course are not so enthusiastic about such problems. This seems to suggest that they do not like novel problems. Relatively small portion of students in both courses definitely agreed that "I think about ways to use what I learn in math lessons in my daily life." This makes us wonder if students do not know when they can apply what they have learned in their daily lives. We need to intentionally discuss situations where what students are learning in mathematics lessons are applicable in the future.

(3) Results and analysis of the readiness test

Administered to all 27 students on May 24

(a) Students placed in *gungun* course (16 students)			
Problem	Purpose	Correct answer (# of Ss) Percent	Incorrect answer (# of Ss)
① 360 ÷ 30 = 36 ÷ □ ② 680 ÷ 40 = □ ÷ 4	Can students apply the property of division to calculate when both the dividend and the divisor end with a 0?	① 3 (14) 88 % ② 68 (15) 94 %	① 300 (1) 10 (1) ② 6800 (1)
Calculate using the algorithm: ③ 9.6 ÷ 4 ④ 47.2 ÷ 8 ⑤ 9.72 ÷ 27 ⑥ 3.36 ÷ 48 ⑦ 55.8 ÷ 124	Can students calculate Decimal ÷ Whole?	③ 2.4 (16) 100 % ④ 5.9 (16) 100 % ⑤ 0.36 (15) 94 % ⑥ 0.07 (15) 94 % ⑦ 0.45 (11) 69 %	⑤ 36 (1) ⑥ 7 (1) ⑦ 00.45 (2) 0.4 (1) 4 rem. 6.2 (1) 0.441 (1)
Calculate using the algorithm: ⑧ 87.6 ÷ 16	Can students calculate Decimal ÷ Whole with a remainder?	⑧ 5 rem. 7.6 (7) 44 %	⑧ 5.4 rem. 1.2 (2) 5.45 rem. 40 (1) 5.4 rem. 12 (1) 54 rem. 12 (1) 6 rem 1.6 (1) 999 rem. 444 (1) 5.475 (1) 5 rem. 0.76 (1)
Word problem ⑨ There is 7.2 L of soy sauce. If we put this soy sauce into 9 bottles equally, how many L of soy sauce will there be in each bottle?	Can students solve a word problem involving division with a decimal dividend (and whole number divisor)?	⑨ Expression 7.2 ÷ 9 (16) 100 % Answer 0.8 L (16) 100 %	
Word Problem ⑩ The length of a red tape is 4 m and the length of a blue tape is 6 m. How many times as long is the blue tape as the red tape?	Can students solve problem involving times as much with decimal number?	⑩ Expression 6 ÷ 4 (15) 94 % Answer 1.5 times as long (14) 88 %	⑩ Expression No answer (1) Answer Incorrect unit (1) No unit (1)

(b) Students placed in *jikkuri* course (11 students)			
Problem	Purpose	Correct answer (# of Ss) Percent	Incorrect answer (# of Ss)
① 360 ÷ 30 = 36 ÷ □ ② 680 ÷ 40 = □ ÷ 4	Can students apply the property of division to calculate when both the dividend and the divisor end with a 0?	① 3 (6) 55 % ② 68 (4) 36 %	① No answer (2) 64 (1) 6 (1) 300 (1) ② No answer (2) 88 (1) 5 (1) 17 (1) 8 (1) 6800 (1)
Calculate using the algorithm: ③ 9.6 ÷ 4 ④ 47.2 ÷ 8 ⑤ 9.72 ÷ 27 ⑥ 3.36 ÷ 48 ⑦ 55.8 ÷ 124	Can students calculate Decimal ÷ Whole?	③ 2.4 (7) 64 % ④ 5.9 (6) 55 % ⑤ 0.36 (3) 27 % ⑥ 0.07 (4) 36 % ⑦ 0.45 (1) 9 %	③ 24 (3) 0.2 (1) ④ No answer (2) 21.4 (1) 59 (1) ⑤ No answer (2) 36 (2) 37 (1) 4.7 (1) 3.6 (1) 1.1 (1) ⑥ No answer (3) 7 (3) 0.01 (1) ⑦ No answer (4) 45 (2) 2.25 (1) 135 rem. 30 (1) 11.2 (1) 4 rem. 62 (1)
Calculate using the algorithm: ⑧ 87.6 ÷ 16	Can students calculate Decimal ÷ Whole with a remainder?	⑧ 5 rem. 7.6 (2) 18 %	⑧ No answer (5) 54 rem. 12 (2) 11.1 (1) 0.5 rem. 7.6 (1)
Word problem ⑨ There is 7.2 L of soy sauce. If we put this soy sauce into 9 bottles equally, how many L of soy sauce will there be in each bottle?	Can students solve a word problem involving division with a decimal dividend (and whole number divisor)?	⑨ Expression 7.2 ÷ 9 (7) 64 % Answer 0.8 L (5) 31 %	⑨ Expression No answer (2) 9 ÷ 7.2 (1) 7.2 × 9 (1) Answer No answer (3) 8 L (1) 1.2 L (1) 9.5 (1) 64.8 (1)
Word Problem ⑩ The length of a red tape is 4 m and the length of a blue tape is 6 m. How many times as long is the blue tape as the red tape?	Can students solve problem involving times as much with decimal number?	⑩ Expression 6 ÷ 4 (6) 55 % Answer 1.5 times as long (3) 27 %	⑩ Expression No answer (3) 4 ÷ 6 (2) Answer No answer (3) 6 times (1) 0.96 (1) 1 rem. 2 times (1) 15 times (1) 1.2 times (1)

Students in *gungun* course appeared to have mastered the basic calculations. However, with the division with remainder (Problem 8) some students missed the problem because they did not see that the question was asking for a "whole number quotient." There are other students who are still having trouble determining the location of the decimal point in the remainder.

Students in *jikkuri* course are still in the process of mastering the basic calculation. Their success rate is better if there is no empty space in the quotient, but some students even quit trying when there is an empty space in the quotient or there is a remainder. These results suggest the need for different types of support are needed by the students in these two groups.

6 Strategies to achieve the research theme

Characteristics of ideal students (Grade 5)

- Students can notice the difference between the given problem and what they have previously learned and identify the mathematical question for the lesson.
- Students can devise a plan to solve the given problem using their prior learning.
- Students can explain their own ideas using tools such as diagrams and equations with words.

<Strategies in this lesson>

[Strategies to help students identify the mathematical question]

⊙ Help students develop the habit of recording "?" that arises naturally when they encounter a novel problem in everyday lessons

When students encounter a novel problem, they naturally mumble to themselves in their minds, "What calculation do I need to do?" "It's going to be difficult with decimal numbers," etc. Those silent mumbles are the natural questions arising from the problem and they lead to the mathematical question for the lesson. Thus, we want students to develop the habit of recording those quiet mumbling in their own notebook. By doing so, students will become more aware of their own questions.

[Strategies to help students make use of their prior learning to solve a problem]

⊙ Have hint cards that are available at any time in the classroom.

Place hint cards that can be used by students in any grade level in the classroom. We will instruct students to think about "What's the hint card we need today?" and select the appropriate one (or ones) on their own. Of course, in such a set of hint cards, some might involve the ideas students have yet to learn. However, we also want students to develop the habit and the ability to reflect on what they have previously learned. Because we started making these hint cards this year, the set is not yet complete, and we plan to keep adding to the set.

7 Unit Plan and Assessment Plan

	Goals	Learning Activities: *Gungun* course	Learning Activities: *Jikkuri* course	Main Assessment Standards
① Division of decimal numbers				
Today	○ Students can set up the appropriate division expression with understanding. ○ Students understand the meaning of dividing by decimal numbers.	• Think about how much a 1 m of ribbon costs if a 2.5 m of the same ribbon costs 300 yen. • Set up an expression and think about why the expression is appropriate by using tools such as number lines and equations with words.	• Think about how much a 1 m of ribbon costs if a 3 m of the same ribbon costs 300 yen. (Re-visiting prior learning) • Think about how much a 1 m of ribbon costs if a 2.5 m of the same ribbon costs 300 yen. • Set up an expression using tools such as number lines and equations with words.	[Mathematical Way of Thinking] Students are thinking about the meaning of division by decimal numbers by connecting to their prior learning of calculation and number line representations.
L2	○ Students can think about ways to calculate Whole ÷ Decimals using their prior learning of properties of division and number line representations.	• Think about ways to calculate 300 ÷ 2.5. • Summarize the method to divide by decimal numbers.	• Think about ways to calculate 300 ÷ 2.5 by making connections to students' prior learning. • Students learn the method to divide by decimal numbers.	[Interest, Eagerness, and Attitude] Students are trying to think about ways to divide by decimal numbers by making connections to their prior learning such as calculation with whole numbers. [Mathematical Way of Thinking] Students are thinking about ways to divide by decimal numbers.

L3	○ Students will understand ways of calculating Decimals ÷ Decimals. ○ Students understand the way to calculate Decimals ÷ Decimals using the standard algorithm and perform the calculations (proper decimal quotients and those calculations involving dividing on by annexing 0's at the end of the dividend).	• Students can set up appropriate calculation expressions and know why they are appropriate. • Think about ways to calculate 7.56 ÷ 6.3. • Summarize how to calculate Decimals ÷ Decimals using the standard algorithm.	• Set up the appropriate calculation expressions using diagrams and equations with words. • Students learn the way to calculate 7.56 ÷ 6.3. • Summarize how to calculate Decimals ÷ Decimals using the standard algorithm. • Solve similar problems to master the procedure.	[Interest, Eagerness, and Attitude] Students realize the merit that division of decimal numbers can be done in the similar way as division of whole numbers. [Mathematical Way of Thinking] Students can think about and represent ways to calculate Decimals ÷ Decimals by making use of the property of division. [Mathematical Skills] Students can calculate Decimals ÷ Decimals using the standard algorithm (proper decimal quotients and those calculations involving dividing on by annexing 0's at the end of the dividend).
L4		• Think about ways to calculate 2.34 ÷ 3.9, 1.8 ÷ 2.4, and 8 ÷ 2.5. • Calculation practices.	• Students learn to calculate 2.34 ÷ 3.9, 1.8 ÷ 2.4, and 8 ÷ 2.5. • Calculation practices.	
L5	○ Students understand that the quotients will be greater than the dividends when the divisors are proper decimal numbers (decimals < 1).	• Compare the quotients and the dividends after calculating 240 ÷ 1.2 and 240 ÷ 0.8. • Summarize the relationship that the quotients will be greater than the dividends if the divisors are proper decimal numbers.	• Compare the quotients and the dividends after calculating 240 ÷ 1.2 and 240 ÷ 0.8 using concrete materials. • Summarize the relationship that the quotients will be greater than the dividends if the divisors are proper decimal numbers.	[Knowledge and Understanding] Students understand the results of calculations such as if the divisors are less than 1, the quotients will be greater than the dividends.

L6	○ Students understand the meaning of remainders in division of decimal numbers, and they can determine remainders.	• Students will set up the calculation expression to solve the problem: How many students will get a 0.7 m piece of ribbon from a 2.5 m ribbon? They can explain their methods for solving this problem. • Summarize how to determine the location of decimal point in the remainder.	• Students will think about how many students will get a 0.7 m piece of ribbon from a 2.5 m ribbon using concrete materials. • Summarize how to determine the location of decimal point in the remainder.	[Mathematical Way of Thinking] Students are thinking about the size of remainders. [Knowledge and Understanding] Students understand where to place the decimal point in remainders using the standard algorithm.
L7	○ Students understand ways to find the quotients using approximate numbers.	• Think about the weight of 1L of sands when 1.5 L of the same sands weight 2.5 kg. • Students will realize the usefulness of using approximate numbers as quotients instead of responding with quotients and remainders. • Summarize that when performing division, we sometimes use approximate numbers as quotients, for example, when there are remainders or the quotients will have many places.	• Think about the weight of 1L of sands when 1.5 L of the same sands weight 2.5 kg. • Students will realize the usefulness of approximating the quotients to the 2nd highest place instead of responding with quotients and remainders. • Summarize that when performing division, we sometimes use approximate numbers as quotients, for example, when there are remainders or the quotients will have many places.	[Mathematical Way of Thinking] Students are thinking about the way to use rounding to approximate the quotients. [Mathematical Skills] Students can respond with the quotients rounded to the specified place.

L8	○ Students will deepen their understanding of how number lines can help them determine appropriate division calculations.	• Students will think about ways to set up appropriate calculation to determine the weight of 1 m of hose and the length of 1 kg of hose when 4.5 m of the same hose weigh 0.9 kg by using number lines.	• Students learn that number lines can summarize the relationship of numbers in problem situations concisely so that setting up appropriate calculation becomes easy by determining the weight of 1 m of hose and the length of 1 kg of hose when 4.5 m of the same hose weigh 0.9 kg by using number lines.	[Mathematical Way of Thinking] Students use number lines to think about and represent appropriate division expressions for problem situations.
② Times as much with decimal numbers and division				
L9	○ Students understand that even when both the compared quantity and the base quantity are decimal numbers, division is still the appropriate calculation to find how many times as much.	• Think about ways to determine how many times as long are 3.6 km and 1.8 km as 2.4 km (2.4 × ☐). • Summarize that even when both the compared quantity and the base quantity are decimal numbers, division is still the appropriate calculation to find how many times as much.	• Think about ways to determine how many times as long are 3.6 km and 1.8 km as 2.4 km (2.4 × ☐). • Summarize that even when both the compared quantity and the base quantity are decimal numbers, division is still the appropriate calculation to find how many times as much.	[Mathematical Way of Thinking] Students are thinking that the way to determine how many times as much even when both the compared and base quantities are decimal numbers is by division as was the case when those quantities were whole numbers.
L10	○ Students understand that even when the compared quantities are decimal numbers times as much, they can use ☐ for the base quantity and set up multiplication equations, and division is the operation to find the value of ☐.	• Think about ways to find the base quantity when 630 g is 1.8 times as much as the base quantity. • Summarize that expressing the relationship in a multiplication equation using ☐ for the unknown base quantity is helpful.	• Think about ways to find the base quantity when 630 g is 1.8 times as much as the base quantity. • Summarize that expressing the relationship in a multiplication equation using ☐ for the unknown base quantity is helpful.	[Mathematical Way of Thinking] Students are thinking that the way to determine the base quantity even when both the compared quantity and how many times as much are decimal numbers is by division as was the case when those numbers were whole numbers.

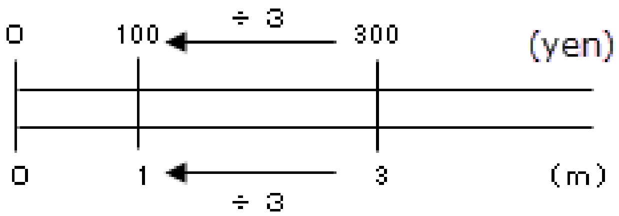
L11	○ Students understand that two quantities can be compared using the difference or times as much.	• Compare how much prices increased by using times as much.	• Compare how much prices increased by using times as much.	[Knowledge and Understanding] Students understand that times as much is an appropriate way to compare quantities based on the purpose of comparison.
③ Summary				
L12	○ Master the content of the unit.	• Engage in practice problems. (Basic Problems)	• Engage in practice problems. (Basic Problems)	[Mathematical Skills] Students can calculate division of decimal numbers accurately.
L13 L14	○ Solidify the understanding of the content of the unit.	• Engage in practice problems. (Advanced Problems)	• Engage in practice problems. (Advanced Problems)	[Knowledge and Understanding] Students understand the meaning of division of decimal numbers.

8 Today's Lesson (Lesson 1 of 14) *Gungun* course

(1) Goal

- Students can think about the meaning of dividing by decimal numbers by making connections to their prior learning of calculation and number lines, and they can explain their ideas logically.

(2) Flow of the Lesson

	Main questions and anticipated responses	⌑ Assessment ◆ Support • Point of consideration ⦿ Teaching strategy
G R A S P	1 Encountering the problem A 2.5 m of ribbon costs 300 yen. How much will 1 m of the same ribbon cost? T: After you read the problem, please write today's "?" T: What were your "?"? C1: What calculation do I need? C2: Will it be easier if we change the decimal number to a whole number? C3: Will it be division? C4: Should it be less than 150 yen? C5: Is it possible to represent with a diagram? T: Do you think you can solve this problem if we change the decimal number into a whole number? C6: For example, if we change 2.5 m to 3 m, it will be simple.  T: Indeed, if you change the decimal number into a whole number, we can solve the problem. So, what will be the calculation expression to find the answer for today's problem? C7: I think it is 300 ÷ 2.5. T: Is "÷ 2.5" the appropriate calculation to find the price for 1 m?	⦿ When students encounter a problem, have them write down their "?" in their notebooks. • By changing the decimal number into a whole number, students can use their prior learning to set up the appropriate calculation expression. • Pose a question to help students develop a new question of their own.

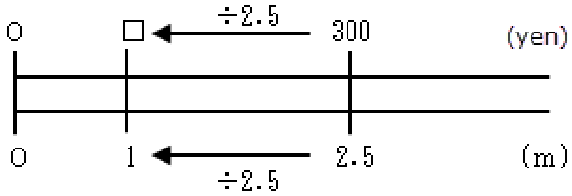
	2 Grasp the mathematical question T: I think we are beginning to see the mathematical question for today's lesson. C8: I want to think about the reason why 300 ÷ 2.5 is the appropriate calculation. C9: I want to explain why dividing by 2.5 will determine the price for 1 m. T: (Listen to students' ideas, and write down the mathematical question for the lesson on the blackboard.) Is 300 ÷ 2.5 the appropriate calculation to find the price for 1 m?	• Treat this discussion carefully so that today's mathematical question will arise from students' questions.
D E V I S E	3 Devise a plan T: Are there things you have learned so far that you might be able to use to explain the reason for the appropriate calculation? C10: I think we can use an equation with words. (Making connections to calculations with whole numbers) C11: I think we can use the idea of multiplication. (Thinking about the inverse of multiplication) C12: I think we can use 0.1 as a unit. (Making use of per-unit quantity) C13: I think we can use diagrams. (Thinking about ways to represent the situation) C14: I tink we can use number lines. (Thinking about ways to represent the situation)	• Help students devise a plan that they might be able to follow through.
S O L V E & D I S C U S S	4 Solve the question and represent T: Please write down your idea in your notebook. (Independent problem solving time) <Anticipated students' responses> (A) Students who can explain the reason for the calculation i. Making use of equations with words With whole numbers, we get 300 ÷ 3 = 100. Because [Price] ÷ [Length] = [Price for 1 m], we can determine the price for 1 m by using division even when the length is a decimal number.	

ii. Thinking in terms of the reverse operation of multiplication

Since we don't know the price of 1 m, we will say it is ☐yen. Since the price of 2.5 m, 300 yen, will be 2.5 times as much as the price for 1 m, we can represent this relationship in the equation, ☐× 2.5 = 300.

Therefore, the calculation to determine the value of ☐must be 300 ÷ 2.5, and we know that "÷ 2.5" is the correct calculation to find the price for 1 m.

iii. From number lines, thinking about the unit quantity.



(B) Students who are thinking more about finding the answer

iv. Think in terms of 0.1 as a unit

2.5 m is made up of 25 0.1 m.

If you divide 300 yen by 25, we get 12. That tells us that the price of 0.1 m of this ribbon is 12 yen.

Therefore, the price of 1 m of this ribbon is 120 yen.

v. Think in terms of 0.5 as a unit

2.5 m is made up of 5 0.5 m.

If you divide 300 yen by 5, we get 60. That tells us that the price of 0.5 m of this ribbon is 60 yen.

Therefore, the price of 1 m of this ribbon is 120 yen.

(Whole class discussion)

C15: (iv)'s idea is how to do the calculation, and I don't think it explains why the correct calculation is division.

C16: I can see why the calculation must be division clearly from the explanations if (i), (ii), and (iii).

T: So, do you think "300 ÷ 2.5" is the appropriate calculation to solve today's problem?

C17: I understood why division is the calculation we need for today's problem by listing to ____'s explanation.

[Mathematical Way of Thinking]

Students think about and explain the meaning of division by decimal numbers by making use of their prior knowledge of calculations and number lines.

(Presentations/Notebooks)

⊙ Have hint cards ready in the room and allow students to use them freely.

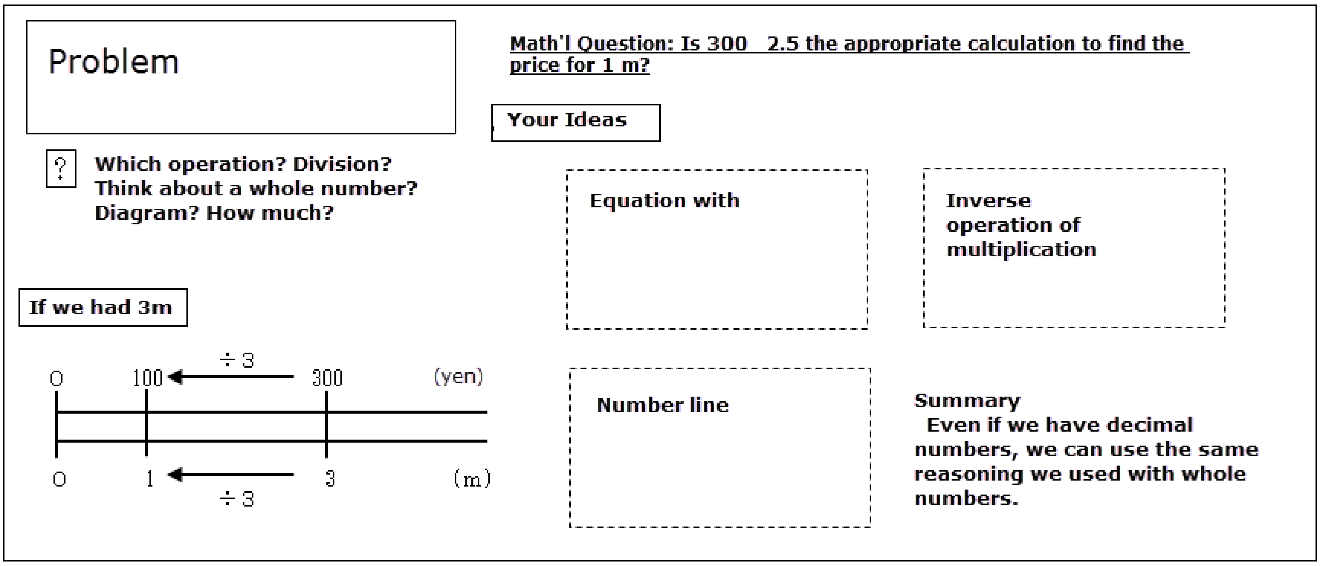
◆ Teacher will suggest an approach if students were having difficulty selecting a strategy on their own.

- Let students who finished writing their ideas to discuss their ideas with each other.
- For (iv) and (v), if any student uses these approaches, select one and have it shared with the whole class.

- Reflect on today's mathematical question and make sure students understood the main idea.
- As a class, affirm that division is the appropriate calculation to solve today's problem.

	5 Summarize T: Today's mathematical question was "Is 300 ÷ 2.5 the appropriate calculation to find the price for 1 m?" Please write the summary of the lesson in your notebook in your own words. C18: Even when we have decimal numbers, we can use the same way of reasoning and use division to solve the problem. C19: When we are finding the amount per 1, we use division even when we have decimal numbers.	• Have several students share their summary, and record the lesson summary on the blackboard based on those summaries.
	6 Think about the mathematical question for the next lesson T: OK, today, we were able to find the appropriate calculation for the problem. What do you want to do next? C20: I want to be able to find the answer for the calculation on my own. C21: I want to think about ways to do the calculation.	• During the whole class discussion, we might have discussed one idea, but it was presented as one idea. We have not yet discussed the validity of the steps. Therefore, we want students to have the desire to figure out ways to calculate and find the answer to the problem.

Board Writing Plan



Lesson Report

Report created by: Alexandra Johansen, Amber Richard, Leila Christensen, Karen Cortez-Ramirez, Lisa Gaglioti

Name of Lesson: Grade 5 - Division of Decimal Numbers
Date of Lesson: June 21, 2017

What are the primary lesson goals?
Students can set up the appropriate division expression with understanding. Students understand the meaning of dividing by decimal numbers.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?
The lesson observed was lesson one of a 14 day unit on division of decimal numbers. The next day's lesson will involve students calculating whole numbers by decimals. Within this unit they will go on to calculate a decimal divided by decimal, and interpreting remainders.

Prior learning includes multiplication and division of decimal numbers by whole numbers in Grade 4. Followed by multiplication by decimal numbers in Grade 5. In Grade 6, students will extend properties of decimal numbers to multiplying and dividing fractions.

Summary of Lesson

Start & End Time	**Lesson Phase**	**Notes**
1:35	**Introduction, Posing Task**	**Strategies to build interest and to connect to prior knowledge** Teacher engages students by bringing a ribbon and proposes a real word problem. She knew the cost of 2.5 meters but only wanted 1 meter. She asked the students to come up with their own math questions. They offered questions and she wrote them on the board. Together, they came up with the question: "2.5 meters of ribbon cost 300 yen. How much will 1 m of the same ribbon cost." The students offered suggestions on how to solve the problem. For example, one student suggested that they change the decimal number to a whole number. Then, teacher puts a 3 on a notecard and puts it in the problem on the board. Teacher writes a tape diagram on the board to represent student thinking.

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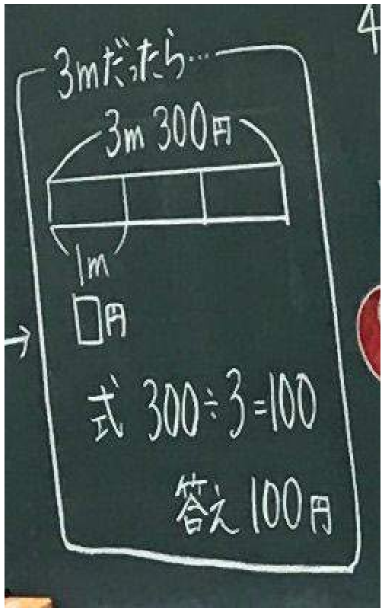

Posing the Mathematical Question

Teacher said, “what’s the mathematical question of the day.”
Students replied, “we haven’t decided if division works to solve the problem.” Then teacher writes on the board: “To find the price of 1 meter, is 300 divided by 2.5 the appropriate calculation?”

Student writes problem

Teacher helps students access prior knowledge by asking them what kinds of things they need to know to help them solve the problem. Students share out ideas and she writes it on the board.
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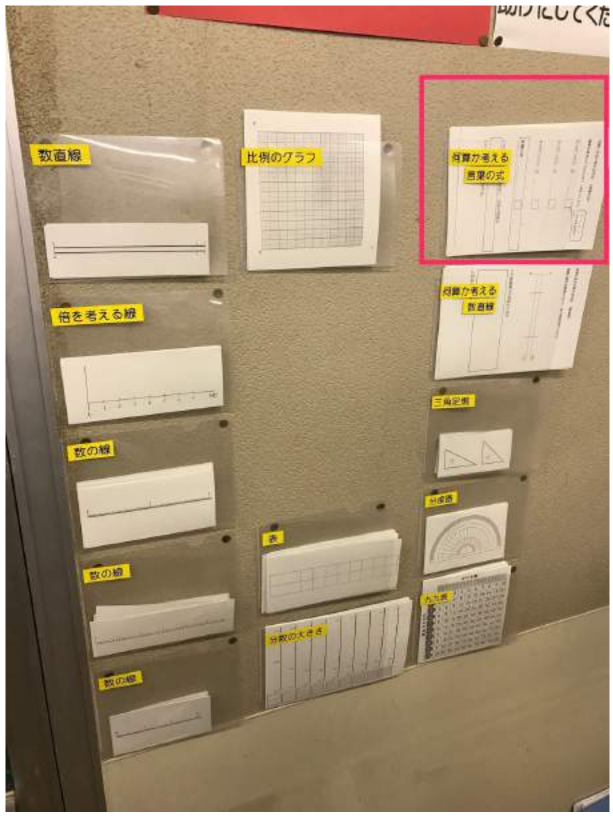
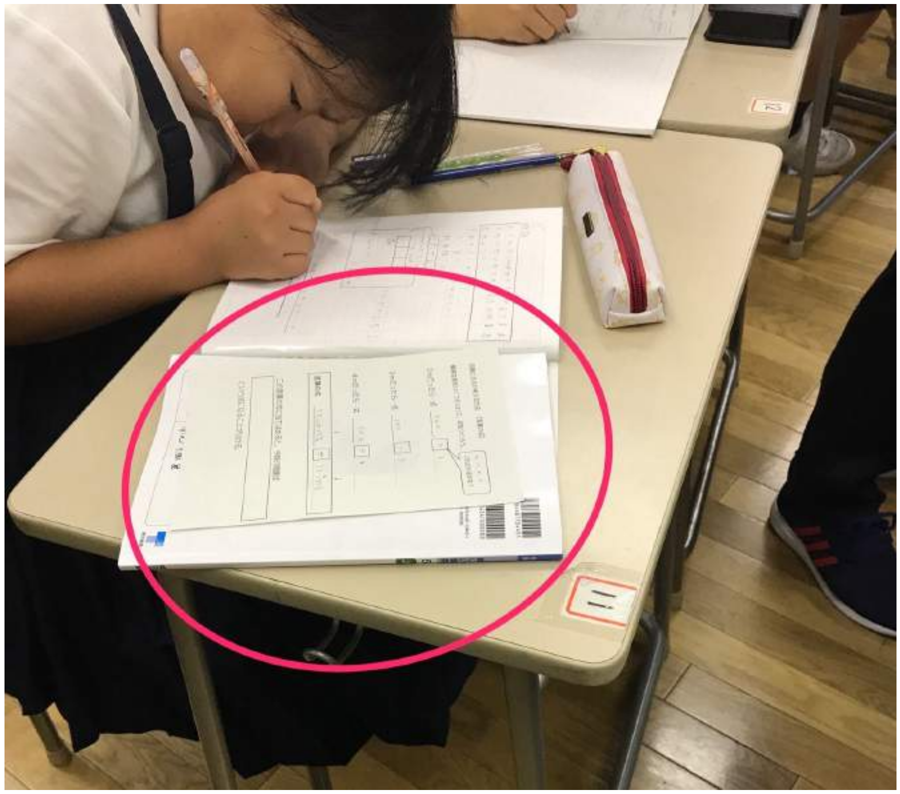
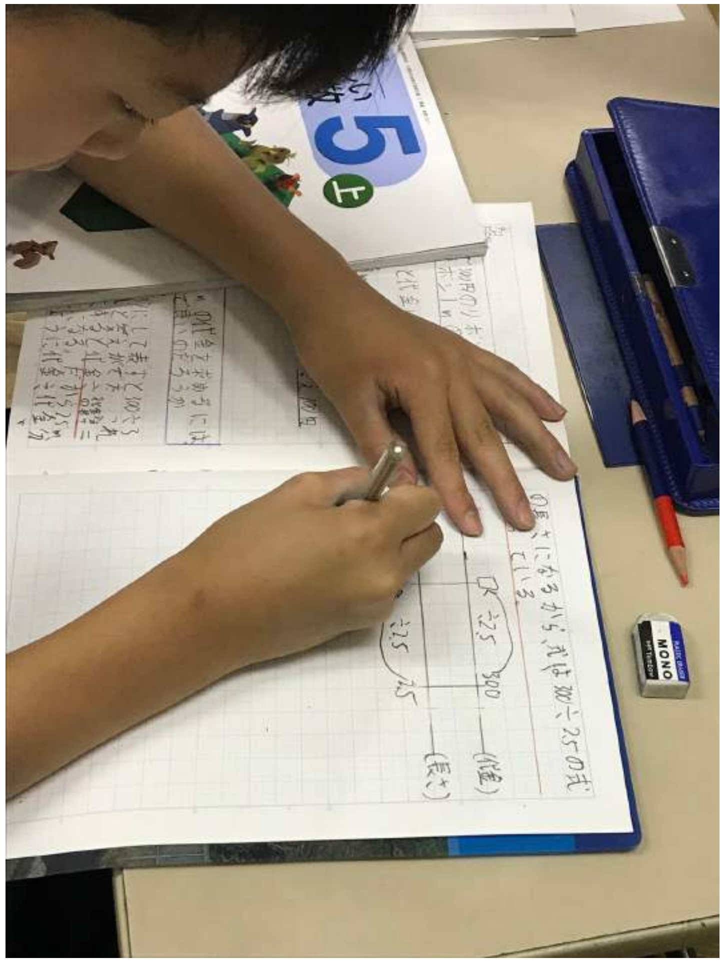
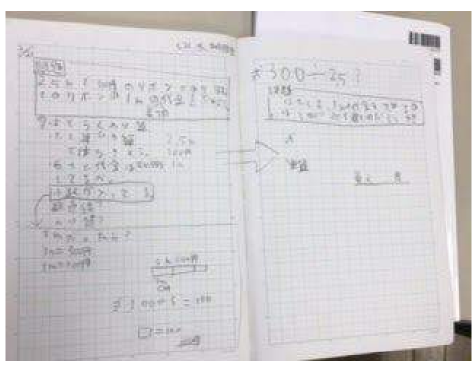
1:54	**Independent Problem Solving**	**Individual, pairs, group, or combination of strategies** **Experience of diverse learners** ● 9 out of 16 students used hint cards ● students were working independently in their notebooks ● Students wrote their thoughts on how to solve the problem in their notebooks.  **Hint cards**

		 **Student using a hint card**  **Student uses a tape diagram to model the division problem with a whole number, but then struggles to use it to represent the problem with the decimal**



Student uses a double number line

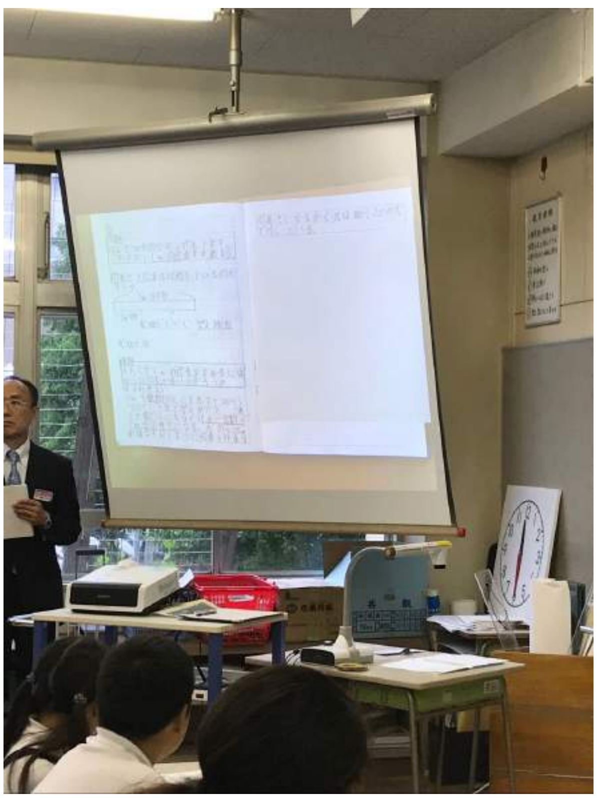


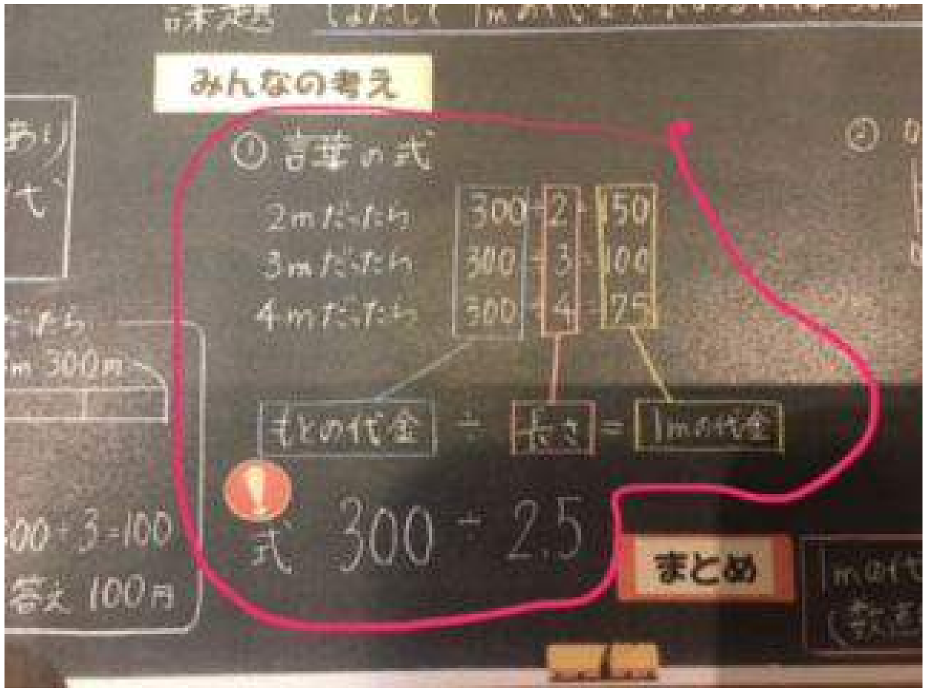
Student uses words to explain their thinking

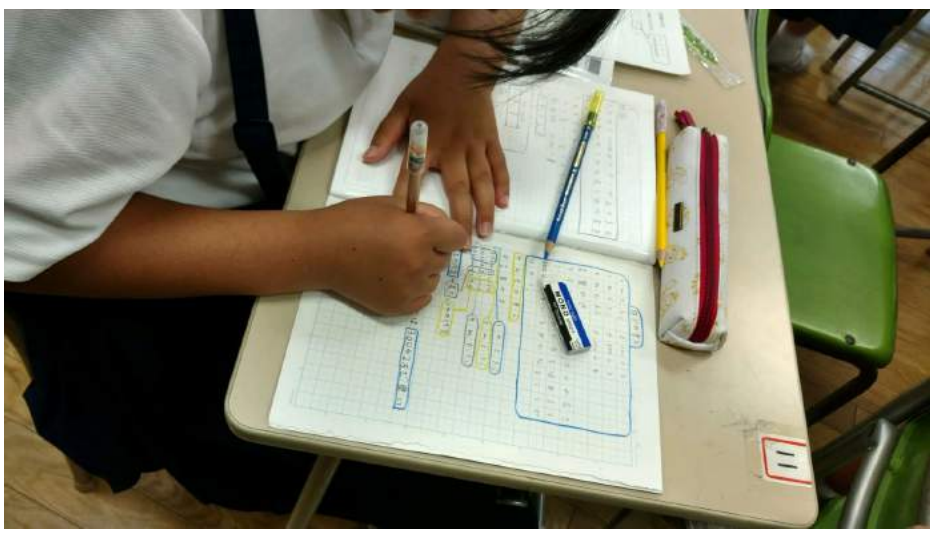
Teacher's activities

- Teacher directs students to use hint cards as needed
- Teacher walked around classroom looking at student work
- Teachers does not talk very much

2:04		Teacher gives them 3 more minutes to finish. If they want, they could get

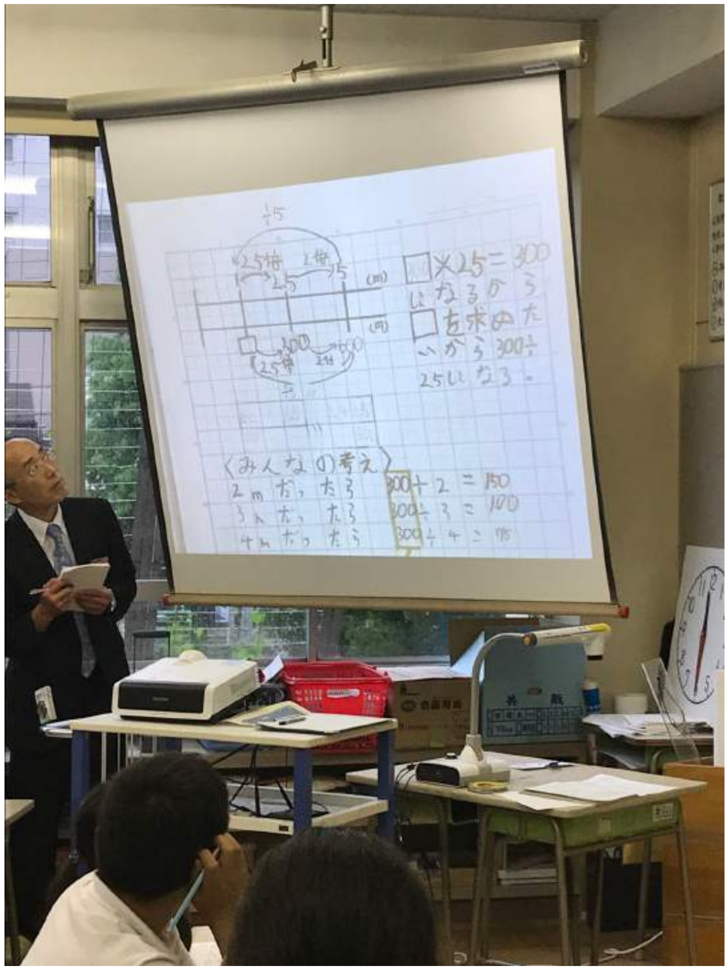
		up to show their classmates their work and discuss their strategy. Less than half the students got up to discuss their work.
2:07	**Presentation of Students' Thinking, Class Discussion**	**Student Thinking/ Visuals/ Peer Responses/ Teacher Responses** **Students' strategies shared whole class include:** Class discussion only included 4-5 students **Student 1: Making Use of Equations with Words I** Price/length= price for one meter **Student 1** notebook projected on document camera to show student's explanation of making the decimal number more accessible by changing it to a whole number. Teacher explained the benefit of using whole numbers to better conceptualize the problem.  **Student 2: Making Use of Equations with Words II** Another student's notebook was projected to show written explanation of the equation with words. Teacher directed the conversation about this strategy whole class and then wrote out the equation of price / length = price for one meter using whole numbers 2, 3, 4.





Student copies friend's idea into their notebook and uses color to differentiate between total price for whole ribbon, set length, and the price for 1 meter.

Student 3: Double Proportional Number Line
Student's notebook was projected to show double number line



Notebook showed price (in yen) on the bottom line and length (in meters) on the top line. Student divided by 2.5 on both part of the double number line to show price per meter.

Teacher explained proportional number line strategy by showing the relationship between multiplication and division through arrows on the top and bottom lines.

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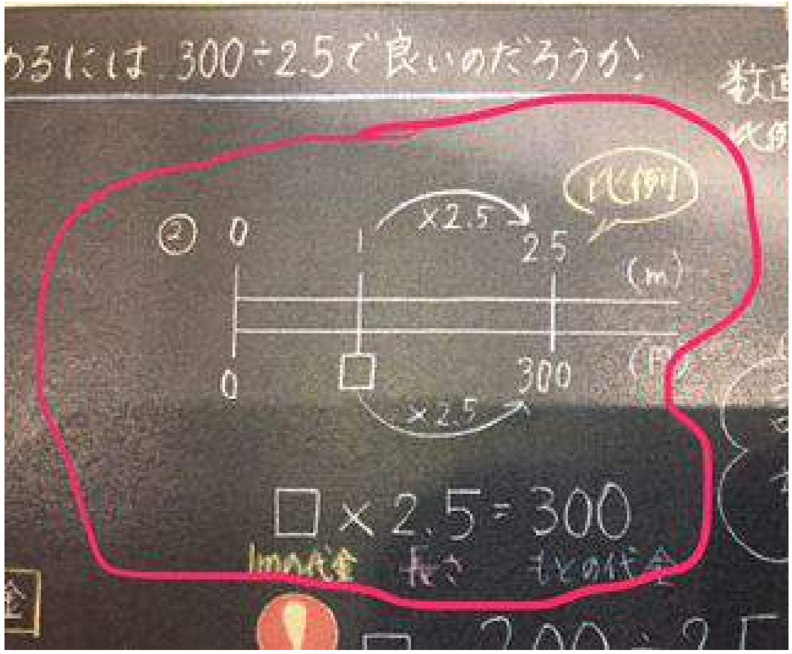
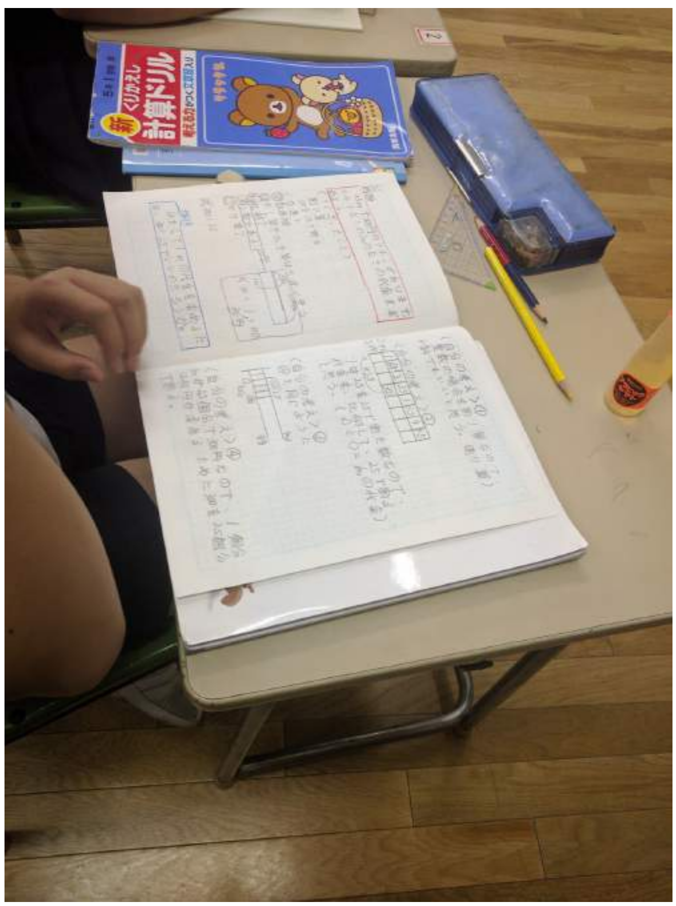
<tr><td></td><td></td><td>



Student recording the proportional double number line</td></tr>
<tr><td>2:19</td><td>Summary/ Consolidation</td><td>Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals</td></tr>
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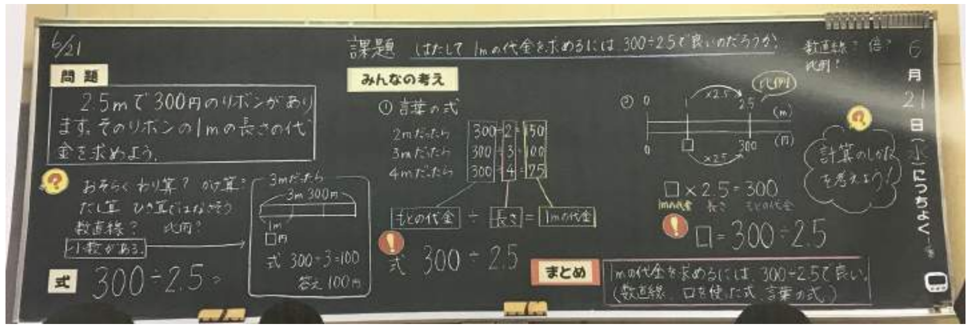


At the end of the lesson, teacher consolidates learning by demonstrating that there are different proofs to validate and explain the expression of 300/2.5 for this problem.

Teacher: Are you okay that this is going to be division? 300/2.5 is appropriate? Do we all agree? Write a summary of the lesson in your own words in your notebook.

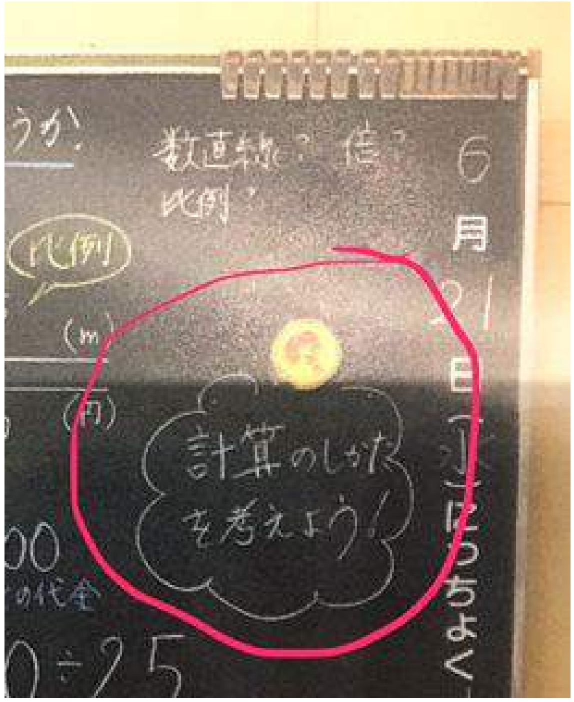
Teacher: I see a lot of you started your summary with 300/2.5 is appropriate- and what's the reason?

Student 1: Number line proved it

Student 2: Equation with words proved it

Teacher: If we have this many proofs, can we say that 300/2.5 is appropriate?

Teacher: (writes summary on blackboard)

		 **Teacher prompts students to come up with the mathematical question for the next lesson:** We found a math equation but still don't know how much to pay her. Let's think about how to calculate tomorrow.

What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?

Different problems and contexts lend themselves to the various types of division. The context for today's lesson would not lend itself to partitive or quotitive division, but rather to unit quantity and proportional reasoning. Typically, teachers choose to introduce division in a partitive context because it is a more concrete visualization of division. However, through the IMPULS participant post-lesson discussion, we discussed the advantages of teaching quotative division first, because this foundational understanding would allow for stronger understanding of unit quantity.

Pedagogically, the launch of lesson provided a real-world context for the students which increased student engagement. More importantly, the teacher led students to construct the task on their own and develop their own mathematical questions.

The teacher’s boardwork allowed for a clear connection between using whole number understanding of per unit division and translating the numbers into words with equations. The second strategy shared was the proportional number line. The inverse relationship between multiplication and division was highlighted. Further, the discussion amongst students and teachers drew out the reasoning behind the operation of division in (eventually) determining the price per unit.

What new insights did you gain about how administrators can support teachers to do lesson study?

Administrators can support lesson study through their own participation in the research team, this could include observation or facilitation during the post lesson discussion. In addition, when planning lesson study cycles, providing appropriate planning/release time for teachers so that cross-grade level teams can participate in the lesson development. This opportunity would allow for deeper analysis of student background knowledge through a study of the scope and sequence.

Administrators can also work to include all staff members in the lesson study observation (art teacher, music teacher, language teachers, etc.**).** Their inclusion would become more purposeful when provided clear observational tools to organize thoughts and focus their observations so that in post lesson discussion they too feel more confident to participate.

How does this lesson contribute to our understanding of high impact practices?

A high impact lesson involves a teacher working hard to listen to and understand the students' ideas. A lesson that may not be as effective involves the students trying to decipher what the teacher is thinking. While this lesson was organized and coherent with a clear objective, the teacher did most of the heavy cognitive lifting, did not deviate from her lesson plan and ignored parts of student's work in their notebooks. Because she carefully guided each part of the lesson so that it went precisely according to the original plan, she may not have brought students' own thinking to the forefront of the learning. This caused the lesson to be less adaptive to the students' thinking and ideas, ultimately leading to a very organized lesson, but one that may not have brought out the most from the students.

June 22

Grade 7, Mathematics Lesson Plan

Thursday, June 22, 2017
Period 6 (14:20 -15:10)
Grade 7, Classroom B
(20 boys and 20 girls)

Teacher's Name: Shou Shibata
Koganei Lower Secondary School
Attached to Tokyo Gakugei University

Location: TGU Conference room

1. The Research Theme and Its Intent

Designing lessons to raise the quality of student understanding of mathematical processes

(1) Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University

This lesson was developed as a part of the activities of the Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University. The purpose and rationale of our research group are as follows: In the Japanese mathematics education, we emphasize not only the mathematical content (results of explorations), but also the processes of exploring mathematical problems and the development of skills and ways of reasoning that are utilized during mathematical explorations.

Even with the emphasis on the process of mathematical explorations in tandem with the emphasis on mathematical content, we are concerned that mathematics teaching overwhelmingly focuses more on mathematical content than process. We are not suggesting that teaching mathematical content should be taken less seriously; nor are we implying that content and process should be considered as separate and distinct from each other. Rather, the concern of our research group results from a question that is critical to mathematics teaching and learning: Are Japanese mathematics lessons indeed emphasizing "mathematical ways of observing and reasoning" or "mathematical explorations/activities?" We have been discussing the importance of processes for a significantly long period of time, but are we seeing a significant emphasis in this domain or do lessons continue to focus primarily on content with process taking a secondary seat?

The idea of emphasizing mathematical processes is well substantiated in the research as the direction of Japanese mathematics education (Nishimura, et al., 2001; Shimizu, 2012; Fujii, 2016). In addition, concrete mathematics learning processes and supporting dispositions and abilities are described in the "Summary of the Discussion" of the elementary and the secondary mathematics working group, which also underscores the importance of mathematical process.

6/22 Lesson

Image of Elementary and Lower Secondary School Mathematics Learning Process

Elementary and lower secondary mathematics processes for identifying and solving problems; Disposition and abilities that foster these processes

Students grasp phenomena mathematically, identify mathematic problem in them, and solve the problems independently or corroboratively.

Students grasp phenomena in daily lives and society, make mathematical meaning of various contexts, and solve problems.

Students think generally and expansively to find solutions to problems.

Thinking and Judgement

A1. Students grasp problems in daily lives and society mathematically.
- Ability to attend to the quantities and quantitative relationships in phenomena; Identify mathematical problems.
- Ability to grasp characteristics of phenomena and express these mathematically. (i.e., represent phenomena)

Mathematically Expressed Problems

A2. Students grasp problems in phenomena mathematically.
- Ability to identify problems from mathematical phenomena.
- Ability to grasp characteristic of phenomena and process them mathematically.
- Ability to generalize and expand ideas based on the results of problem solving.

B. Students have foresight and plan for problem solving using mathematics.
- Ability to grasp the essence of mathematical problems (insight).
- Show foresight for solving mathematical problems (planning).

Mathematical Phenomena

Daily or Societal Phenomena

Focused Problems

D1. Students reflect on the solution processes and identify the meaning of the results, and apply the solution methods to other problems.
- Ability to revisit the original phenomena and think about the meaning of the results.
- Ability to apply specific solution methods to various phenomena.

C. Student solve focused problems.
- Ability to use numbers, expressions/equations, diagrams, tables, and graphs to mathematically process, solve procedurally.
- Ability to process accurately and efficiently based on mathematical point of view and thinking.
- Ability to reason logically using induction, deduction, or analogical thinking.

D2. Students reflect on the solution processes and create mathematical concepts and systems.
- Ability to identify the merits of mathematical points of view and reasoning.
- Ability to critique solution results and develop the ideas systematically and constructively.
- Ability to connect the results with the previously learned knowledge to expand and deepen the understanding of the mathematical concepts.
- Ability to think generally and expansively.

Results

Expression

◆ These abilities are not only fixed in the places. They may appear in other locations.

E. Students express and exchange ideas using mathematical language and reasoning.
- Ability to understand and evaluate explanations with mathematical expressions.
- Ability to explain their own ideas to others using mathematical expressions that respond to objectives of problem or study.

Human Quality

E. Students attitude and disposition for mathematical problem solving
- Ability to examine the problem-solving process and results; demonstrate a disposition for evaluating solutions and reasoning to improve one's grasp of the problem(s).
- Ability to think from multiple points of view and the discipline to identify and solve a range of problems.

Emphasizing the importance of mathematical process means caring about key aspects of engaging in mathematical processes, such as identifying mathematical problems from phenomena, using mathematics to solve problems, and creating and applying mathematics. As a multi-faceted endeavor, we can consider these processes as "mathematical activities" and the observations and reasoning used in these processes as "mathematical ways of observing and reasoning." Therefore, our research group uses the inclusive term of "mathematical processes" to represent the totality of the processes involved in identifying problems from phenomena, using mathematics to solve problems, and the processes of creating and applying mathematics.

The objective of our research group is to find and map the direction for how to create (materialize) lessons that intentionally raise the quality of mathematical practices. Our research group is pleased to take advantage of this research conference where lower and upper secondary school teachers of Tokyo Gakugei attached schools and regular lower secondary school teachers are here together to observe and discuss the lesson that was designed to focus on raising the quality of "mathematical processes." From this important opportunity as a community of educators, our research team looks forward to investigating a direction for designing lessons that raise the quality of mathematical practices.

(2) The Intention of the Lesson

Our intention with this lesson is to create and foster students who can conduct high quality mathematical processes, namely their ability to demonstrate the processes they couldn't conduct well in the past. However, achieving high quality mathematical processes does not mean only that students can solve problems they couldn't solve before. The deeper meaning is that students can revisit the problem-solving processes after having gained a new point of view or purpose, and they generalize and extend the ideas gained from their discussions.

To improve the quality of the mathematical processes, it is necessary to teach the related mathematical content, as well as the process skills. Thus, to plan such lessons, we need to set up problem-based instructions that require students' learning about problem solving in general. But, we also need to prepare problem based instructions that are particular to the mathematical content of the lesson.

Therefore, in the process of *kyozaikenkyu* (investigation of instructional materials), we must carefully focus on discussions of the mathematical processes, including (a) thinking about students' anticipated responses for the problems/tasks/materials, (b) identifying necessary student skills for conducting mathematical processes and setting these as one of the objectives of the lesson, and (c) thinking about what supports we can provide to students so they will successfully conduct or demonstrate high quality mathematical processes. In this lesson, *kyozaikenkyu* focused on mathematical processes. I will anticipate students' mathematical processes based on the topic I select, identify students' necessary skills for conducting mathematical processes, set these skills as objectives, and think about what support(s) I need to provide in to develop such skills. By doing so, I would like to show that I can plan a lesson that provides evidence of high quality mathematical processes.

2. “Mathematical Processes” of this lesson

(1) About the Unit Plan and the “Mathematical Processes”

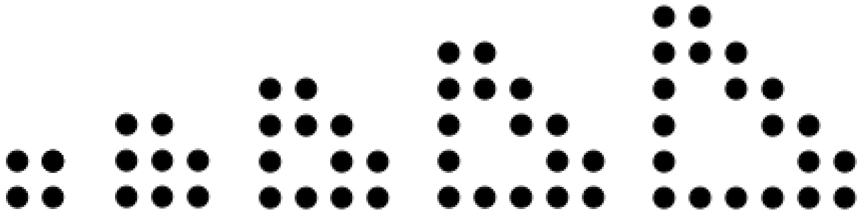
This lesson is in the part of the unit entitled “Letters in Algebraic Expressions.” In elementary school, students had used symbols, such as ☐ and Δ in equations; for example, 5 + ☐ = 8 and 3 × Δ = 24. These symbols helped students grasp the relationship between addition and subtraction or multiplication and division. Students learned to represent the relationships among quantities within the context of word sentences, such as (speed) x (time) = (distance) and learned to interpret and understand the meaning of given expressions and equations.

In lower secondary school, as a foundation for the study of algebraic expressions with letter symbols, students learned that letters such as a and x may be used in place of ☐ and Δ. They also learned how to express direct and inverse proportional relationships using algebraic expressions. Moreover, students have experience using quasi-variables; e.g., thinking about ways to calculate division of fractions or expressing relationships/patterns in numbers. Building on mathematical studies in elementary school, lower secondary school students will learn not only about using letters as representations and manipulating them, but also how to manipulate and interpret letters as variables, unknowns, and representations of a set. Finally, instead of simply introducing letters and studying calculations involving algebraic expressions with letter symbols, we will introduce letter symbols starting with the examination of quasi-variables (numbers that act like variables) and then, through activities of interpreting algebraic expressions and their structures, students will use algebraic expressions to represent mathematical generalizations.

Using letter symbols, students experience numbers not as specific numbers (such as 1, 3, or 0.7), but as a general object of study. They can also express various phenomena as relationships in the mathematical world. Furthermore, by transforming the given algebraic expressions or equations, new interpretations may become possible. The intention here is that through the study of letter symbols, students' mathematical explorations are deepened and become more refined. Such explorations typically take place when students try to prove conjectures and/or utilize the ideas of equations and functions. However, in this unit, instead of simply positioning the current study as the preparation for those future explorations, the main purpose is for students to experience mathematical manipulations and interpret their results by (a) representing a relationship in a real-world phenomenal as an algebraic expression, (b) transforming it to reflect their own thinking, and (c) interpreting and understanding the algebraic expressions and equations that other students express. These are all examples of the mathematical processes that students will experience by the end of the unit.

(2) About *kyozai* (instructional material) and anticipated “mathematical processes”

The instructional material I prepare for this lesson is the go stones where the number of them increase gradually as shown below.

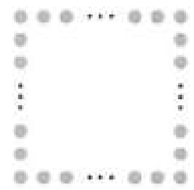


The value of this instructional example is the fact that the actual number of go stones in each stage of the growing figure is the same as the number of go stones when the original figure is transformed into a square whose side is the same as the original side at the base. In other words, the number of total go stones used for the nth number of the figure can be expressed as $4n$, therefore, students may notice that these figures can be transformed into squares. In this case students are not only interpreting the equations and understanding the solution processes within which they are engaged, but they are also interpreting the equations and imagining the transformed figures as squares from the expressions. This additional thinking process helps students to develop deeper understanding of the reciprocal relationship between expressions and phenomena. Although the importance of interpreting expressions and understanding corresponding phenomena has been stressed previously and often, the process itself is not easy for most students to do.

Students have experienced processing expressions formally and they have interpreted expressions to understand the corresponding phenomena; however, they have not experienced transforming the original phenomena into new form based on the interpretation of the expressions. Nor have they transformed the expressions based on the interpretation of the phenomena.

In this unit, the problem asking students to interpret expressions and think about solution ideas is the “application problem” in the section called “utilizing algebraic expressions with letters."

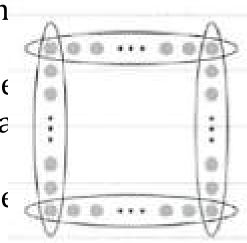
< Application Problem>



We are making squares by lining up *n* go stones on each side as show to the right. Sakura's idea for finding the total number of the go stones was shown below."

Sakura' idea:

"When I circle in the go stones as shown to the right, each circle contains *n* number of go stones. There are four such circles, so the total number of the go stones can be expresse as 4*n*. However, the go stones at the vertices of the square a counted twice, so the total number of the go stone is 4 less than 4*n*. Therefore, the total number of the go stones can be show as 4*n* – 4."



Yuto expressed the total number of the go stone as 4 (*n* – 2) + 4. Explain Yuto's idea by drawing circles on the diagram of go stones, just as Sakura had shown.

The textbook includes several problems that ask students to interpret and describe what a given expression is representing and to identify the corresponding phenomenon. This problem, however, is new to students. It asks students to examine a given expression, interpret the structure, then create a diagram to explain the thinking behind the expression. This problem provides an opportunity for the students to think about and understand Yuta's method; yet, I don't think it is enough experience for students to develop a deeper understanding of phenomena like this.

If we look at the result of calculations in today's problem, 4*n*, it is not easy for students to define what 4 and *n* are representing in the phenomenon (original geometric figure). By going back and forth between the phenomenon and the expression, students need to recognize that 4*n* could be representing a square. From this recognition, I want students to realize that the original figure can be transformed into a square.

To interpret expressions and understand corresponding phenomena well, it is necessary for students to interpret and analyze the expressions and understand the corresponding phenomena, and then transform phenomena based on the results and interpretation of expressions.

The problem in today's lesson provides opportunities for the students to think and discuss where 4*n* is represented in the figure and deepen their understanding of algebraic expressions with letters and their meanings. This is where I believe, the value of the *kyozai* (instructional material) is most evident. The problem helps deepen students' understanding of phenomena and expressions. The experience that students gain from this lesson must help them in the future study of algebraic expressions with letters, through this experience of expressing expressions and phenomena by transforming them appropriately based on their way of thinking. Since the students have been practicing calculating linear expressions, I decided to use this problem in this lesson.

In this lesson, I decided to ask students to think about the number of go stones in the 10th figure instead of asking them to think about the number of go stones in the nth figure. The first reason for that is there are some students who are not used to working with algebraic expressions with letters. Although drawing the 10th figure is cumbersome, it is still possible to draw. So the problem is still accessible for all students. It gives students the chance to produce their own expressions and represent their own ideas, so they are more likely to participate in the presentation and discussions. The second reason involves the number of stones for the 10th figure, which becomes 40 stones. The number is simple, so students may think there is some mystery behind the number. This students' thinking might motivate them to investigate after finding the solution, $4n$.

I am going to explain the mathematical processes that I anticipate during the lesson. The actual students' mathematical processes are not easily described, because their processes vary greatly and may be complex, including students who go back and forth between ideas and different solution methods. Therefore, what I will describe here is an ideal interaction. The ideal mathematical processes that students think through by trial and error until they resolve the problem.

The first step is to give students time to observe and grasp the phenomena. For example, students observe the diagrams of the geometrical arrangement of the go stones, i.e., the first figure uses 4 stones, second figure uses 8 stones, third figure uses 12 stones. "If you know what number the figure is in the sequence and how many stones increase each time in this function, you can draw the conclusion that with each increment, "There is 4 times as many as the previous number in the sequence." If we apply this thinking (function), the number of stones in the 10th figure can be determined as 40 stones. The process of finding the number of stones in the 10th figure can be expressed as 10 x 4 using an expression. Then if we generalize the method, by replacing 10 with n, we can establish an algebraic expression with the letter symbol, $4n$.

Depending on what counting method a student uses, the algebraic expression with letters can vary. However, when these algebraic expressions are simplified, all the expressions become $4n$. $4n$ is a simple expression but when you look at the original figure, it is not easy to identify what 4 and n are representing in the figure. On the other hand, if you think about what $4n$ might represent without thinking about the original figure, the figure that comes to mind is a rectangle that has 4 stones as its width and n stones as its length, or a square that has four equal n number of stones in each side. This line of thinking - the interpretations of the expression $4n$, leads students to realize they could transform the original shape into a rectangle or a square.

In general, a proof using expressions with letter symbols, "expressing," "transferring (processing)," and "interpreting" of an algebraic expression with letters could become an issue for students to conduct. In this lesson, "interpreting the algebraic expressions and understanding the corresponding phenomena" could be an issue. The difficulties of conducting the mathematical processes described above are not explained in figure 1, but these are issues specific to the topic of algebraic expressions with letters. Therefore, it is very meaningful to examine the mathematical processes in the context of algebraic expressions with letters through *kyozaikenkyu*.

3. About This Lesson:

(1) Methods for Raising Quality of Mathematical Processes:

During the Research Conference at Tokyo Gakugei University attached Lower and Upper Secondary School in 2016, I had proposed following four points for the methods for raising quality of Mathematical Processes.

① Having a long-term view for fostering students' quality and ability to conduct the mathematical processes.
② Actualizing the mathematical processes and establishing time for evaluating and improving the mathematical processes.
③ Identifying the ideas that promote the mathematical processes
④ Summarizing the lesson focused on the mathematical processes

Using this as the guide, I have written the lesson plan so that it raises the students' quality and frequency of use of the mathematical processes.

① Having a long-term view for fostering students' quality and ability to conduct the mathematical processes.

Students' quality and ability to conduct the mathematical processes cannot be developed within one lesson. This quality and ability has to fostered by consistent instruction over a long period of time. In our research group, we proposed that "having a long-term view" is the key for developing students' use of mathematical processes.

To show evidence of this key point, I expect to see the following students' behaviors: The students in the class will have practiced describing their own ideas using algebraic expressions with letters. They also interpret other ideas from their algebraic expressions. If students have developed these types of abilities, they should be able to apply the same processes in this lesson. However, even with questioning by the teacher, there will most likely be only a few students who recognize the relationship between the 4n and squares. I think many of the students are capable of interpreting algebraic equations and understanding the phenomena related to them; however, some students will have difficulty conducting and demonstrating this level of understanding.

The actual state of my students reflects an accumulation of students' learning experiences in my everyday instruction. There facility also represents the natural development of skills that are acquired through many experiences of consistent instruction and practice. If I think about this point, I realize also that the mathematical process should be developed over a long time of period; therefore, my instruction needs to be very intentional in regard to planning and practicing instructions that focus on interpreting algebraic expressions with letters and understanding corresponding phenomena. These considerations and careful attention need to be demonstrated, also, in instructions in the previous grade level as well as in future grade levels.

② Actualizing the mathematical processes and establishing time for evaluating and improving the mathematical processes.

In order to develop students' skills to conduct the mathematical processes, it is important to help students think consciously about the importance of the mathematical processes during lessons, evaluate the processes carefully, and improve the process if necessary. Needless to say, in order to do this, we need to make sure that the mathematical processes are realized in everyday practice, observe the lessons with others, and discuss the lessons and students' progress together.

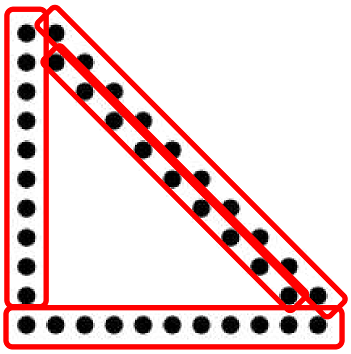
In this lesson, students will find the number of go stones at the 10^{th} figure. Then students will compare the segment when students discuss various different methods and the segment when students find the solution $4n$. Students will realize that although they were interpreting various algebraic expressions and relating them with phenomena (how the stones are arranged to find the number of stones), they have not thought about the calculation results of $4n$ in terms of its relationship to the phenomenon. This realization will help students realize that the algebraic expression 4n can also be interpreted and applied to real phenomena. This discovery naturally will lead the students' motivation to use this new process of thinking. The total process of thinking -- not only thinking about the algebraic expression $4n$, but also the process of thinking about solving for the 10th figure -- will help engage them in reflecting on their own thinking process, as well as provide opportunities for improving their thinking.

In order to carry out the discussion of the mathematical processes, I believe that *bansho* (board writing), not only helps visualize the thinking process, but it is the key to supporting the mathematical processes. *Bansho* provides more than a record of the results of the problem solving. It represents and provides a record of the thinking process, allowing students to jointly share their thinking and understanding; consequently, students can more easily clarify the discussion and state the agreement clearly.

③ Identifying the ideas that promote the mathematical processes.

I have talked about ideal mathematical processes before, but in reality, students will develop the skills necessary to conduct the mathematical processes after many twists and turns over a long period of practice. Therefore, it is important not only to teach the importance of the mathematical practices, but also to teach the ideas for helping to do the process well. These are ideas to carry out problem solving. As I mentioned

before, in order for students to conduct mathematical practices for interpreting algebraic expressions and understand the corresponding phenomenon (ways to find number of the stones with the diagram), it is important to compare different ideas that students represent in their algebraic expressions. In addition, students need to think about making a correspondence between algebraic expressions and phenomena.



For example, students may count the total number of go stones as shown to the left. In this case, by adding the number of stones in the vertical rectangle, horizontal rectangle, inner diagonal rectangle, and outer diagonal rectangle sides, students may represent the counting methods as 10 + 11 + 10 + 9. To get the total 40, students may think of 10 x 4, or 10 + 10 + 10 + 10. In this case, students could also see the expression as moving one stone

from the vertical side of the rectangle to the inner diagonal side of the rectangle, and realize they can make 4 groups of 10 stones as the 10th figure.

To find this solution, the students draw rectangles to create several groups, such as 10, 11, and 9. They use these numbers to create an expression. Then, students interpret the expression and deepen their understanding of the phenomena by connecting the idea represented in both the expression and the diagram. The reason that students can do this is that they have been experiencing this kind of thinking consciously before, including experience in elementary school mathematics. However, they have not had much experience transforming the original phenomenon into a new phenomenon from the expression. For this reason, in this lesson, the students need to do the following: first, recognize the connection between expressions and phenomena; second, discuss and understand the correspondence between expressions and phenomena; and lastly, discuss how to interpret $4n$ which is the result of calculation of an algebraic expression.

Transforming algebraic expressions and understanding of phenomena are the most important ideas students learn in the unit of Algebraic Expressions with Letters. I say this because the practice helps students develop understanding of algebraic expressions and phenomena. I underscore how critical this is by asking you to compare the understanding of a student who has memorized the formula for the composition of trigonometric ratios versus the student who understands the relationship geometrically. It is the latter student who has a much deeper understanding of the phenomena.

④ Summarizing the lesson focused on the mathematical processes.

According to Nishimura (2011), the summary of the lesson should be focused on the mathematical processes. In short, the lesson is summarized by reflecting and highlighting the ideas used to conduct the mathematical processes. In this lesson, for example, the important point of the summary is to discuss how students need to think to conduct a high quality “interpretation of the expressions and the corresponding phenomena;" therefore, a discussion needs to summarize “comparing the cases that only use numbers to solve the problem and those that use variables (letters) to solve the problem and “corresponding the result of algebraic expression to the phenomena." I believe this type of summary helps students realize this important way of thinking and how it can establish deep learning.

The above discussion on points about the methods for rising quality of mathematical processes can be summarized as follows:

(A) Having a long-term view for fostering students' quality and ability to conduct the mathematical processes.
- Having a stance of long-term view for fostering students' quality and ability to conduct the mathematical processes

(B) Actualizing the mathematical processes and establishing time for evaluating and improving the mathematical processes.
- Devising questions and *bansho* (board writing) to support the actualization of the mathematical practices.

(C) Identifying the ideas that promote the mathematical processes.
- Comparing expressions with numbers and algebraic expressions with letters
- Transforming algebraic expressions and connecting the phenomena.

(D) Summarizing the lesson that is focused on the mathematical processes.
- Reflection of ideas use in the mathematical processes.

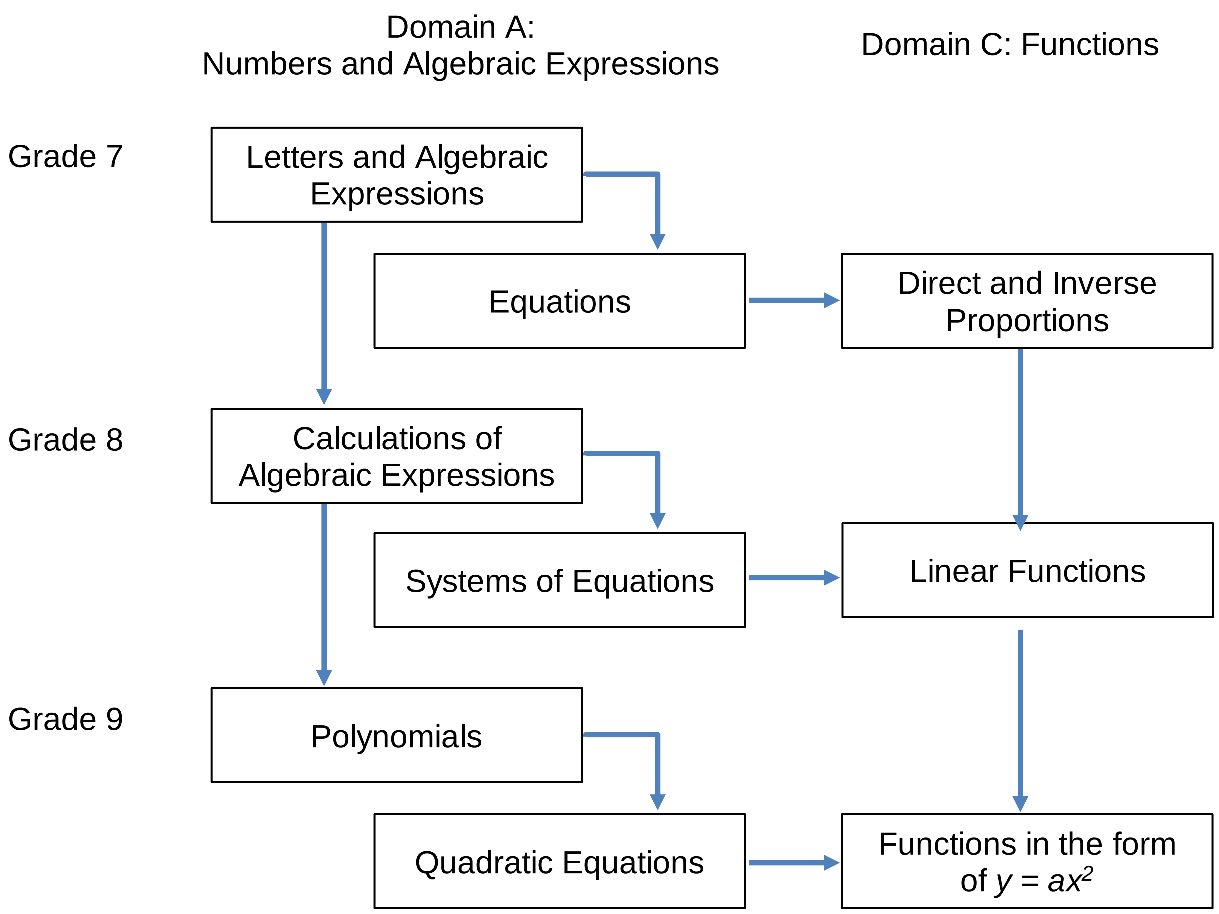
(2) About the students:

This year, students studied positive and negative numbers (integers) in a unit where students learned to view negative numbers in the same way they view whole numbers. Moreover, by studying calculations with integers, including patterns and properties of operations, they studied what numbers are. During these lessons, I tried to help students by asking them to "think about 'what needs to be considered;'" to "reflect on your solution processes;" and to "explain your thinking process in words."

In general, students' mathematical achievement levels are high. For example, when examining calculations with integers, some students were able to use 3 and -2, as quasi-variables. Most students were able to understand their mathematical explanations, indicating that most of them understood the notion of quasi-variables. Therefore, I anticipate that few students will have difficulty generalizing numbers.

Although few students consider mathematics to be difficult, there are some students who find it difficult to explain their ideas. Yet, they have experienced how their mathematical understanding was deepened by clarifying questions other students had. Therefore, I believe there is a classroom culture where students feel safe to admit openly something they don't understand. Moreover, many students are willing to share their ideas in whole class discussions, and they do not hesitate to share even simple ideas.

(3) Scope and sequence in lower secondary school:



(4) Unit Plan:

	Content	Anticipated Process	Main Evaluation Points
I	**<u>Section 1: Algebraic expressions with letters</u>** • Merits of using letters in place of numbers • Representing various quantities using letters • Learning how to write algebraic expressions with letters • Interpreting algebraic expressions with letters • Substituting values in letters of algebraic expressions and the meaning of the value of an algebraic expression	• Grasp the phenomenon and represent it with numbers and symbols. • Identify variables in numbers and represent it with a letter. • Establish an algebraic expression with a letter • Interpret an algebraic expression • Substitute numbers with the letter (variable)	• Students are interested in the necessity and merit of using letters to represent relationships/patterns among quantities generally, and they try to use algebraic expressions with letters to represent relationships/patterns and interpret the given expressions. [Interest, eagerness, and attitude] • Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning]

II	**Section 2: Calculations of algebraic expressions** • Relationships between terms and coefficients • Combining like terms • Addition and subtraction of linear expressions (distributive property) • Multiplying and dividing linear expressions (commutative and associative properties)	• Find the properties of calculations of linear expressions. • Identify preconditions (properties) for calculating four operations in the domain of linear expressions.	• Students know how to represent multiplication/division within algebraic expressions with letters and they try to use them to manipulate expressions. [Interest, eagerness, and attitude] • Students are able to think about ways to calculate algebraic expressions with letters by realizing that calculations with algebraic expressions as analogous to calculations with numbers. [Mathematical ways of observing and reasoning] • Students can use algebraic expressions with letters involving multiplication and division by following conventions appropriately. They can add and subtract simple linear expressions. [Mathematical representations and manipulations]
III	**Section 3 Applications of algebraic expressions with letters** • Quantities represented by algebraic expressions • Algebraic expressions that represent relationships	• Grasp the phenomena and represent phenomena with numbers and symbols. • Identify variables in numbers and represent it with a letter. • Establish an algebraic expression • Interpret an algebraic expression and understand corresponding phenomenal • Using letters to represent the relationship of quantities with an algebraic expression	• Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning] • Students are able to represent quantities and relationships/patterns among quantities in phenomena using algebraic expressions with letters, and they can interpret given algebraic expressions. [Mathematical representations and manipulations] • Students understand that by using letters quantities, and patterns, quantitative or functional relationships can be represented generally or interpreted from the given algebraic expressions with letters. [Knowledge and skills about numbers, quantities, and geometric figures]

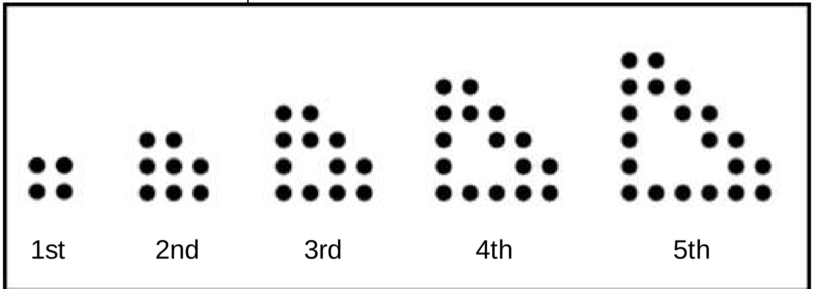
(5) About this Lesson:

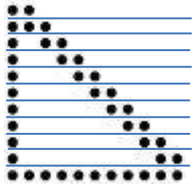
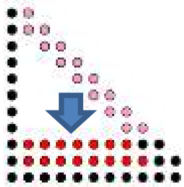
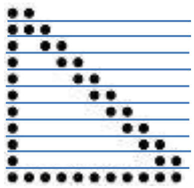
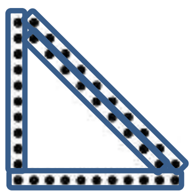
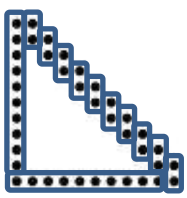
① Goals of this Lesson

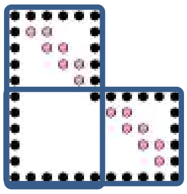
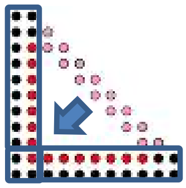
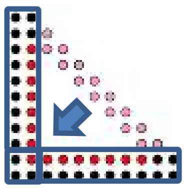
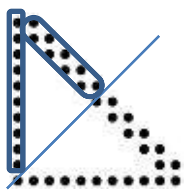
- Students are able to express the quantities, relationships among quantities, and rule of patterns in a phenomenal using algebraic expressions with letters. In addition, they are also able to interpret the algebraic expressions with letters and understand the quantities and quantitative relationships of the phenomena.
- Students understand the importance of comparing and discussing quantities and letters in algebraic expressions, and making connections between/among the transformation of algebraic expressions and phenomena. Because these actions help them conducting high quality mathematical processes.

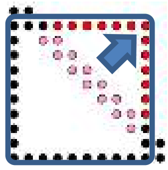
② Flow of the Lesson

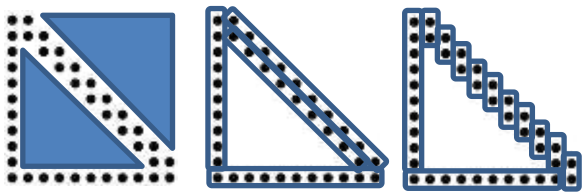
<table>
<tr><td>Time</td><td>Main Learning Activities</td><td>Students' activities and their anticipated responses</td><td>◆ Things need to remember
○ Evaluation</td></tr>
<tr><td>5 min.</td><td>[Introduction]

Posing problem</td><td></td><td></td></tr>
<tr><td></td><td colspan="2">
</td><td>◆ Paste the poster of the problem on the board.</td></tr>
<tr><td></td><td>"How many stones are there in each figure?"</td><td>"The first one has 4 stones."
"The second one has 8 stones."
"The third one has 12 stones.
"It looks like the number of stones increases 4 stones each time. "</td><td>◆ Try not to discuss how the number of stones increases because it will have and effect on students' problem solving.</td></tr>
<tr><td>10 min.</td><td colspan="2">Hatsumon: How many stones are in the 10th figure? Show how you can find the number of stones using an expression.</td><td></td></tr>
<tr><td></td><td>[Solving the problem on their own]</td><td>(1) Count by putting check mark.
(2) Use table</td><td>◆ Walk around the classroom and monitor how students are solving the problem.</td></tr>
<tr><td></td><td></td><td colspan="2"><table>
<tr><td>No. of figure</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>No. of stones</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr>
</table></td></tr>
</table>

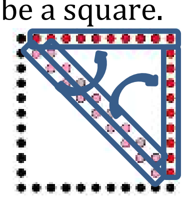
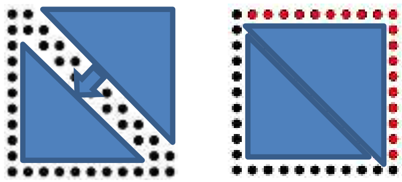
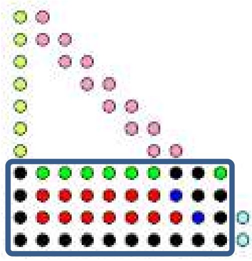
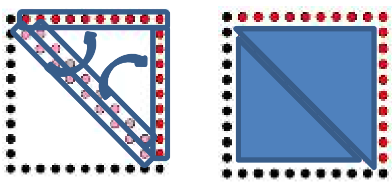
		(3) Slice horizontally and add 2+3+3+3+ … +3+11 	◆ If see students who are not showing their work, ask them to think about how they counted the stones and if they can represent the counting method into an expression. Or at least, write about their thinking.
		(4) Move stones to the bottom 11+3+3+3+ … +3+2 	◆ The stones could be mover to the left side instead of to the bottom. The expression remains the same.
		(5) Slice horizontally and use multiplication at the middle part 2+3 x (6-1) +7 	◆ Confirm with the students that the methods (3), (4), and (5) are basically the same method.
		(6) Split vertical, horizontal and inner and outer diagonal parts 10+11+10+9 	◆ It is easier for students to make a group of 10 when counting. Be sure to notice this method. Even if none of the student show this idea, the teacher will share the idea.
		(7) Split vertical, horizontal and diagonal parts 10x2+2x10 	◆ It is easier for students to understand to transfer from 10 groups of 2 (2x10) to 2 groups of 10 (10x2), therefore, the method (7) will be shared to the students, even if none of the students came up with this method.

		(8) Subtract overlapping square part form the two rectangles. (wrong solution) (11x2+4x2)x2-5x4  (9) Move stones from the diagonal part into the vertical and horizontal rectangles. (9x2)x2+4  (10) Similar to method (9). Subtracting the overlapping square part. (11x2)x2-4  (11) Cut the figure into two halves. Add vertical and diagonal parts, then double it. At last add two stones that are not in the boxes. (10+9)x 2+2  (12) (10+9)x2 (incorrect solution) Using the same method as (11), but forgetting to add two remaining stones at the end.	◆ The method (8) shows an incorrect solution. If none of students came up with the idea of transforming the shape into rectangle or square, this method will be shared even though it is an incorrect solution. ◆ If students do not come up with the method (13), the methods (9) or (14) will be sheared.

		(13) Make a square by filling with stones, then subtract the two triangular parts. 11x11-9x9  (14) Adding 2 extra stones at the top and at the side 9x4+4 	◆ The method (13) will be shared because it will help students to notice the methods related to the square. If none of the students came up with this method and students do not think about changing the shape into a rectangle or square, the teacher will share the method.
		(15) Count by two's (This method cannot be generalized) 2 x 20 (16) 4+4x10 (wrong solution) Using the table to solve the problem, but miscounts the number of increases.	◆ This method cannot be generalized using algebraic equations with letters.
		(17) Wong solution of 11+11+10+9. Using the method (6) but counts the left bottom corner stone twice. (18) (10x2+8x2)x2-9x4: an incorrect solution. Using the method (8) but makes mistakes counting the number of stones in the vertical side and horizontal side.	◆ If the method (6) did not come from the students, use this method. However, this is an incorrect solution, so it is important to make sure to explicitly discuss what the misunderstanding is.

10 min.	**[Sharing and Discussing]** “What is happen to the number of stones when it is in *n*th figure?”	 • It will be 4n • It will be n+(n-1)+n+(n-1) • It will be $(n+1)^2(n-1)^2$ • If we calculate all of them, they all become 4n.	◆ It is important to focus on students’ interpretations of the expressions. Thus, the student presentation will be done by sharing expressions. Then, ask other students to explain the presented solutions. (I use this practice in everyday classroom.) ◆ Confirm with students that the result of calculations is 4*n*. The students do not know how to calculate method (13) to get 4n since they have not learned factorization. At this point, simply tell students the result of calculations should be 4n.
5 min.	“When we discussed the expressions that use numbers, what did you do?”	• I thought about where the numbers are represented in the diagram. • I looked at the expression and thought about how the person counted the stones. • I wondered if there is any other way to write the presented expression. • I thought about whether or not the presented expression is correct. • I wondered if I could extend the presented expression.	◆ I’m hoping that students will come up with the anticipated responses to the left. If there is no response, I will ask the students what they did in order to understand other students’ expressions.

Hatsumon: Where 4n is represented in the diagram? Let’s think about what 4n mean in the diagram.

10 min.	**[Solving problem on their own]**	• If we move the stones at the diagonal part to the top and to the right, the figure will be a square.  • If we move the triangle part that do not have stones, I can make a square.  • Move the stones to create a rectangle whose height is 4. Move pink stones to the red circles Move yellow-green stones to the green circles Light blue stones to the blue circles 	◆ If I see students who are not working, I will ask them to think about where the 4n is coming from and how we are getting the 4n. ◆ If it is necessary, I will ask the same questions to the class, even if students could not finish the solution completely.
5 min.	**[Sharing and Discussing]**		◆ Make sure to bring up these two methods on the left. ◆ If the idea of square does not come from the students, ask students if they could they make a rectangle with four groups of *n* stones. ◆ If students came up with the idea of using rectangle, make sure to choose and discuss the method.

5 min.	**[Summarizing]** “What did you think about? What did you learn?”	• When we simplify the algebraic expressions, we can get the simple algebraic expression. When we think about the simple algebra expression with the original figure, we can see that we are able to transform the original figure to the different figure. • First, I did not understand how people find the number of stones from the expressions, but when I look at the diagram and think about how the numbers are represented in the diagram, I felt it is possible to understand.	◆ While students write their reflections, walk around and read there writing. Identify good responses and ask these students to share what they wrote.

HOW DO YOU DESIGN LESSONS TO RAISE THE QUALITY OF STUDENT UNDERSTANDING ON MATHEMATICAL PROCESSES?

Keiichi Nishimura

Professor of Mathematics Education

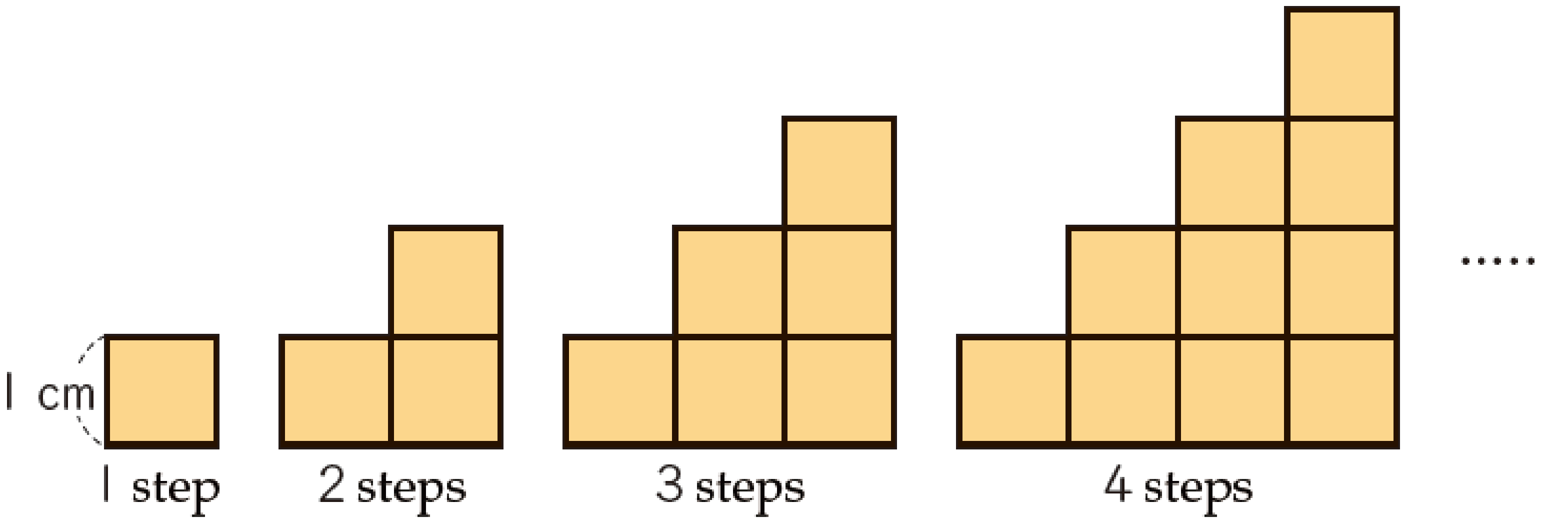
HOW DO THE STUDENTS LEARN ALGEBRAIC EXPRESSION IN THE TEXTBOOK BASED ON NATIONAL CURRICULUM IN JAPAN?

Year4

Investigating Changes

How Do Quantities Change?

3 Squares with 1 cm sides are arranged into staircases that have 1, 2, ... steps, as shown below.
Find the length around a staircase that has 20 steps.



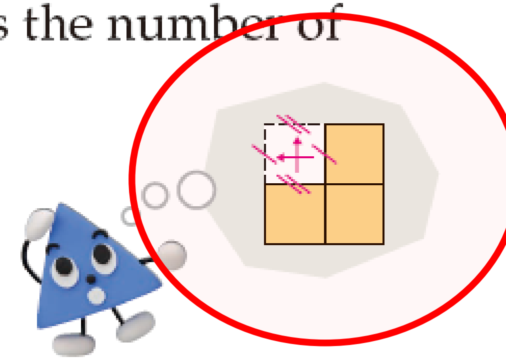
It will take too much time to draw a picture of a 20-step staircase.

I wonder what the relationship between the number of steps and the length around the staircase is like.

1. Record the number of steps and the length around the staircase in the table below.

Number of steps (steps)	1	2	3	4	5	6	7
Length around the figure (cm)							

2. How does the length around the figure change as the number of steps increases by 1?



3. How many times as many is the number for the length around the figure as the number of steps?

4. Calculate the length around the figure when there are 20 steps in the staircase.

$$20 \times \square = \square$$

5. Suppose there are □ steps, and the length around the figure is ○cm. Write a math sentence to express the relationship between □ and ○.

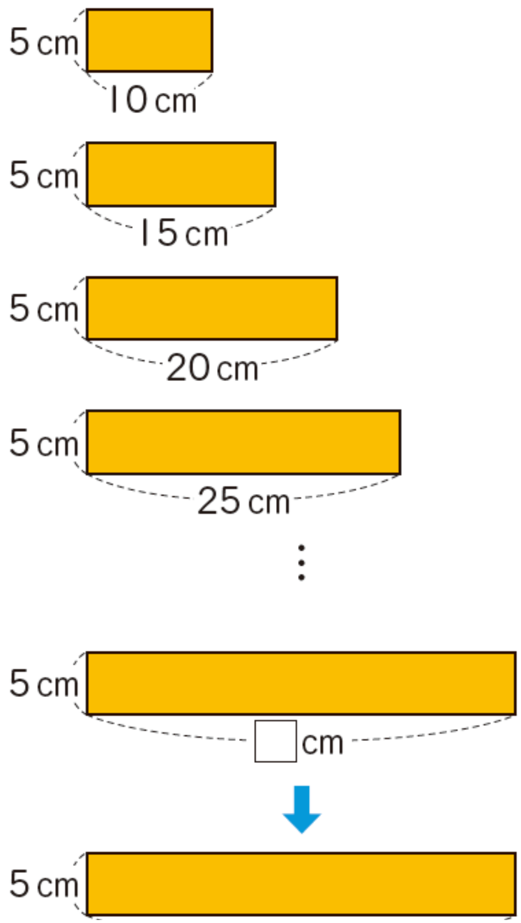
Number of steps × 4 = Length around the figure

□ × 4 = ○

Year6

Investigating Changes

How Do Quantities Change?



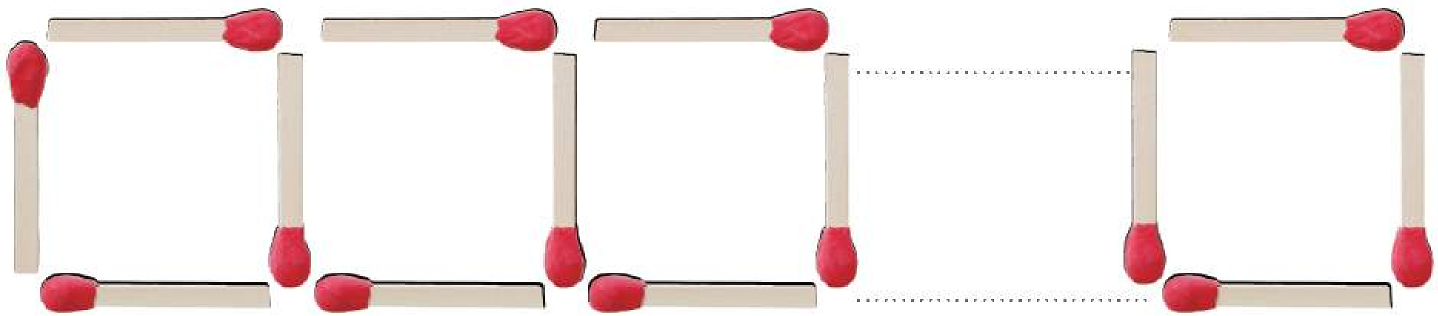
	Length	×	Width	
If the piece is 10cm	5	×	$\boxed{10}$	(cm^2)
If the piece is 15cm	5	×	$\boxed{15}$	(cm^2)
If the piece is 20cm	5	×	$\boxed{20}$	(cm^2)
If the piece is 25cm	5	×	$\boxed{25}$	(cm^2)
⋮	⋮			
If the piece is ☐ cm	5	×	☐	(cm^2)
↓	↓			
If the piece is $x$cm	5	×	x	(cm^2)

2 In the math sentences above, which number stays constant? Which number varies?

Year7

Letters in Algebraic Expressions

We are making squares by lining up matchsticks as shown below.
When we make 20 squares, how many matchsticks will we need?



① ② ③ ⑳

Should I actually use the matchsticks and try to line them up?

Sakura

Yuto

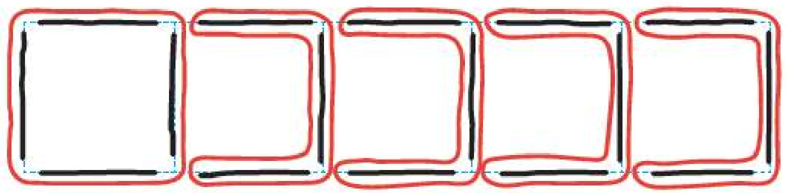
It is not easy to count them one by one, is it?

Fill in the $\boxed{\quad}$ with the appropriate math sentence in Sakura's idea as shown below.

Sakura

If I think about it by dividing it like below, I can represent the number of matchsticks with the following math sentence.

$\boxed{\qquad\qquad\qquad\qquad}$



$4+3(x-1)$

How did Yuto think to come up with the math sentence $1+3\times5$?
Think about it using the diagram below.

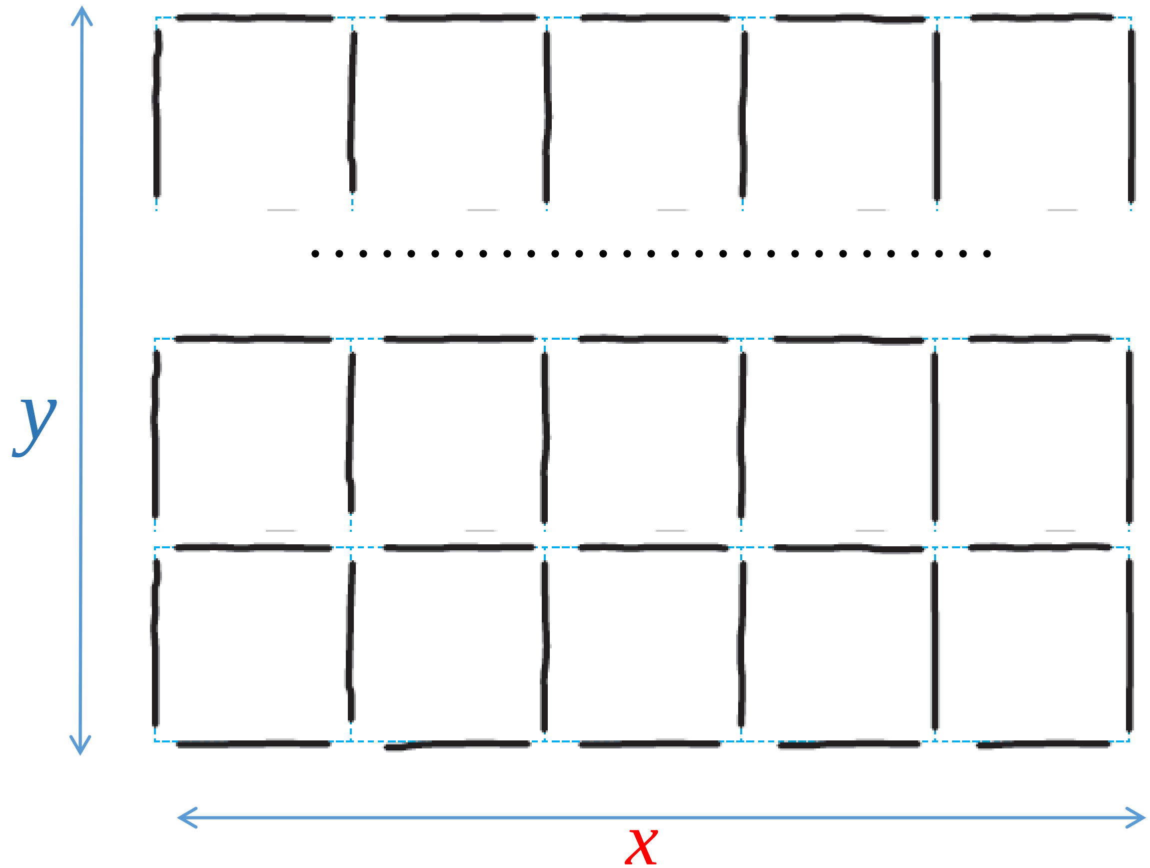
|| ?

Yuto

The number of matchsticks can be found with the following number sentence.

$1+3\times5$

$1+3x$



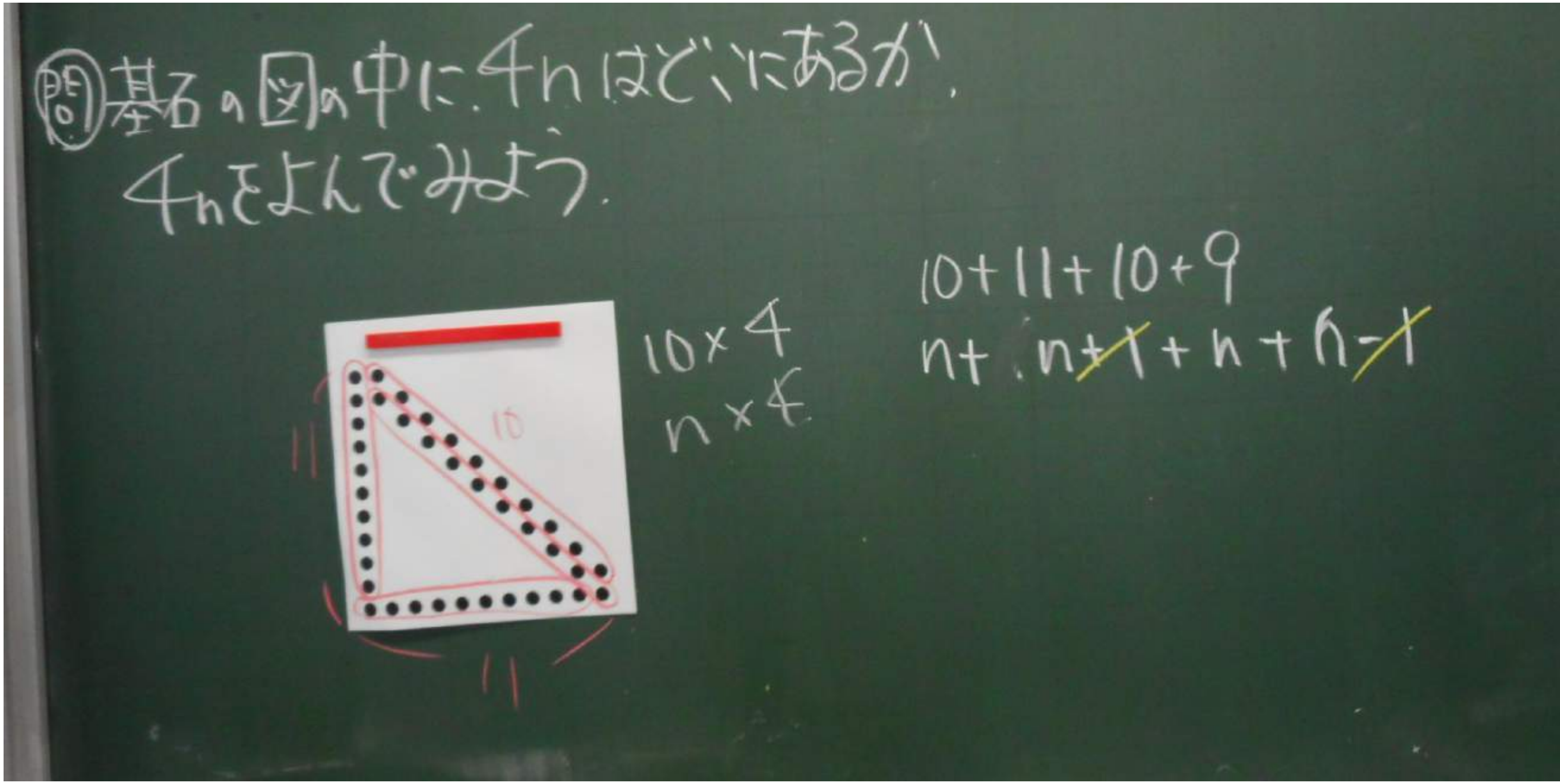
$$(1+3x)\times y - x(y-1)$$

Year8

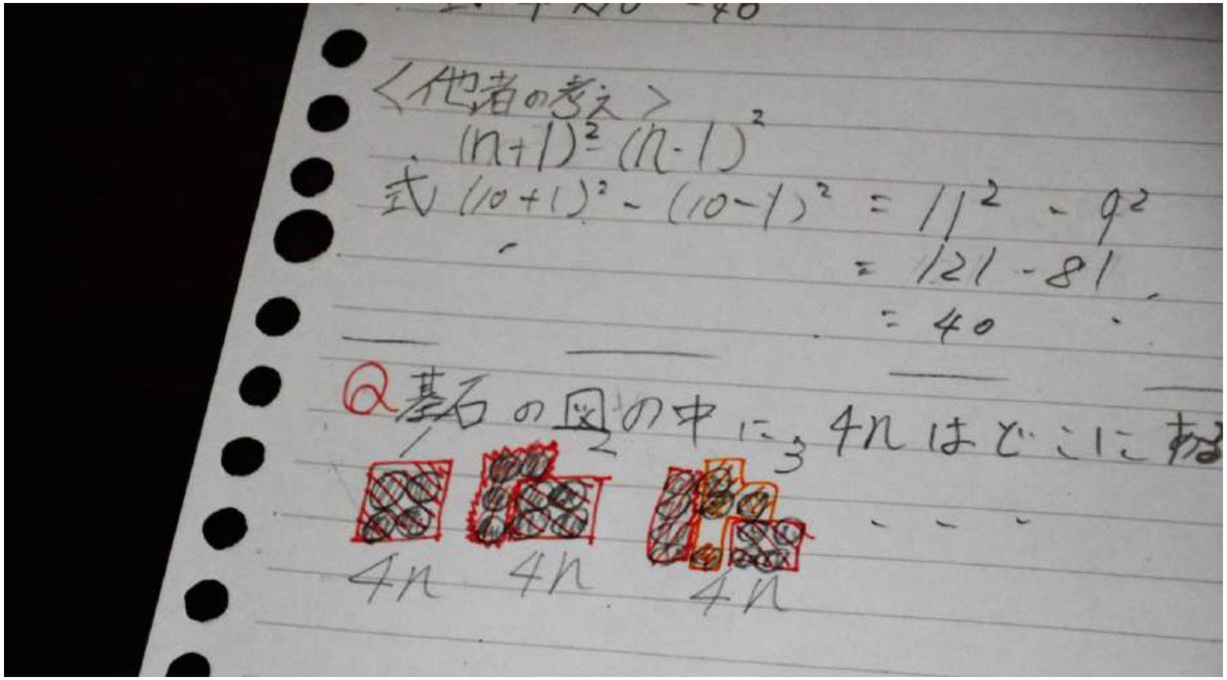
$2x+1+3x+2=5x+3$
$2x+y+3x+2y=5x+3y$

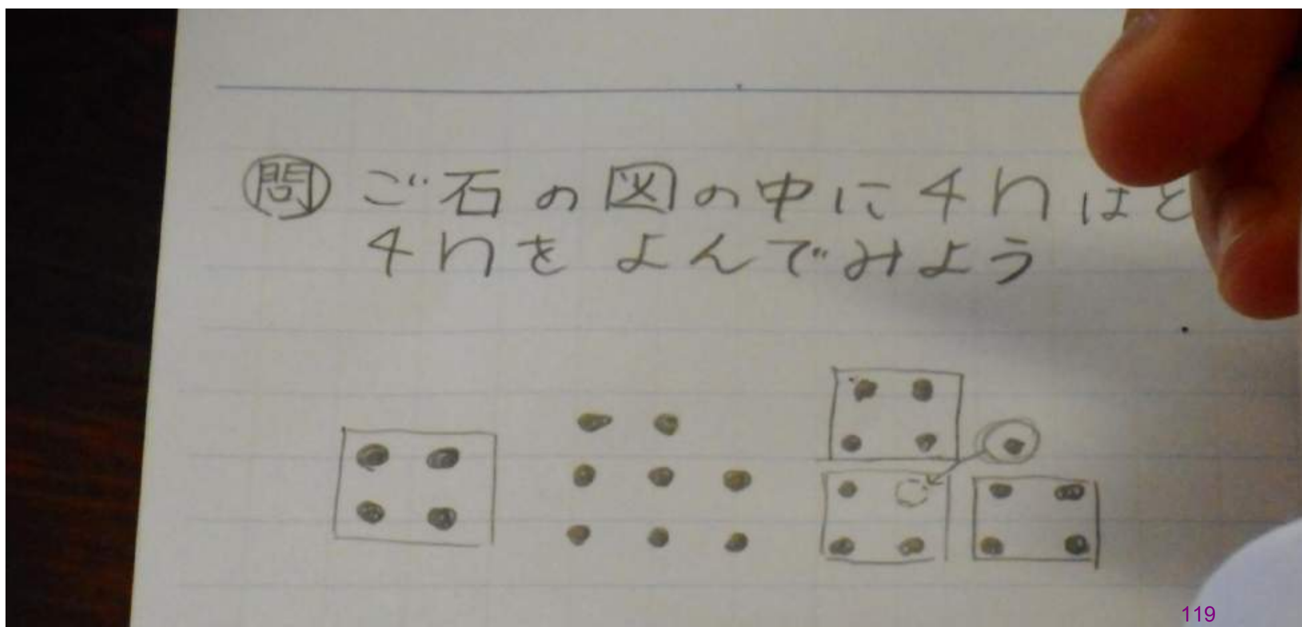
These calculation is quite similar but the meaning and complexity in the phenomena is very different.

Key point in this lesson

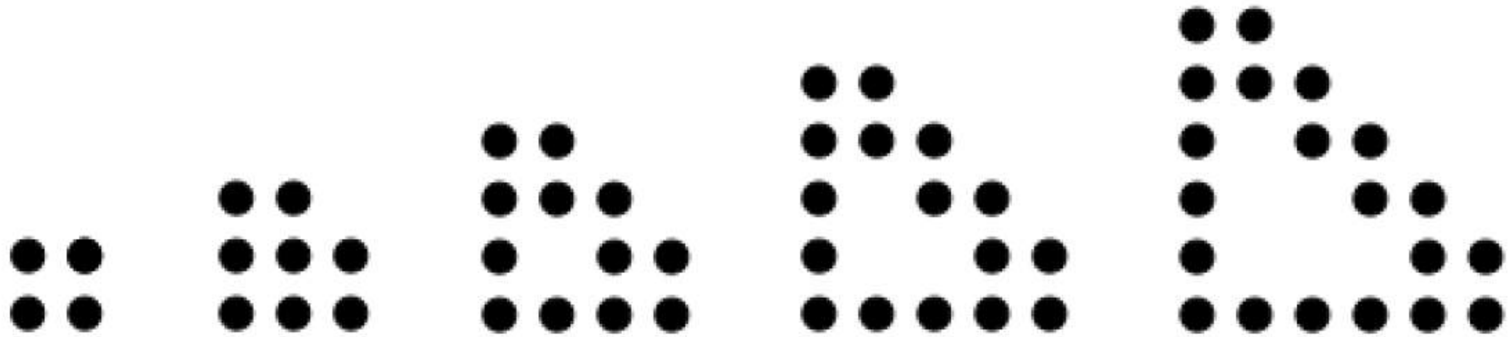


Where 4n is represented in the diagram?


〈他者の考え〉
(n+1)² (n-1)²
式 (10+1)²-(10-1)² = 11² - 9²
= 121 - 81
= 40
Q 碁石の図の中に 4n はどこに
1 2 3
4n 4n 4n


問 ご石の図の中に 4n はど
4n を よんでみよう

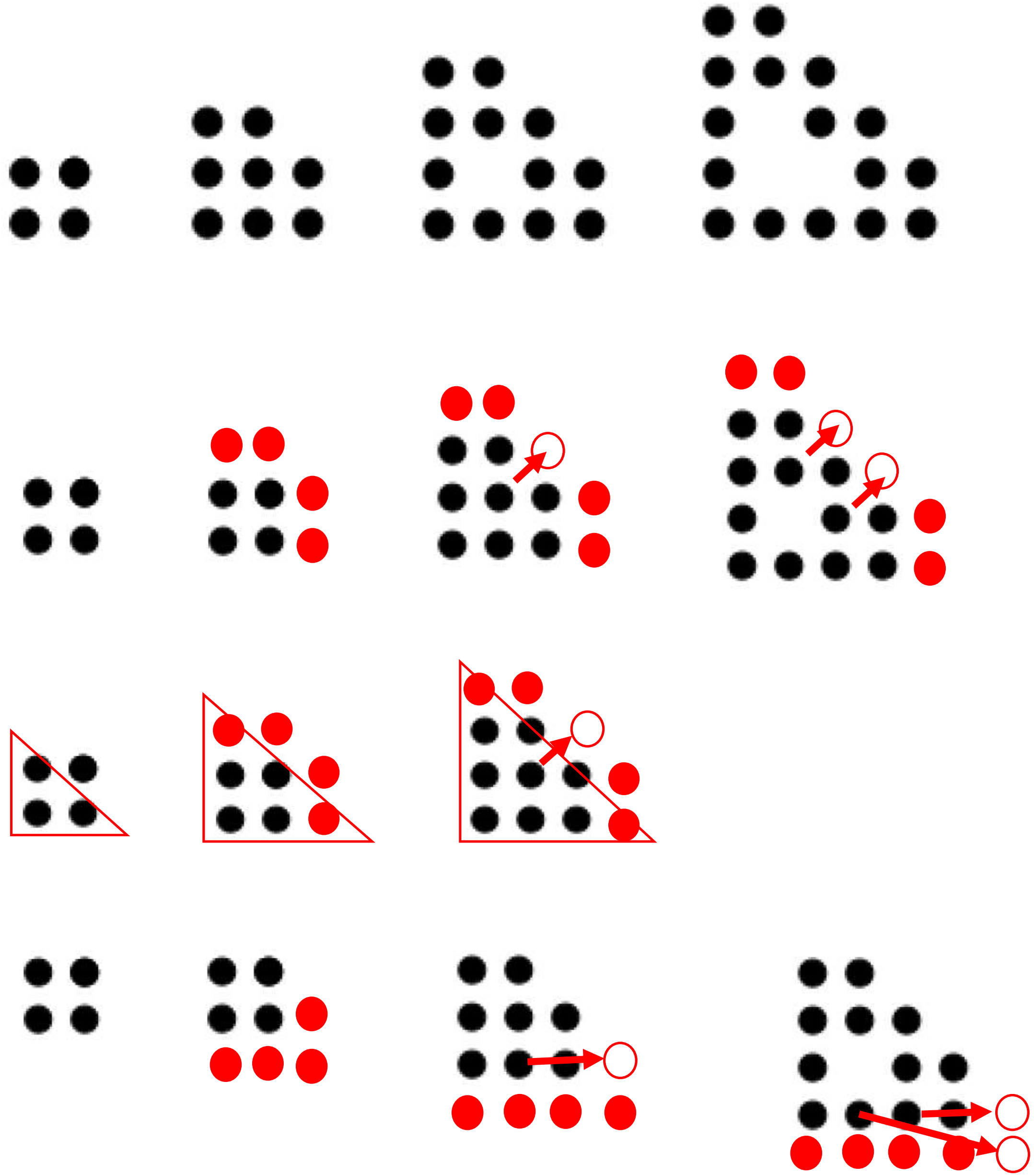
The instructional material for this lesson is the go stones where the number of them increase gradually as shown below.



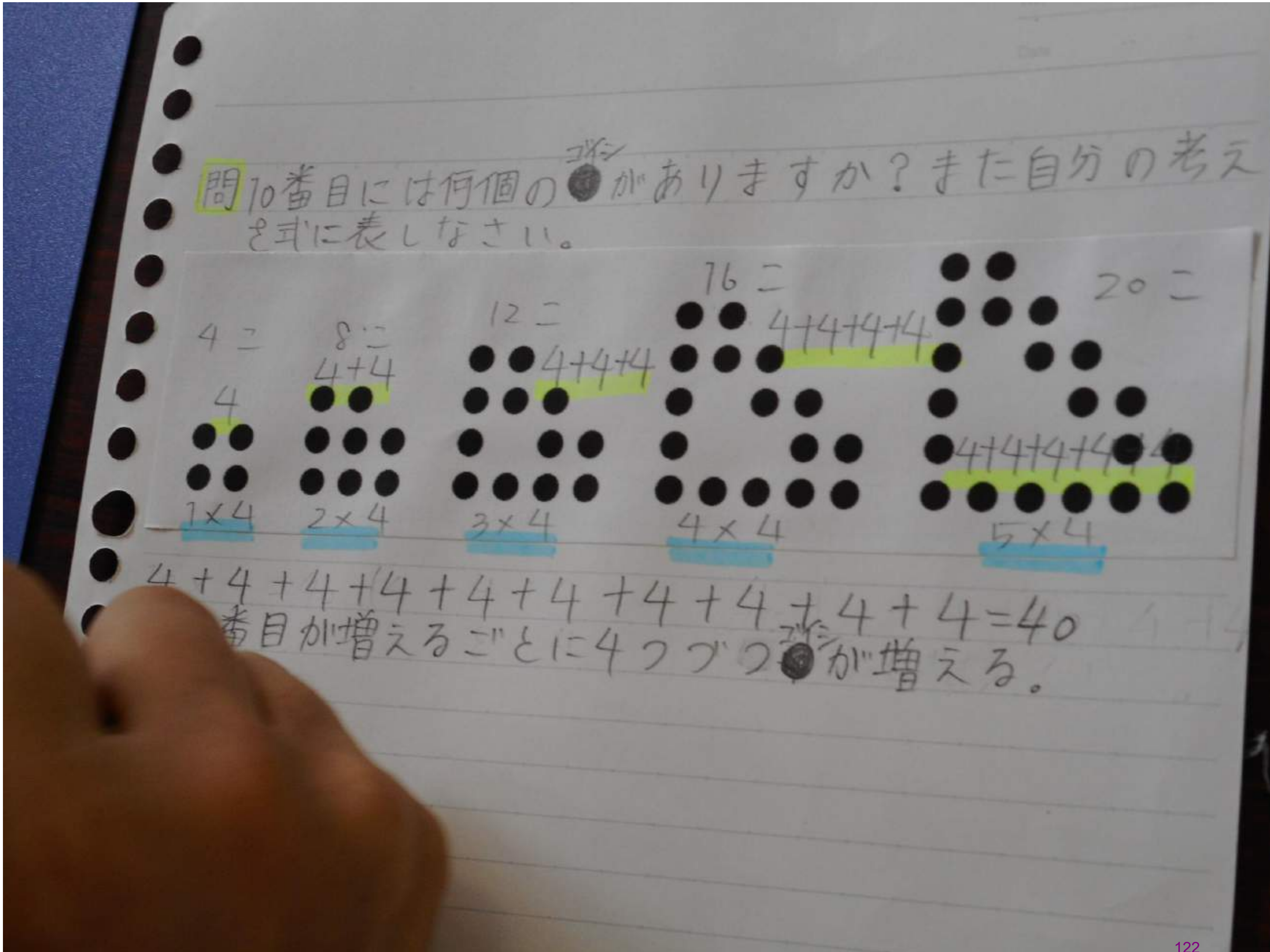
How many stones are in the 10^{th} figure?

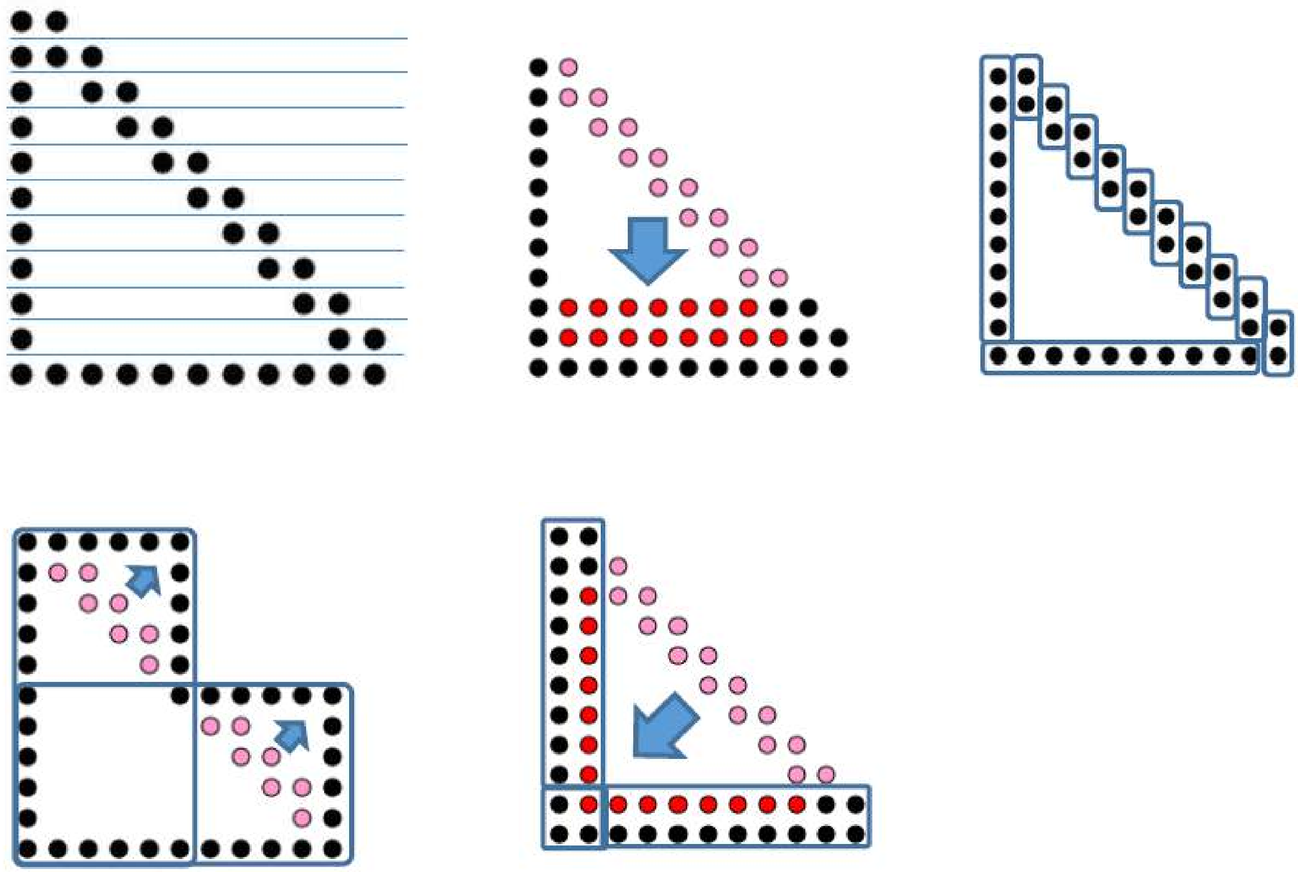
The students have to find the rule of increasing, because they cannot imagine the 10^{th} figure.

My Kyozai Kenkyu



They found out *4 ×10* based on these rules.

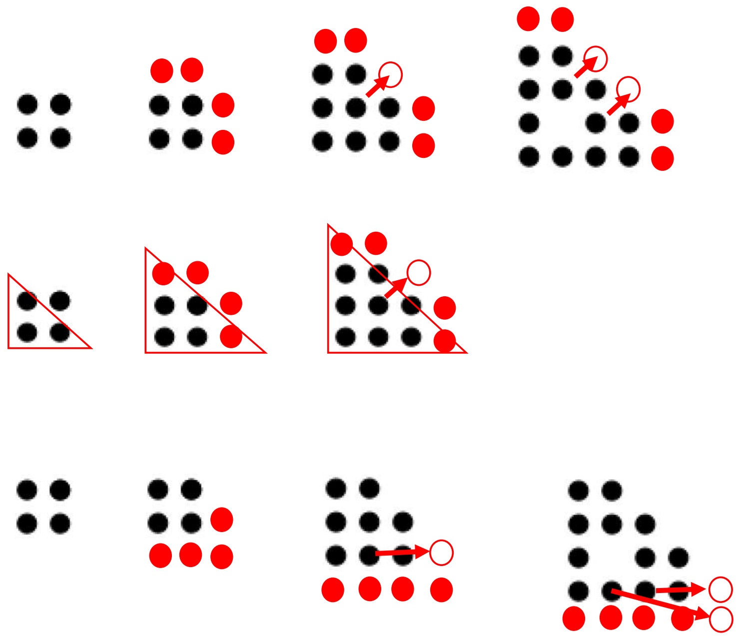

問 10番目には何個の●がありますか？また自分の考えを式に表しなさい。
ゴイシ
4こ
8こ
12こ
16こ
20こ
4
4+4
4+4+4
4+4+4+4
4+4+4+4+4
1×4
2×4
3×4
4×4
5×4
4+4+4+4+4+4+4+4+4+4=40
番目が増えるごとに4コずつ●が増える。



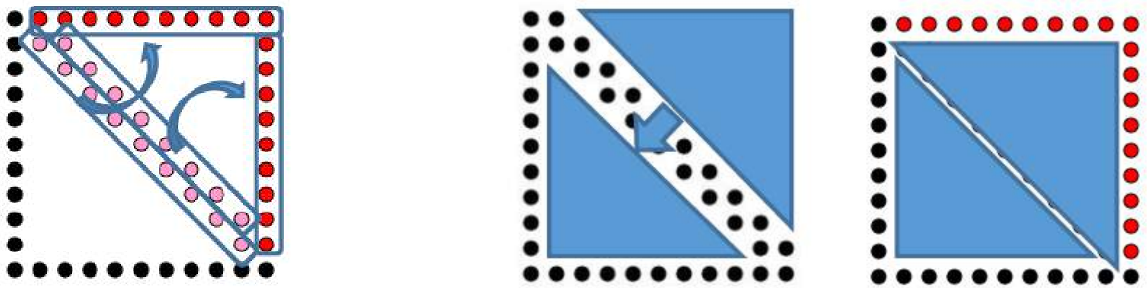
These ideas are on the 10th figure and different from the increase image.

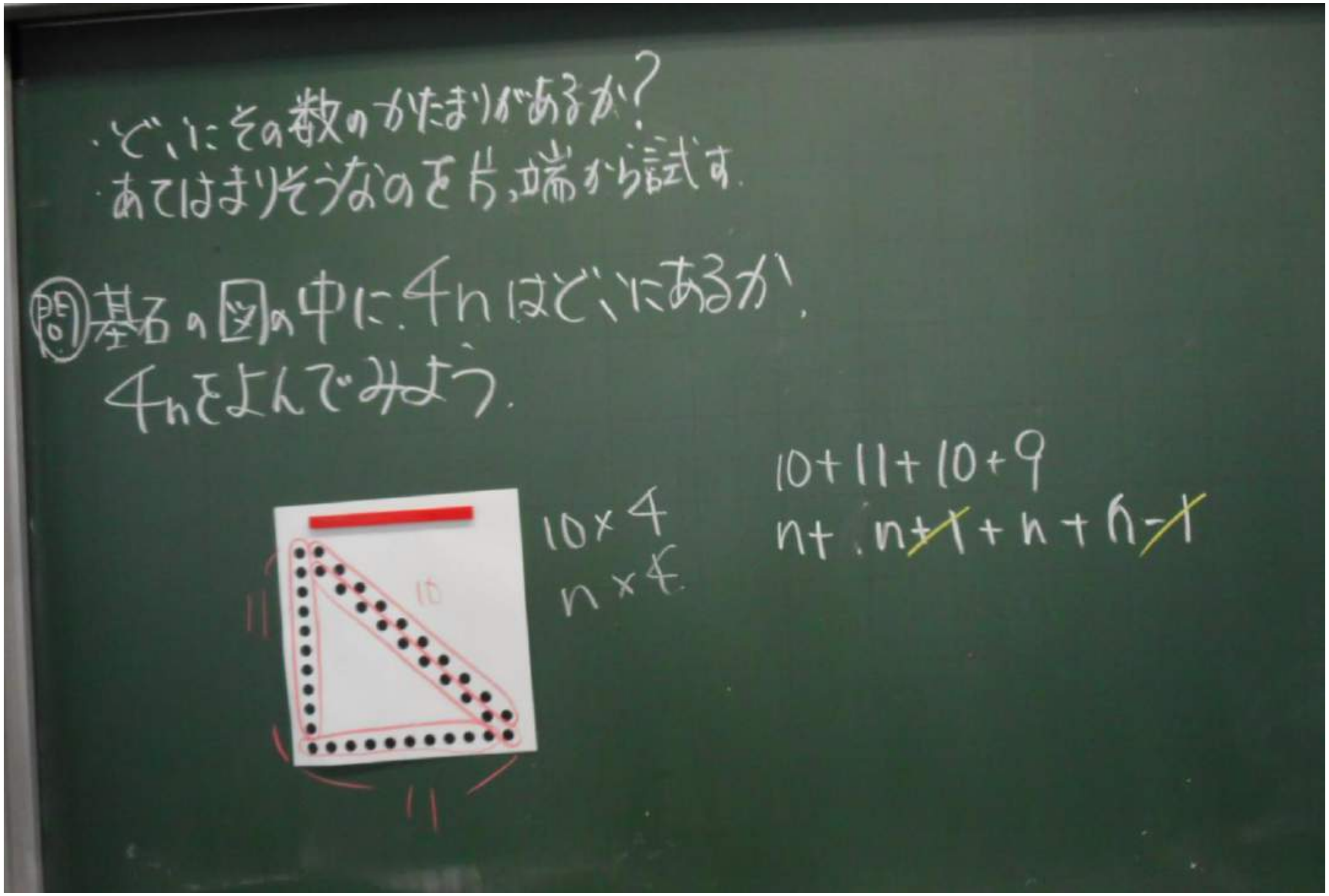
Where 4n is represented in the diagram?

- $4 \times n$



- $n \times 4$





$$4n$$
$$=(n-1)+n+n+(n+1)$$
$$=(n-2)+(n-1)+(n+1)+(n+2) \qquad \text{etc}$$

Why is this important?

Is 396 a multiple of 9？

$3+9+6 = 18$

18 is a multiple of 9.

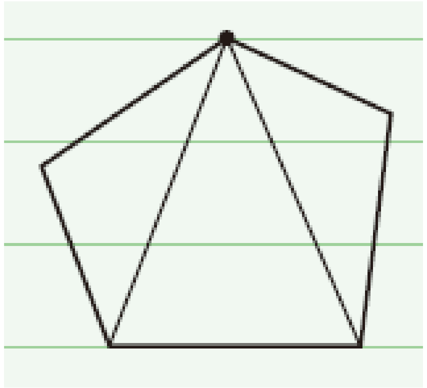
So 396 is a multiple of 9.

- $100a+10b+c$

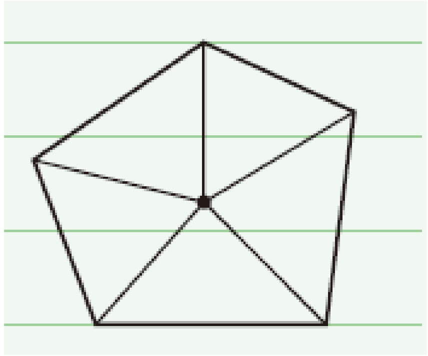
$= 99a+a+9b+b+c$

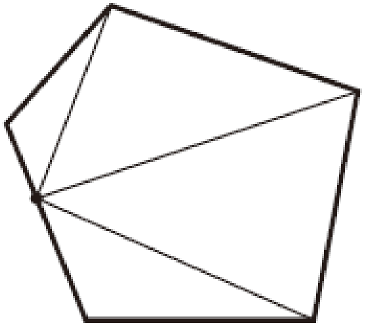
$= 99a+9b+a+b+c$

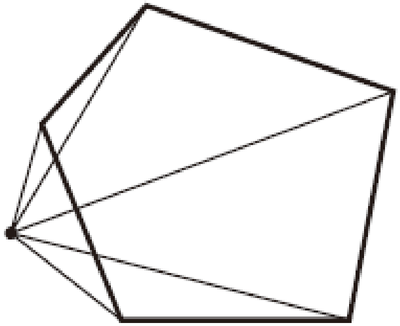
$$180(n-2)$$



- $180n$-360
- 180(n-1)-180
- 180(n-3)+180



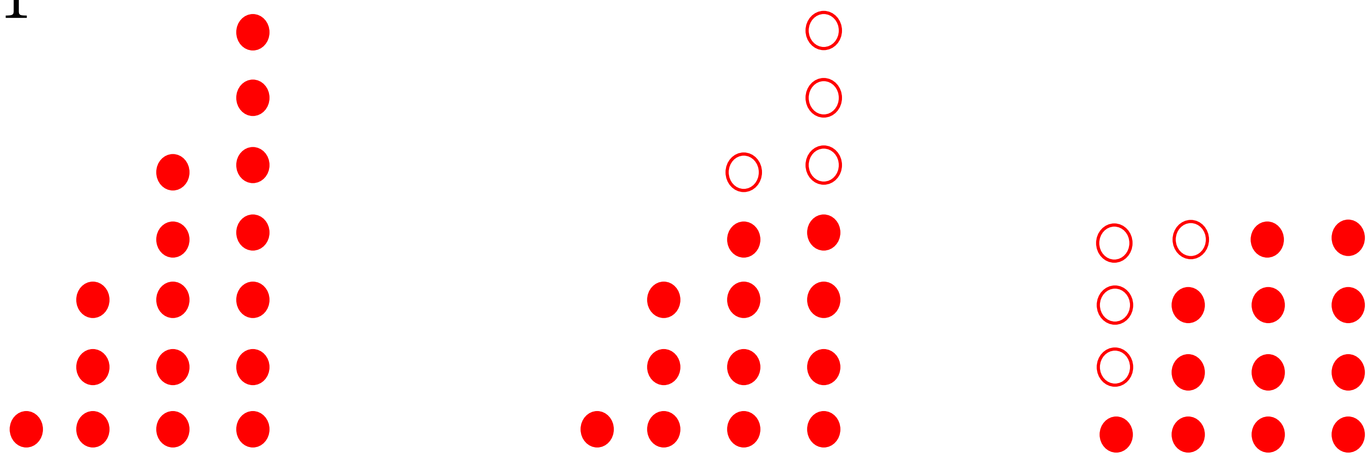




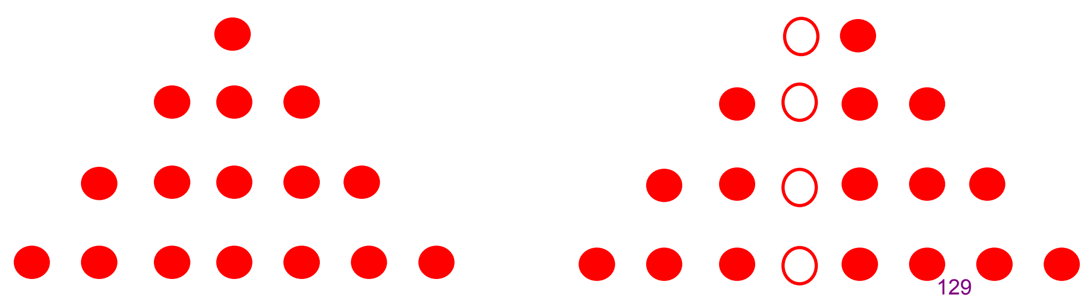
Year11

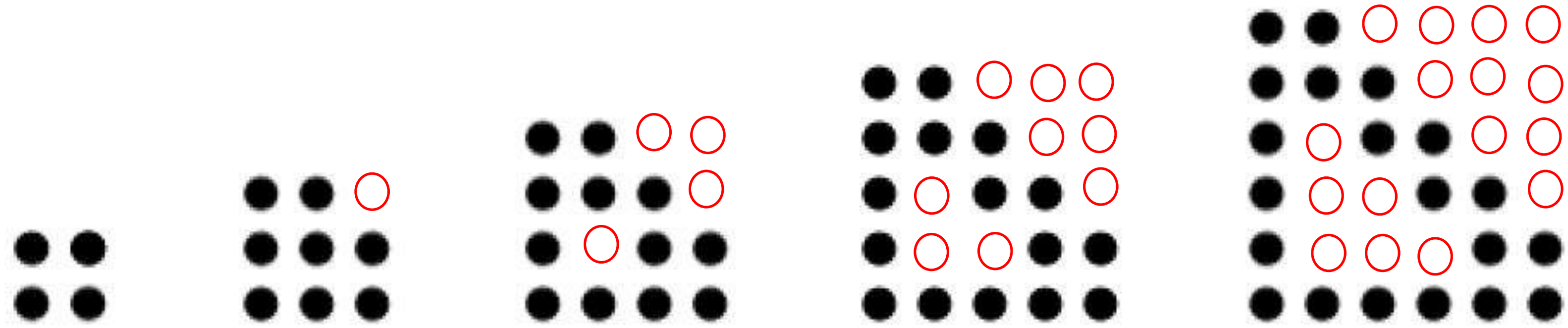
$1+3+5+7+\cdots$ numerical sequence

$$\sum_{k=1}^{n}(2k-1) = n^2$$



$$= 2\times\sum_{k=1}^{n}k - n$$





2^2-0^2 3^2-1^2 4^2-2^2 5^2-3^2 6^2-4^2

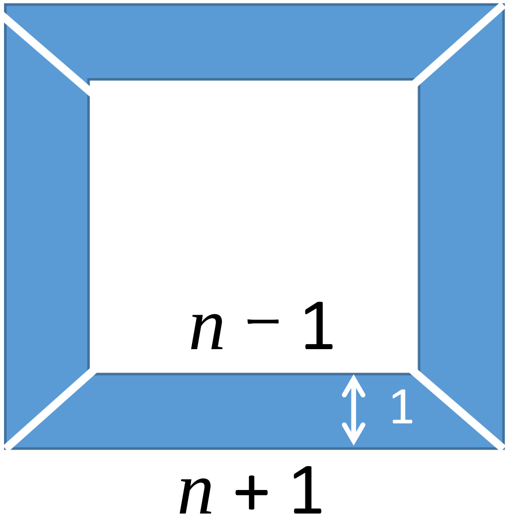
$$(n+1)^2-(n-1)^2=4n$$

Year9

(5)About this Lesson:

① Goals of this Lesson

・・・ In addition, they are also able to interpret the algebraic expressions with letters and understand the quantities and quantitative relationships of the phenomena.

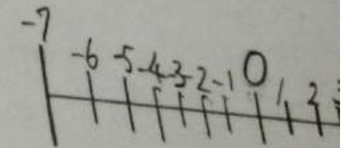


$$\{(n-1)+(n+1)\}\times 1\times\frac{1}{2}\times 4$$

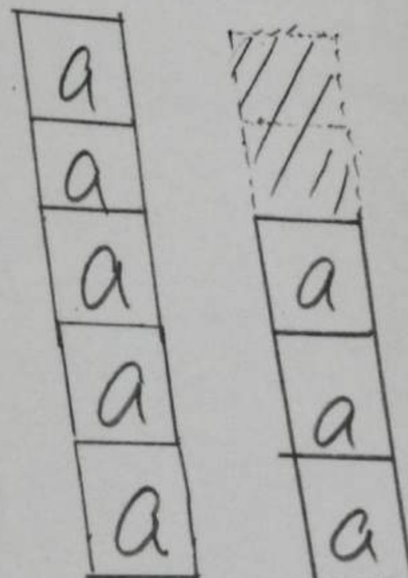
Year7

〈a が正の場合〉

$3a = a + a + a$



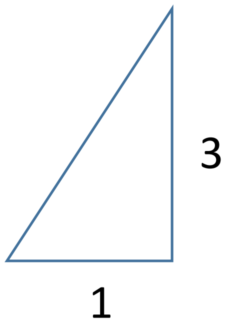
$5a = a + a + a + a + a$

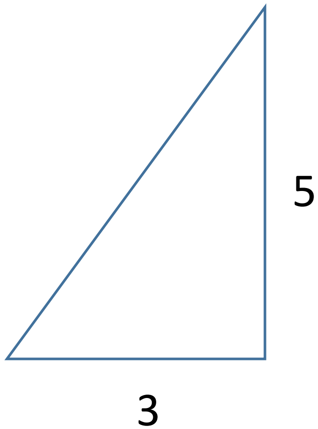


$5 \times a = 5a$

$3 \times a = 3a$

If you emphasis on understanding the quantities and quantitative relationships of the phenomena,・・・





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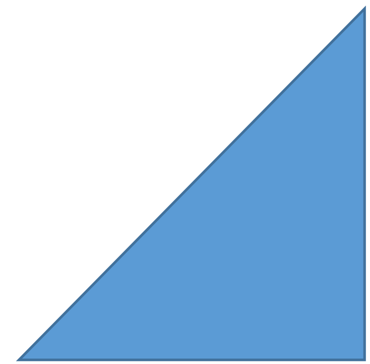
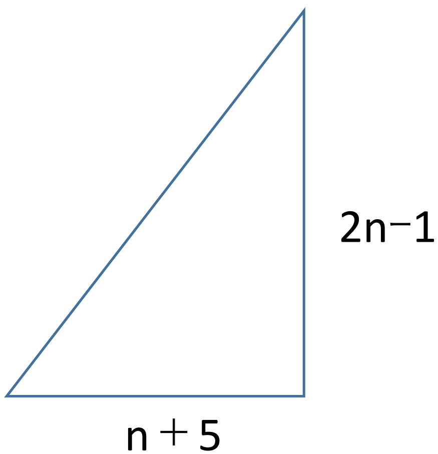
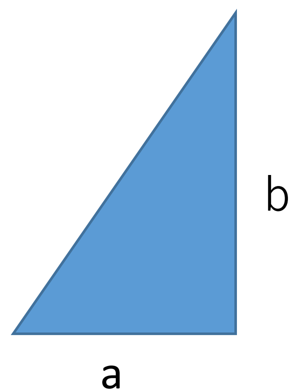


Isosceles triangle





$$\lim_{n\to\infty} \ ?$$

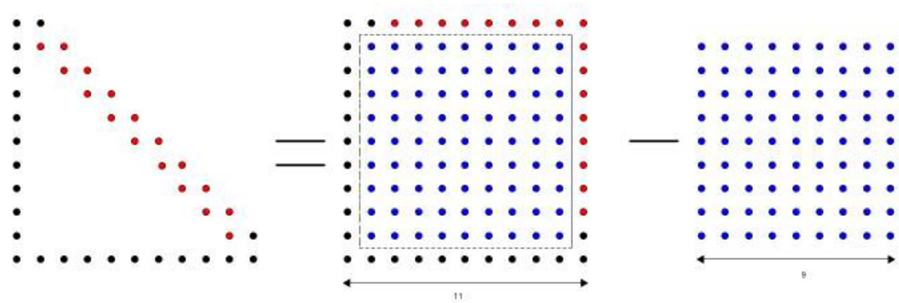
a：b＝1：2

Lesson Report

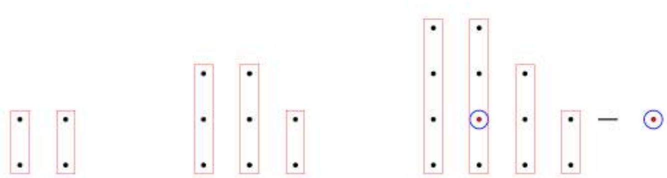
Report created by: E Southall, S Hironaka, D Correa
Name of Lesson: Sequences
Date of Lesson: 21/06/17
Grade Level: 7th grade

Background to the Lesson
In this lesson, students work with the following pile pattern:

One way to look at this pile pattern is to transform each figure in the sequence into a square, from which there are multiple ways to calculate the total from that. (See diagram below, red stones have been moved to make a square. Then one calculation is to subtract an imaginary inner square of stones (9x9) from the outer square (11x11).



However, it is also possible to conceive a pattern without manipulating the stones:



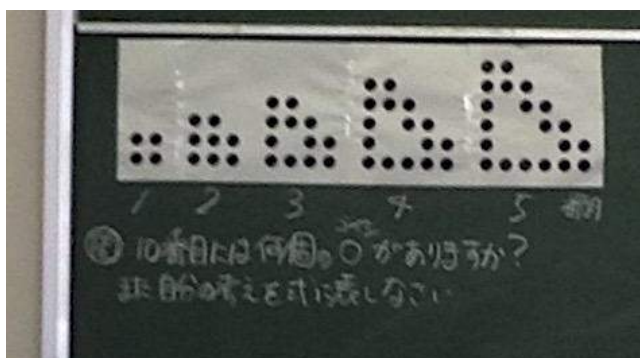
What are the primary lesson goals?
Students will be able to
- calculate the number of stones in the 10th figure in the pile pattern.
- use the relationships among quantities in a pile pattern to create an algebraic expression with letters.
- make connections between other equivalent algebraic expressions in relation to the pile pattern

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

In elementary school, students learned about variables using symbols, such as 5 + □ = 8. In lower secondary, students began replacing these symbols with letter variables, such as *a* and *x*. This lesson is in the unit, “Letters in Algebraic Expressions.” The unit consists of three sections: (1) algebraic expressions with letters, (2) calculations of algebraic expressions, (3) applications of algebraic expressions with letters. This lesson is placed in section 3 of the unit.

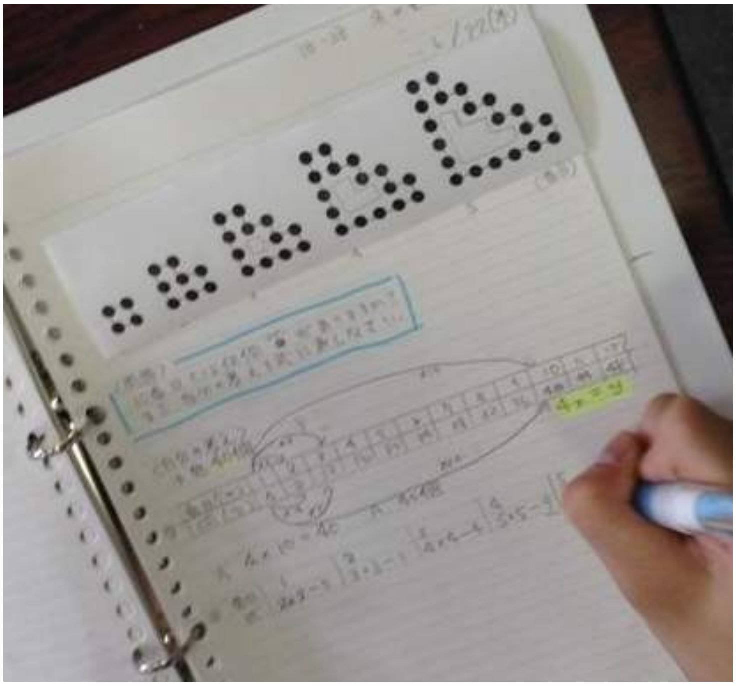
In the future, this will prepare students for 8th grade math, where they will learn calculations of algebraic expressions, systems of equations, and linear functions. In 9th grade, students learn polynomials, quadratic equations and functions in the form of y=ax^2.

Summary of Lesson

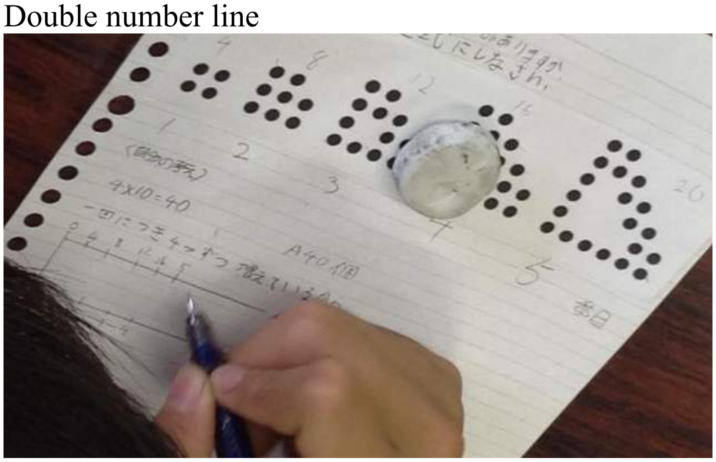
Time	Lesson Phase	Notes
2:28 - 2:31	**Introduction, Posing Task** *Four arrays following a sequential structure are shown to students on the blackboard. Students are asked how many dots are in each figure, followed by how many dots would there be in the tenth figure?*	**Strategies to build interest and to connect to prior knowledge** The figures are provided to students as handouts, and they are encouraged to annotate them and write notes to help them structure their thinking. The patterns are interesting to look at, and whilst counting the dots in the provided figures is straightforward, finding the number of dots in the tenth figure is not.  *“How many stones are in the 10th figure? Show your own thinking.”*
2:31-2:43	**Independent Problem Solving** *Students are given twelve minutes to calculate how many dots there would be in the tenth figure, and encouraged to reason their answer.*	**Individual, pairs, group, or combination of strategies** ● **experience of diverse learners** ● **teacher’s activities** Students work individually, with no specific guidance from the teacher. The teacher does not specify any strategies or give hints at how to solve the problem. Instead they encourage students to keep thinking, and those that believe they have an answer are encouraged to think of a different approach in order to convince themselves that their answer must be true.

Student Approaches in independent problem solving

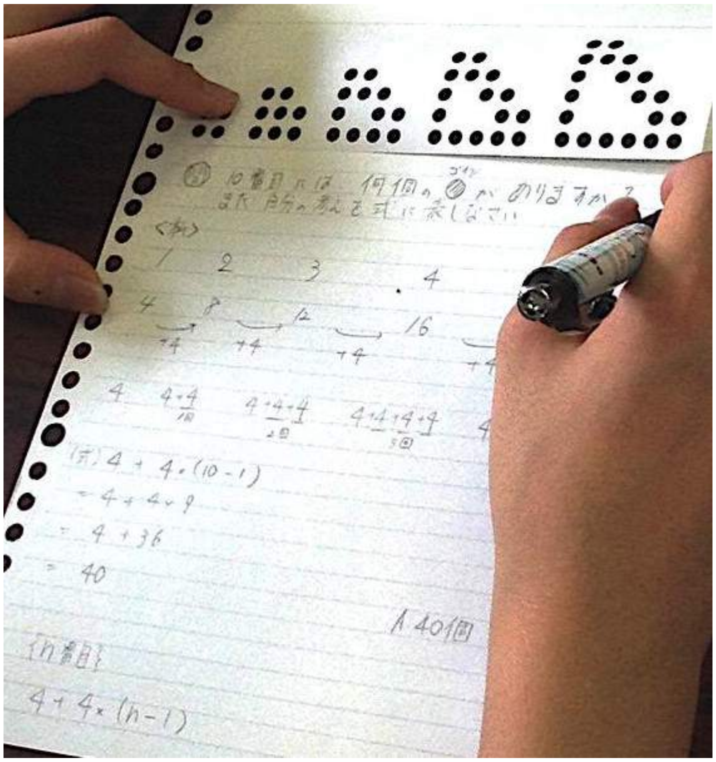
Table



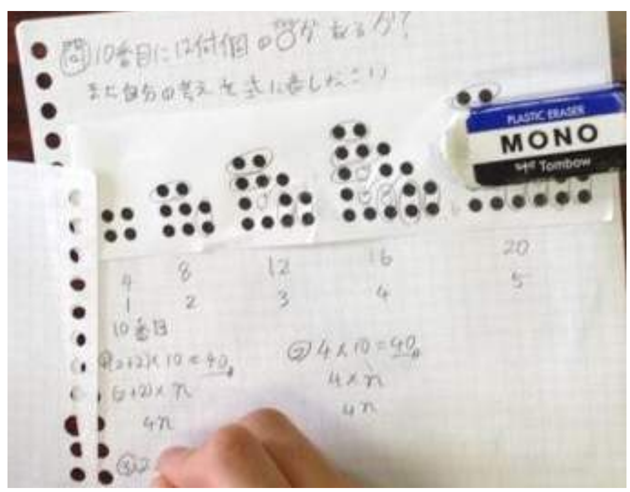
Double number line



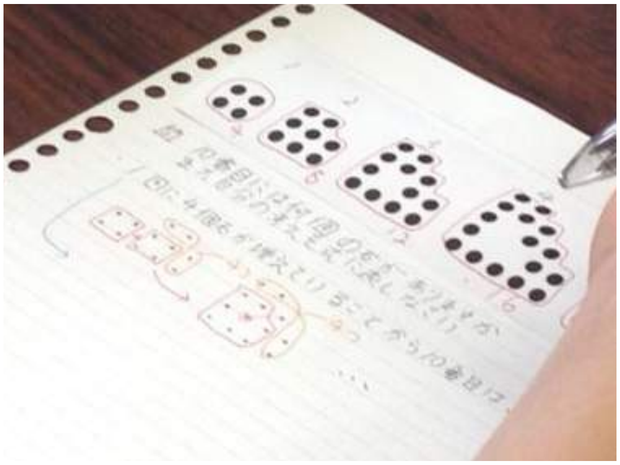
Identifying the +4 pattern

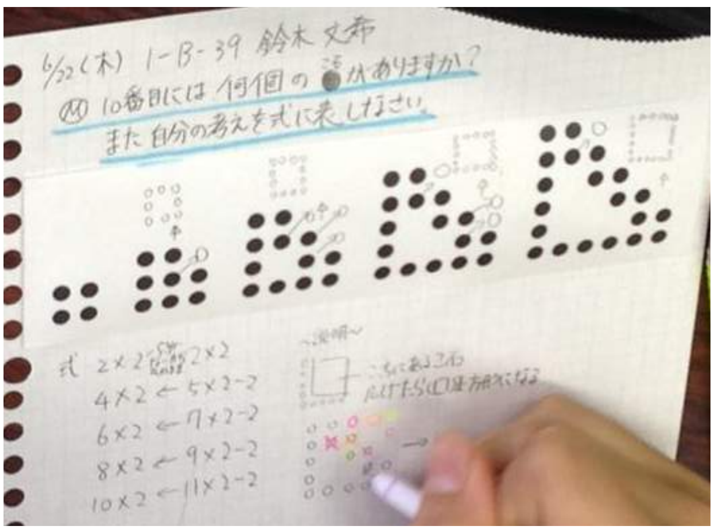


Algebraic Reasoning



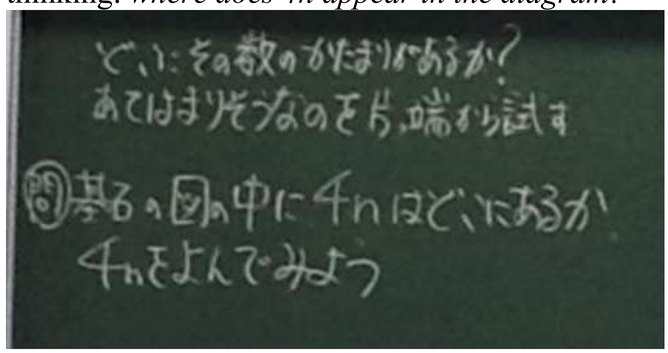
Moving stones to new formation





2:43 – 3:12	**Presentation of Students' Thinking, Class Discussion** *Student responses are gathered onto the board in the form of mathematical expressions that point towards how the students thought around the problem, rather than just the answers.* *Each different response was written onto the board, and then students were encouraged to try to understand what the thinking was behind the approaches of their peers.*	**Student Thinking/ Visuals/ Peer Responses/ Teacher Responses** **Student 1:** *4 x 10* *(The student was seeing the problem as a linear numerical sequence, largely ignoring the arrays.)* **Student 2:** *10 x 4* *(The student saw the problem in the same was a student 1)* **Student 3:** *4+4x(10-1)* *(The student saw the problem as adding (n-1) groups of 4 to the original 4)* **Student 4:** *21x2-2* *(The student rearranged the 10^{th} figure into a square and counted up the vertical and horizontal dots (summing to 21), doubled them, and subtracted the cross-over (2))* **Student 5:** *(10+1)^2-(10-1)^2* *(The student envisioned the array as a square like student 4, and squared the side length, then subtracted the inner, 'empty' square.- see diagram 1 below)* **Student 6:** *2+3(10-1) + (10+1)* *(The student explained that each row has 3 dots on it except the first row (it has 2 dots) and the last row (it has "10-1" dots) – see diagram 2 below).* **Student 7:** *20x2* *(The student saw the problem as a numerical sequence and was doubling the number of dots in the fifth figure.)* **Student 8:** *11x11 – 9x9* *(The student saw the problem in the same way as student 5)* **Student 9:** *2+3+4+…+11 + 11 – (1+2+3+…8)* *(The student counted the number of dots in each column (right to left) and then subtracted the number of 'missing' dots from right to left – see diagram 3 below)* **Student 10:** *10+11+10+9* *(The student circled four groups of dots in each figure, 2 of length 'n', 1 of length 'n-1' and one of length 'n+1')*

<table>
<tr>
<td>3:12-
3:14</td>
<td>Summary and new question to extend student thinking</td>
<td>Teacher leads class discussion and summarizes that the simplified expression is 4n. Teacher emphasizes that 4n is not particularly useful way of representing the diagram because ‘4n’ is not easily conceivable just by looking at the diagram. However, the (non-simplified) equivalent expressions that students had come up with earlier better related to the diagram.

Teacher then poses question on the board to extend student thinking: where does 4n appear in the diagram?

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<tr>
<td>3:16
–
3:21</td>
<td>Independent student work time</td>
<td>Students begin to examine the pattern sequence and add to their work to answer the question.

As shown in the students work examples below, generally students searched for four stones in the figure, rather than manipulating the figure into a new formation (the square) and deriving 4n from the square.</td>
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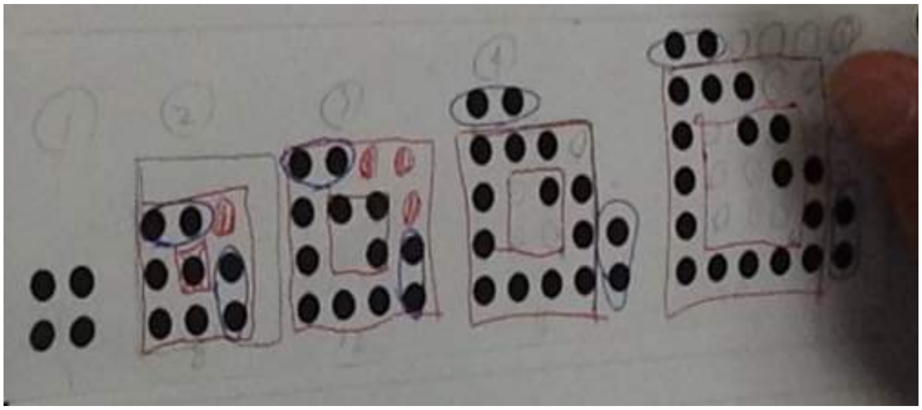
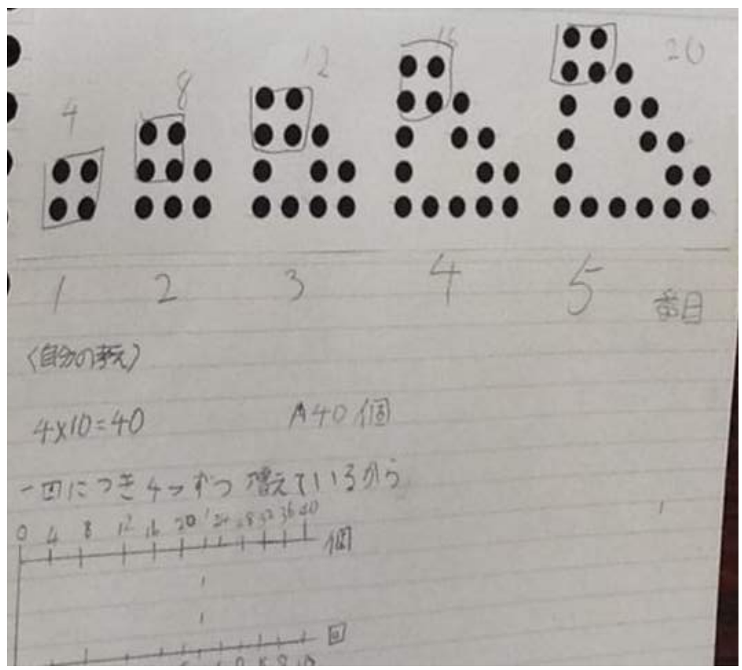
</table>

Student approaches to the extension question

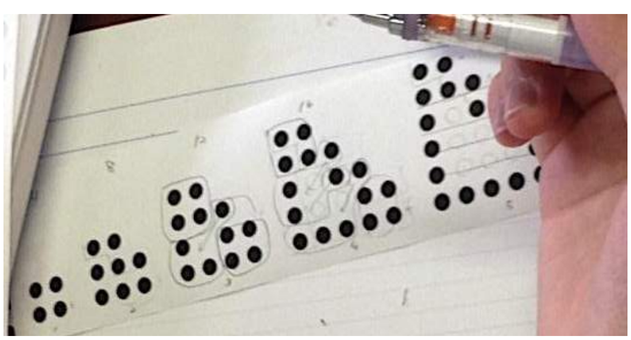


Box formation (this student had originally rearranged the stones in a box formation in the first independent problem solving time. The student is has now circled two pairs of stones, and is working through some errors with identifying how big the box should be for each figure.

This student had initially used a double number line, and now circled one group of four in each diagram. The student sat and looked at the diagrams for the remainder of the worktime, possibly unsure what else to do.



This student identified groups of four in each figure.



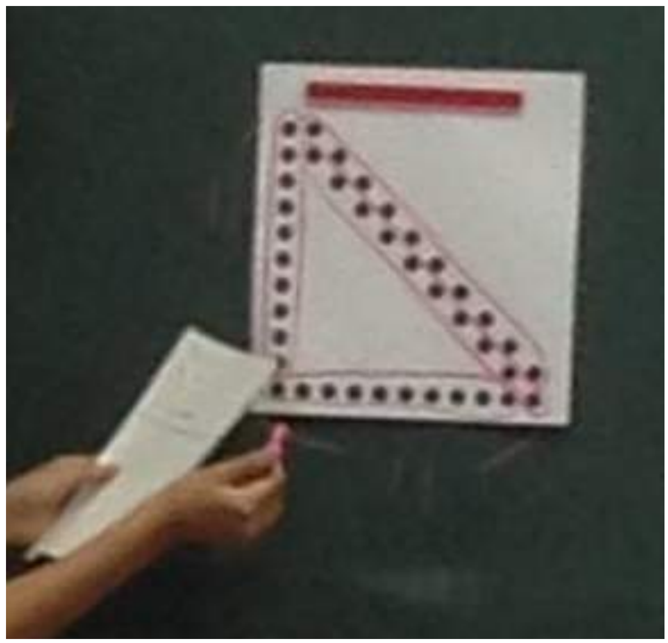
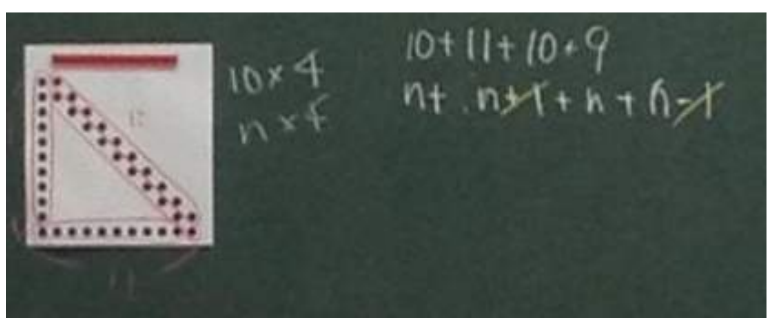
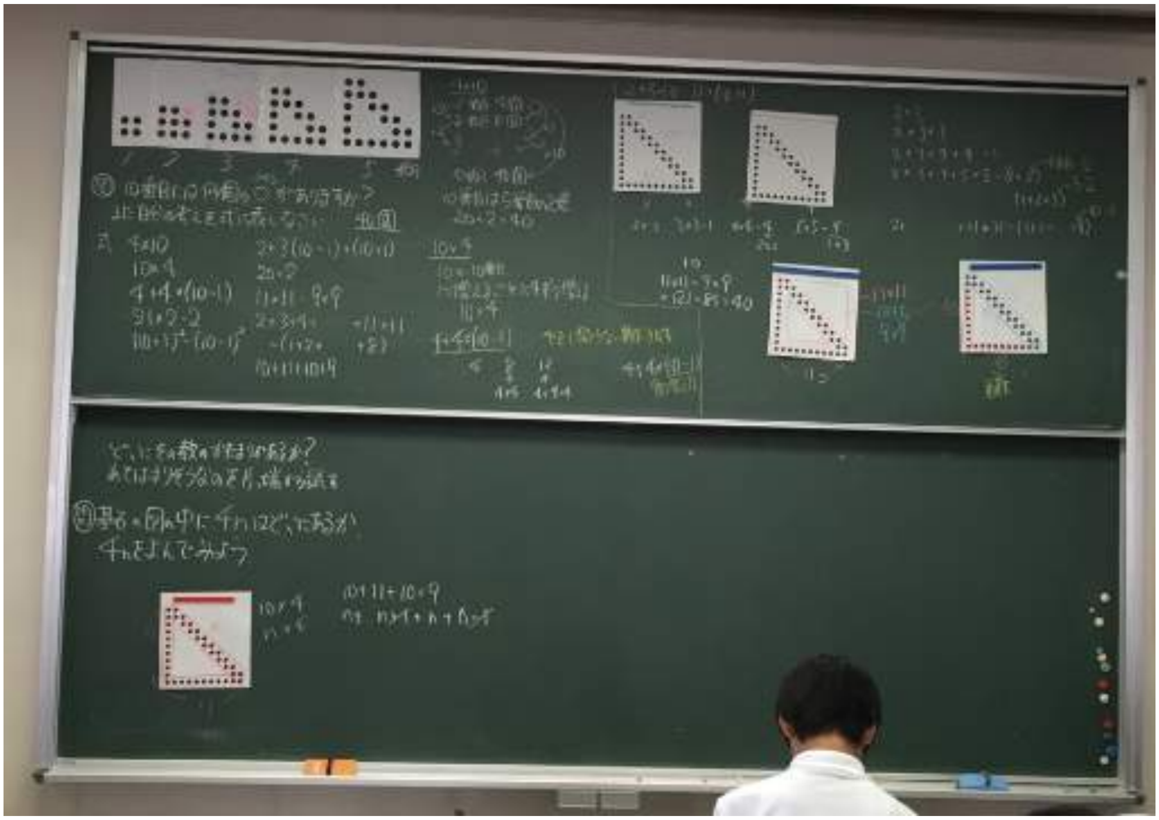
3:21 – 3:25	**Summary/Consolidation of Knowledge**	**Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals** In final class discussion, a student comes to the board to annotate how she saw the 4n in the 10th diagram. The student identifies 10 on the bottom, left side and 2 groups of 10 on the diagonal. This same student, in her original independent problem solving, had manipulated the diagram into a box formation, but now is conceiving the diagram in a different way.  Teacher connects the above student's strategy to another strategy and demonstrates how it simplifies to 4n.  Teacher prompts students to write their concluding statement for the day in their math journals.

Photo of the blackboard at the end of the lesson:



What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?

One of the most interesting elements of this lesson was the emphasis on analyzing student responses and their impact on whole class understanding. The post lesson discussion pointed out that the segment of the lesson where the teacher was trying to tie together the notion of 4n and the diagram required more time, and that several students were unable to see directly how the two were related. In addition, Dr. Keiichi Nishimura (the final commentator for the post-lesson discussion) gave an insightful presentation highlighting why what the teacher was trying to achieve was so important for the students, by showing us where the topic would naturally lead to in higher grades, and how the skills acquired would benefit their problem solving skills.

One question that was raised in the post-lesson discussion was that students were struggling with how the pattern grew in the diagram itself. Discussants questioned whether students understood the pile pattern well enough to draw the 10th figure. Although knowing how to draw the figures was not the goal of the lesson, without this understanding, it made it difficult to for students to see how the “4n” appeared in the pattern, when the teacher posed the extension question of “where does 4n appear in the diagram?”. Additionally, students who immediately settled on the expression 4n, were not basing this on the diagram, but rather the pattern they may have generalized from a table. The final commentator speculated that because the teacher did not emphasize connection between the table and 4n, it made it harder for students to see the 4n in the extension question. The lesson goal was that students would manipulate the figure to see 4 groups of n. However, after the teacher posed the extension question, students tended to look for n groups of 4.

From this lesson, we developed a more nuanced understanding of pile patterns as a mathematical task. If a pile pattern is growing by four for every subsequent figure, we are used to thinking of "n groups of four," where a pattern adds 4 more stones in a clear and predictable way. In this particular lesson's pile pattern though, identifying where the four stones were added was not as clear, even though students were able to identify +4 pattern in their individual thinking. This pattern is better conceptualized as "4 groups of n", where the number of groups stays constant, but the number of stones in each group changes. We thus recognize the limitations of thinking about pile patterns as n groups of 4, and recognize the opportunities for further exploration when there are"4 groups of n."

What new insights did you gain about how administrators can support teachers to do lesson study?

Administrators were supportive of the teacher and whilst they had critical feedback, it was never received poorly or as a slight on the ability of the teacher. There was a general feeling of community and group benefit to the discussions, rather than a feeling of being singled out and criticised openly.

Administrators can support a rich post-lesson discussion by seeking final commentators with vast knowledge of the scope and sequence of the research lesson's math content. The final commentator for this research lesson provided an insightful overview of they ways that sequences appear in different grade levels before and after 7th grade. This overview helps contextualize the lesson's applications and provides insight on what aspects of sequences we should prioritize in future teaching of this content.

In the post-lesson discussion, one administrator wondered if the whole class discussion had related students' tables with the diagram (so that they understood how the diagram was growing by 4,) students may have been able to make more connections in the extension question. This suggestion for an impromptu instructional moves pushes teachers to consider where it is useful to stick to the plan, and where it is necessary go off the script in order to better build upon students' current thinking. Administrators can pose these questions to support teachers in making connections between student work and the goals of a lesson.

How does this lesson contribute to our understanding of high impact practices?

There were several elements of this lesson that we feel could be effective in our classroom environments – particularly the emphasis on processes over answers. Often as teachers we spend a large amount of time ensuring a student has the tools to gain the correct answers to questions, but a relatively small amount of time in trying to get students to understand where the answers come from, and why we use the methods that we have chosen. These particular elements are critical for student understanding and the ability to apply skills to new and unfamiliar problems. Understanding these processes will benefit students' work with sequences in future mathematics, such as finding the total angle measures in a polygon, summation notation, and limits.

June 23

Grade 4, Mathematics Lesson Plan

Place: Showa City Oshihara Elementary School
Teacher's Name: Toshio Ohma

1. **Name of the Unit:** Pay Attention to Commonalities (Thinking with Diagrams)

2. **Goals of the Unit:**
   - Students recognize the merits of using diagrams that help them visualize the relationships between quantities clearly and easily in the context of word problems.
   - Students are able to solve word problems by paying attention to the difference between two quantities, a difference that results from splitting one quantity into two parts or moving one of the two parts to the other part.

3. **About the Unit:**

The problem of today's lesson is as follows:

> Riko and Kota split 60 sheets of origami paper to make paper cranes.
> Riko has to have 12 more sheets than Kota.
> How many sheets of origami paper will they each have?"

The objective of this task is to foster students' problem-solving skills by helping them understand/see the relationships of quantities involved in a story problem by reading the problem carefully and representing it using a line segment model.

At our school, we have been working on developing students' communication skills through English language lessons and activities developed under the school-based research theme, "Fostering Students' Ability to Thrive in a Global Society." When we develop students' communication skills in Mathematics, we keep in mind the descriptions of the basic educational direction of mathematics education in the 2008 Mathematics Course of Study:

> Mathematical thinking and expression play an important role in rational and logical thinking as well as in intellectual communication. … We will enrich the kind of teaching where students are taught to think systematically, in logical steps, by reasoning, and to understand the connections among words, numbers, algebraic expressions, figures, tables, and graphs. This kind of teaching will also allow students to learn appropriate usage, problem-solving, how to explain one's ideas clearly, and how to express and communicate one's ideas to others.

Active participation of students, such as explaining their own ideas to others and listening to understand their friend's ideas, is the connection to fostering students' communication skills. To develop lessons where the students participate actively, I pay attention and care about the "important questions" which are the students' own questions. That is, questions such as "I wonder if we could use something I learned before" and "I wonder if we could use this thinking to understand and solve other problem situations."

Nakamura (1989) identified and underscored different types of student questions; namely, questions that engage the following:

- a student's previous learning;
- other ideas different from what the student has done;
- the basis for mathematical reasoning;
- commonalities and similarities among solutions, ways of thinking;
- differences among solutions, ways of thinking;
- generalizations;
- expandability/application of reasoning; and the
- merits of a solution pathway.

The questions (wonderings) I care most about in this unit include:

- "If we show it [the relationship] using a diagram, what would the relationship look like?"
- "I wonder if we could solve it by calculating."
- "If we make the amounts of the two quantities the same, I wonder if we could solve the problem."
- "Is there any other way to solve the problem by making the two quantities the same?" and
- "Could we make Kota's quantity the same as Riko's?"

I would like to practice the following instructional moves or decisions:

- During the "grasping the problem" phase of the lesson, I would like to provide opportunities for students to split a quantity (60 sheets) freely into two quantities in order for them to understand the meaning of splitting in two. This activity provides an opportunity for students to understand that although a quantity is split into two, the sum of those two smaller quantities will always be the same (60 sheets). Depending on how a quantity is split in two, the two quantities that result could produce a difference (parts that are different in size). After students understand this, I will propose a difference of 12 sheets between the two quantities. Students will consider this difference of 12 and demonstrate their understanding by drawing a line segment model and using it to solve the problem on their own.
- At the start of the "independent work" phase (when students solve problem on their own), I will ask students to think about what mathematical expressions they need to establish. Then I will ask them to solve the problem freely.
- During the "comparing and discussing" phase, I will first select the incorrect solution that involves students adding or subtracting 12 sheets from the equally split number of sheets, 30 sheets ($60 \div 2 = 30$, $30 - 12 = 18$). When 30 sheets are split in two, each person will receive 30 sheets. Then, when Kota gives 1 sheet to Riko, the difference in the number of sheets between the two students is now 2 sheets. By discussing the wrong solution first, students may discover their misunderstanding and lead themselves to think about and reach the correct solution. During the discussion, I hope students will be able to discuss the misunderstanding not only verbally but also using the line segment model. These diagrams represent the relationship of quantities involved in story problems clearly and easily. Therefore, using diagrams in class discussions helps students see the merit of using this model. The visual clarity and simplicity of diagrams makes them useful for understanding and explaining quantitative relationships and solutions.

Next, I will turn the students' attention to the solution that starts with removing the difference (12 sheets) from the total number of the sheets (i.e., 60 – 12 = 48). After subtracting 12 sheets from the total number of sheets, Riko's number is now equal to Kota's number of origami paper sheets. Since 48 ÷ 2 = 24, both boys will have 24 sheets each. I would like to see the students use the line segment model to describe this process. Lastly, I will ask students to think about the method that made the number of Kota's and Riko's sheets the same.

- During the "understanding the problem" phase, students represent the relationship of quantities involved in the problem using a line segment model, think about establishing equations from the diagram, and explain the meaning of these equations using diagrams. In order to develop students' communications skills, I want to hear the students share their ideas with each other using terms they have been using in mathematics class, including the words introduced in this unit, such as "difference" and "make both quantities the same."

4. About This Lesson:

(1) Date and Time: June 23, 2017 (Friday)

(2) Place: Showa City Oshihara Elementary School, Grade 4 Class 2 Classroom

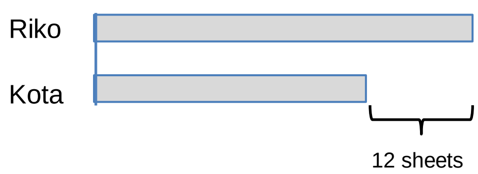
(3) Objectives:

- Students recognize the merit of using diagrams to solve word problems, because diagrams help them see the structure of problems easily.
- Students are able to solve word problems by paying attention to the difference of two quantities that results from splitting a quantity into two quantities/parts or moving one of the two quantities/parts to the other quantity/part.

5. About This Lesson:

Time	Learning Content (○ Anticipated Student Questions)	Instructional Points to Remember
15 min.	1. Grasping the task (1) Teacher pose the problem	
	Riko and Kota split 60 sheets of origami paper to make paper cranes. Riko has to have 12 more sheets than Kota. How many sheets of origami paper will they each have?”	
	• Kota gets 30 sheets, Riko gets 30 sheets. • Kota gets 20 sheets, Riko gets 40 sheets.	• In the beginning, the gray-shaded sentence will be covered with a strip of paper, so the students can experiment with splitting the origami sheets freely. Students confirm that the sum of the two sets of origami sheets is always constant (60 sheets).
	Riko has to have 12 more sheets than Kota.	

	• What do you think about the meaning of the sentence “12 more sheets than Kota?”
	  • Riko will have more than Kota.
	  • It is not easy tell.
	
	○ “I wonder if we show the problem with a diagram, what will it look like?”
		• Peel off the strip of paper to show the sentence, “Riko has to have 12 more sheets than Kota.”

	(2) Representing the problem situation using a diagram 	• Help students foresee that if they represent the problem situation with a diagram, they can solve the problem more easily.
	• The difference in the number of sheets between Riko and Kota is 12 sheets, so if we take 12 sheets away from Riko, the number of sheets for both of them becomes the same. • The difference of the number of the sheets between Riko and Kota is 12 sheets, so if we add 12 sheets to Kota’s number, the number of sheets for both of them becomes the same. ○ “I wonder if we could calculate and solve the problem.”	• Help students to become aware that both ideas (“if we take 12 sheets from Riko” and “if we add 12 more sheets to Kota’s”) make make the number of Riko’s and Kota’s sheets the same (equal).

5 min.	2. Solving the Problem on Their Own Anticipated Solutions: (a) First, split the 60 sheets equally among two people, then add 12 sheets to Riko's and subtract 12 sheets from Kota's number (a wrong solution) $60 \div 2 = 30$ $30 + 12 = 42$ $30 - 12 = 18$ <u>Answer: Riko will have 42 sheets and Kota will have 18 sheets.</u> (b) First, split the 60 sheets equally among two people, then add 6 sheets to Riko and subtract 6 sheets from Kota (a correct solution) $60 \div 2 = 30$ $12 \div 2 = 6$ $30 + 6 = 36$ $30 - 6 = 24$ <u>Answer: Riko will have 36 sheets and Kota will have 24 sheets.</u> (c) Subtract the difference of 12 sheets from 60 sheets in order to make the number of Riko's sheets the same as Kota's. $60 - 12 = 48$ $48 \div 2 = 24$ $24 + 12 = 36$ <u>Answer: Riko will have 36 sheets and Kota will have 24 sheets.</u> (d) Add the difference of 12 sheets to 60 sheets in order to make the number of Kota's sheets the same as Riko's. $60 + 12 = 72$ $72 \div 2 = 36$ $36 - 12 = 24$ <u>Answer: Riko will have 36 sheets and Kota will have 24 sheets.</u>	

20 min.	3. Comparing and Discussing (1) Discussing the incorrect solution method (a) First, split the 60 sheets equally among two people, then add 12 sheets to Riko's and subtract 12 sheets from Kota's quantity (a wrong solution) ○ The difference between Riko's and Kota's will be 24 sheets. ○ The answer is wrong. What was the cause of this mistake? ○ "I wonder if we could solve this problem by making the number of sheets of Riko's and Kota's the same." (2) Review the diagram • When we move 1 sheet from one person to the other person the difference in the number of sheets becomes 2 sheets, not 1 sheet. • Therefore, when we move 6 sheets the difference will be 12 sheets. • We can use equations to show what is happening: 30 + 6 = 36 and 30 – 6 = 24 --- (b) method (3) Discuss other solution methods ○ "I wonder if there are other ways to make the number of sheets of Riko's and Kota's the same." Subtract 12 sheets to make the number of Riko's sheets the same as Kota's. --- (c) method • If I use the equations to explain … • If I use the diagram to explain … ○ "I wonder if there is a way to make the number of sheets of Kota's as same as Riko's. Subtract 12 sheets to make the number of Kota's sheets the same as Riko's. --- (d) method • If I use the equations to explain … • If I use the diagram to explain … • The answer became as same as when we make the number of sheets of Riko's as same as Kota's.	• Help students be aware that if they split the total number of sheets into two equal quantities, then subtract 12 sheets from one part and add 12 sheets to the other part, the answer will be wrong. Lead them to look at the diagram carefully and rethink this incorrect solution. • Help students to recognize that when 1 sheet was moved from one person's quantity to the other person's, the difference in the number of sheets become 2 sheets. • Be sure to confirm with students that when the difference of 12 sheets is subtracted from the total number of sheets, the numbers of sheets for Riko and Kota become the same using not only equations but also using the diagram. • Be sure to confirm with students that when the difference of 12 sheets was added to the total number of sheets, the numbers of sheets for Riko and Kota become the same … using not only equations, but also using the diagram.
5 min.	3. Looking back and summarizing (1) Solving an application problem (2) Writing a "reflection of learning" • I could make two quantities the same by adding or subtraction the difference. • It is easier to think about the solution method using a diagram.	• Using the "reflection of learning", assess students' learning and the lesson.

6. **Evaluation of the lesson:**
    - Did the students understand the merit of using diagrams that clearly show the structure of problem situations?
    - Were the students solving word problems by paying attention to the difference of two quantities that resulted from splitting a quantity into two quantities/parts or moving one of the two quantities/parts to the other quantity/part?

7. **Board Planning:**
    The board plan will be added here later.

Oshihara Elementary School, School-Based Lesson Study

Observation and Discussion Points

< Focal Points of Student Observation > (Record in chronological order using the 2nd page)

- How students were solving the problem on his/her own (grasping problem, solution method(s), writing in notebook)
- How the whole class discussion was conducted (understanding other students' ideas, change in thinking, speaking, writing in notebook)
- How students were reaching to the goals of the lesson (speaking, writing in notebook, solving application problem)

< Focal Points of Discussion >

1. Were the students trying to solve the problem by paying attention to the difference between two quantities, difference that results from splitting one quantity into two parts or moving one of the two parts to the other part?

2. Do the students recognize the merits of using diagrams that help them visualize the relationships between quantities clearly and easily.

3. Was the lesson appropriate for supporting the students to develop their communication skills?

< Lesson Observation Sheet >

Time	Learning Activities	How students were learning	Other notes

Project IMPULS Lesson Report
23 June 2017 - Grade 4 - Thinking With Diagrams

(Annotate with pictures, quotes, student work examples, board work etc.)

Report created by: Marty Garrett, Nicole May, Kim Towsley, Stephanie Moore
Name of Lesson: Pay Attention to Commonalities (Thinking with Diagrams)
Date of Lesson: 23 June 2017

What are the primary lesson goals?

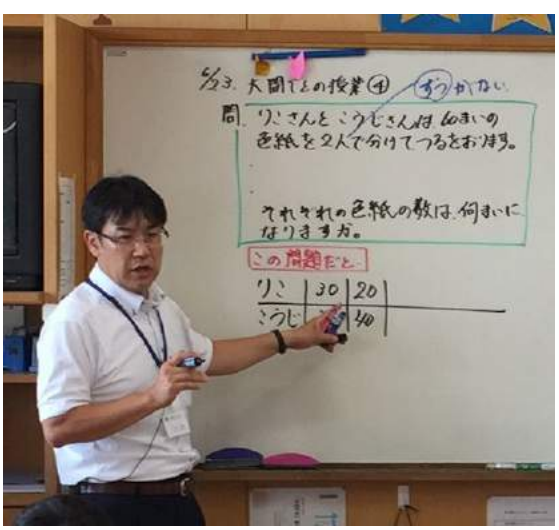
- Students recognize the merit of using diagrams to solve word-problems, because diagrams help them see the structure of problems easily.

- Students are able to solve word-problems by paying attention to the difference of two quantities that results from splitting a quantity into two quantities/parts or moving one of the two quantities/parts to the other quantities/part.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)? We were unable to find which lesson this was in the unit on the lesson plan, but there was an expectation that students should be somewhat familiar with tape diagrams.

Summary of Lesson

Start & End Time	**Lesson Phase**	**Notes**
1:53pm 1:55pm	**Introduction, Posing Task** 	**Strategies to build interest and to connect to prior knowledge** T: starts lesson by asking students what they think the bag is inside? S: say sweets Then gets origami cranes out and asks students what it is. Students know bag is from the popular Nakamura Dept. Store and they respond that they are paper cranes. T: asks how many of the students know how to make an origami crane Mostly all raise their hand T: places two on the board

		T: I want to talk about folding an origami crane
1:57pm		T: explains that two friends were folding origami paper and begins writing math story on the board T: reminds students that it is the 4th lesson with him, and when he writes on the board that they should be writing too. S: begin writing T: prompts S to make space for two lines (later to be filled in with math details)
2:00pm		T: has S choral read question: Riko and Koji were folding origami paper. There are 60 pages in total. T: asks how many they think riko will have. Several S say 30.
		T: then writes their response on the board and creates chart riko 30, koji 30 T: asks if there is another way to show S: say no other way T: asks if they notice something in the problem One S says “it just says to split 60, nothing about it being equal, just divide in two S: the equal is not there

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2:08pm</td>
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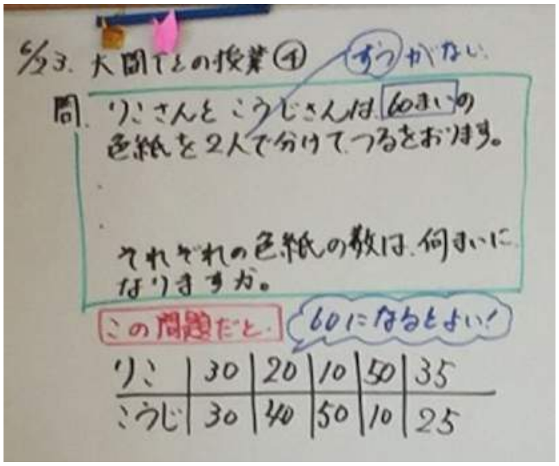

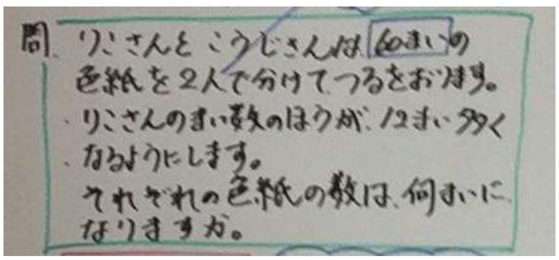

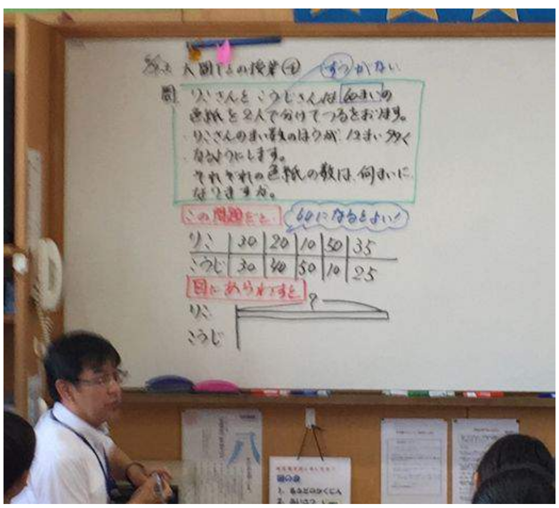
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Students then respond with 20/40, 10/50, 50/10
For 10/50, 50/10 T asks why do you think is okay to show this? S hesitate, then T Says the total becomes 60.
One S says 35/25. Then T asks “can you say what he said?”
S: says “as long as 60 sheets, it is okay.
T: highlights 60 in the question
T: asks if there are any more ways
S: says 60/0, another student disagreed and said that then you aren’t splitting 60.
T: didn’t write on chart

T: moves to two spaces to fill in remaining portion of math problem/story.
Riko has to have 12 more sheets than Koji

S: choral read problem twice
T: says “So, what should we do?”
Pointing to the chart: Can anyone tell immediately? Let’s look here at the chart. What do we need to know?
S: maybe we use a diagram
T: says, “If we don’t know the answer, we show it with a diagram.”
T: begins to create a tape diagram and students write it in journal.
Puts ? mark on length of riko, and asks student to draw length of koji.
S: comes to board and draws tape diagram too long, other students help correct to show less than riko
T: asks what S think, they say it is good
T: say why do you think it is good and asks where can they find the info in the problem.
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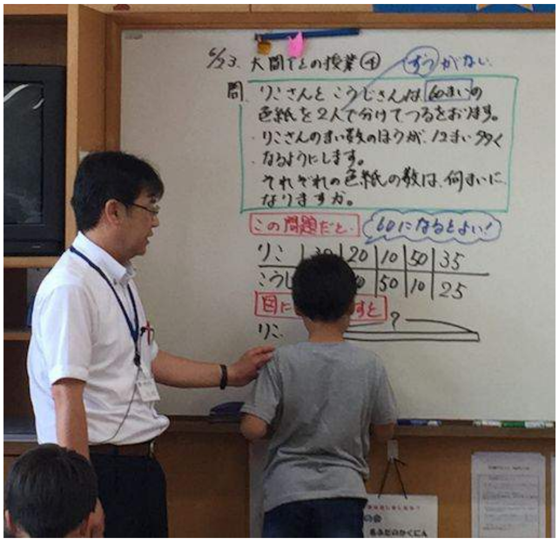
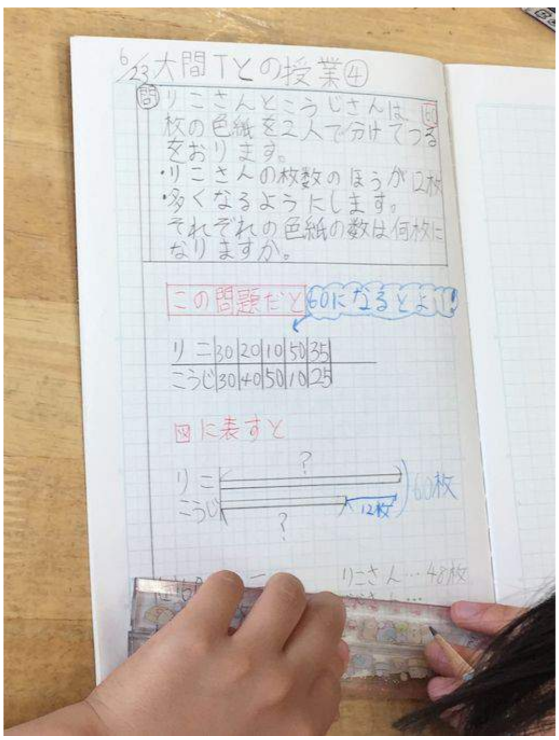
		T: then moves back to diagram and says, where can you see the “plus 12” then draws the box in as the 12 being the difference between riko and koji.
2:15pm	**Independent Problem Solving** 	**Individual, pairs, group, or combination of strategies** · **experience of diverse learners** · **teacher’s activities** T: asks students to solve for how many riko and koji have. S: begin working T: goes around checking S Student 1 Notebook

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<td>Student 2 Notebook</td>
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2:22pm</td>
<td>Presentation of Students' Thinking, Class Discussion


</td>
<td>Student Thinking/ Visuals/ Peer Responses/ Teacher Responses

T: counts 12 students have answers.
Then begins documenting the different answers for riko: 42, 48, 36, 38
Then asks to defend answers, tell me the math expression you used.
S: 60/2=30, 30+12=42, 30-12=18
T: "something's not working here"
Completes diagram with +12 on riko and -12 for koji to reveal difference being 24
T: "so what should we do?"
One S says let's take 10 instead of 12
One S says "i think it's 6"
T: says are you really sure?
Why is it if you do 6?
S: if you do 6+6=12</td>
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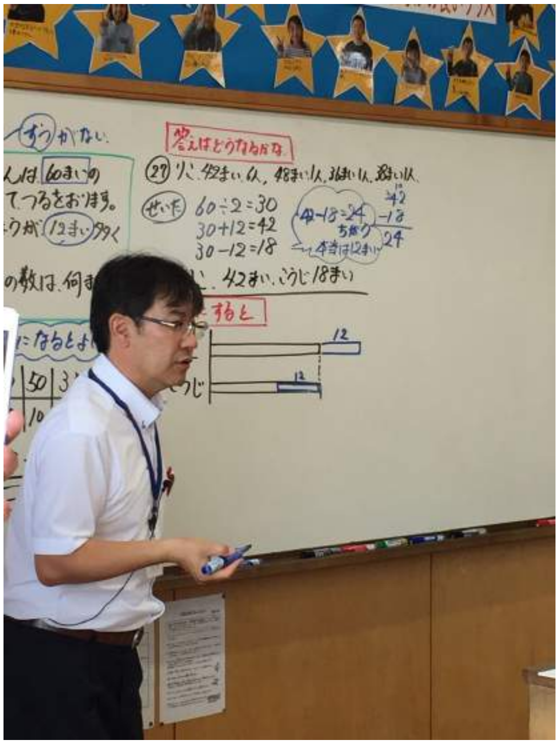

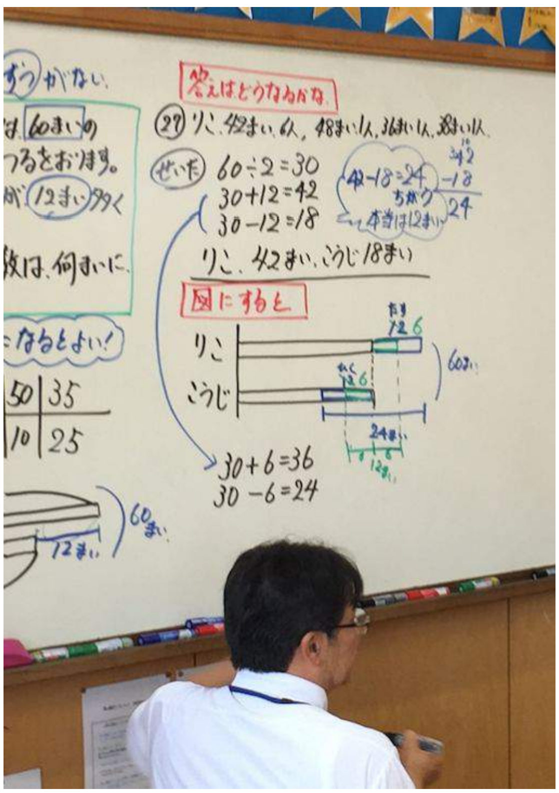

</td>
<td>let ‘s fix
60/2=30, 30+6=36, 30-6=24
T: “let’s see if that’s right” “what is the difference between 36&24?”</td>
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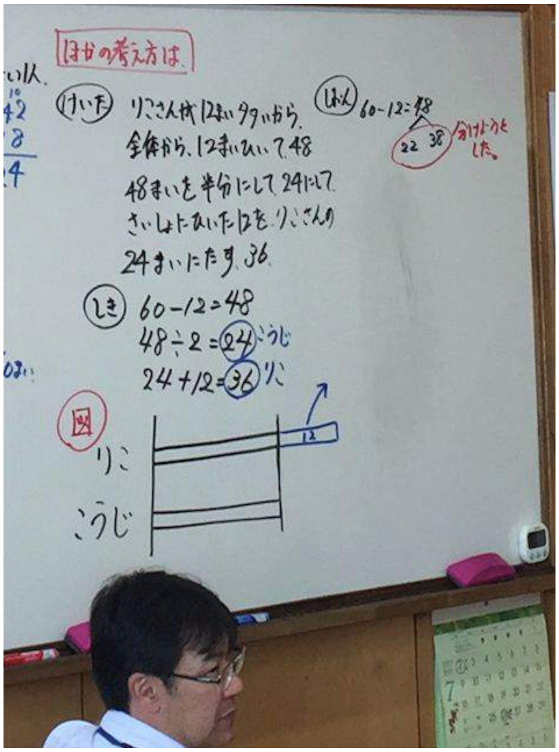
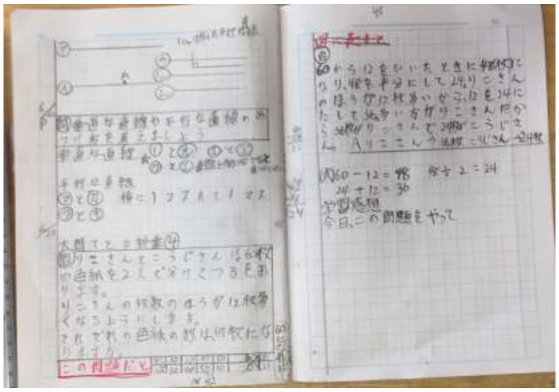
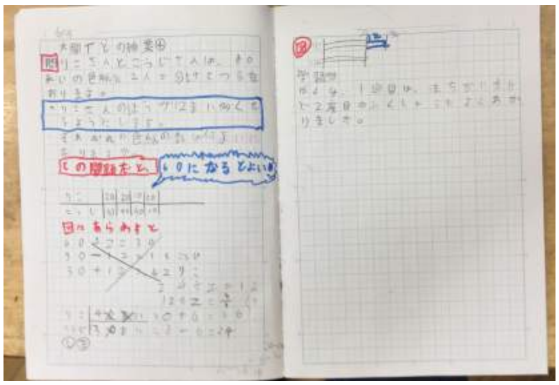
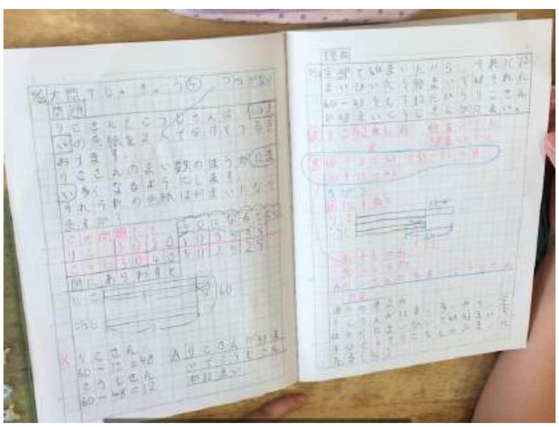
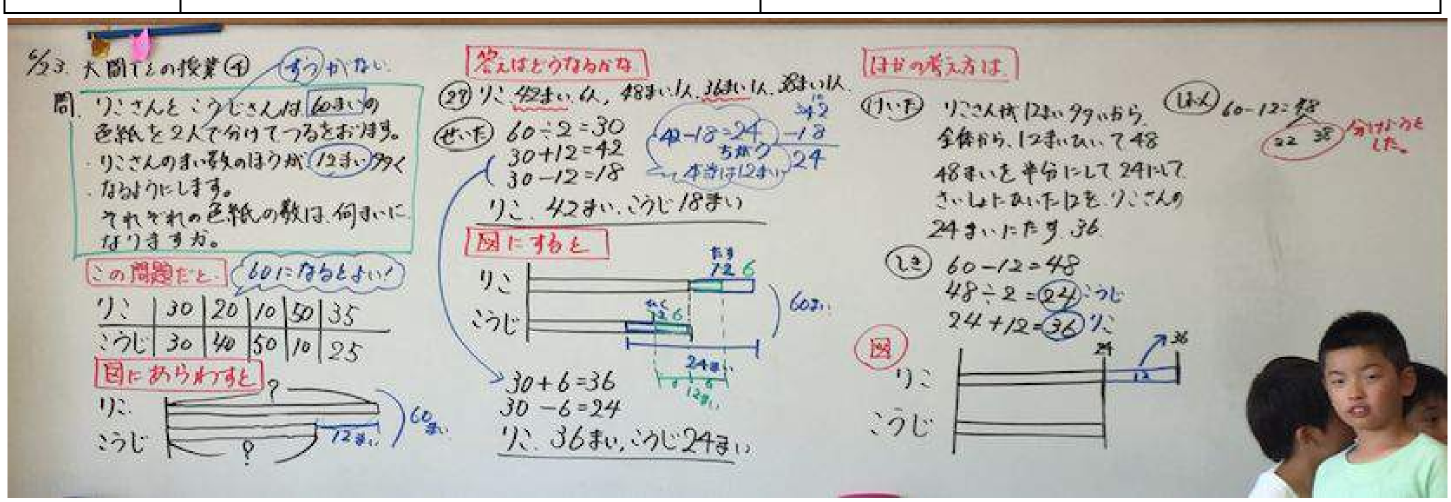
2:50pm		Other student answers: expressions with words: Riko has 12 more that koji, 60-12=48, then i divided 48 by 2 and got 24 for koji. Riko has 12 more so i put 12 more back to riko, she has 36. Another S: Similar 60-12=48, then split 48 into 22 & 38 T: let’s explain with math expression: 60-12=48 48/2=24, 24+12=36 T: “can anyone else explain? I want to draw diagram” T: does double tape diagram

2:55pm	**Summary/Consolidation of Knowledge**   	**Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals** T says: I want you to write what you learned today in your notebook. S1: I made a mistake at first, but after fixing it, I understand S2: I didn't understand diagram, but I listened to other ideas so I now understand S3: I made a mistake at first, but I got it the second time. S4: So many guests, and I was nervous, but I did Ok.
3:00PM		We are out of time, let's doing closing ritual then continue writing.



<u>What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?</u>

Within our group, we focused our discussion around several parts of this lesson. We had questions even before the lesson was taught as to whether the teacher introducing the tape diagram at the beginning of the lesson was going to steer students in a particular direction and it later seemed to become a problem during the lesson. There was discussion during the post-lesson commentary, as well as from members in our IMPULS debrief, as to whether there should have been a more open-ended invitation for students to solve the problem using whatever strategy they found helpful (manipulatives, table, guess and check, etc.). For example, the teacher could have used origami paper for the students to find the difference of 12 through trial and error. This strategy would have enabled students to construct meaning on their own, and then from there the teacher could have created the table with the group based off of how they split the sheets. If the goal of the lesson was to show that diagrams can help you solve word problems, then students needed to be able to come up with diagrams on their own since they had already had several math experiences using diagrams. It was brought up during the final commentary that the student who did NOT copy down the teacher's diagram was able to think more openly and clearly and thus solved the problem on his own. Different strategies could have been connected at the end of the lesson, and students could have discussed the benefits of using a diagram strategy to solve this problem.

Additionally, there was discussion about the table used at the beginning of the lesson when the 60 sheets were divided between the two students (without the additional caveat of a difference of 12.) Tables are a great way for students to organize information, but, again, the table didn't go far enough in helping student understanding. The students might have been helped by adding another row that showed that the two numbers added up to 60, and a fourth row that showed the first and second numbers with a difference of 12 (this could also be put into a separate table.) These additional rows may have made a difference in many of the students making sense of what the problem was asking. It was also noted by a commentator that the teacher could have used a number smaller than 60.

There was also discussion about what to do when students are "stuck". When the problem with the difference was given, students tended to do one of two things. Some of them split the 60 in half and then subtracted 12 from one student and added that 12 to the other student (difference of 24). Most of the rest of the students did 60-12, and then couldn't seem to figure out what to do from there. This was an opportunity for the teacher to have students express in words what they did not understand, and then for him to go back to the table/diagram to make use of the students' ideas. Was there a teachable moment there to stop the lesson, and ask some questions to help move the students? Or, was there a chance to have students do some comparison of their answers to see if they could ask think about the problem differently?

The final commentator also noted that is was important for the teacher to be able to make use of students incorrect answers and to value their thinking. For example, many students did 60-12=48 and 30-12=18. This was another opportunity for the teacher to take these incorrect answers and go back to the table/diagram to help students figure out whether or not they were correct and what they might do next. Here, the commentator stated, was an opportunity for students to think about how to overcome challenges.

Another topic that was noted was the amount of "teacher talk" that occurred during the lesson. In what we can presume was an effort to finish the lesson and get to the summary, the teacher took over the end of the lesson, and pushed forward the diagrams, trying to show how to get the answer using them. It was raised in the expert commentary that the teacher "stopped listening" to the students. When the students got to the point in the whole group discussion where there answer of 18 and 42 showed that the difference was 24 and not 12, it could have been a great learning opportunity for the students to have worked in pairs or groups to figure out why their answer didn't work. They could have discovered the generalization about moving one (sheet of paper) actually creates a difference of two (etc.). Students might have been ready then to see how this would look on a diagram (if one of the students had used or could use it.) At the end of this lesson, it did not seem from their notebooks and body language as if they understood the diagram showing a difference of 12 by splitting it into 6 and 6. It was interesting to note that several of the translated comments from the students' summary said that they now understood what to do, or that they made a mistake the first time, but they got it right the second time. But it was unclear to us, as observers, if they really understood the problem or the diagrams. There could have been some great follow-up questions or problems that would see if students had understood the math involved. It would be interesting to see if students could test out whether all numbers would work in this situation (only even numbers with the stipulation of 60 sheets and a difference of 12), or to see if they could rewrite a situation where odd numbers could work.

<u>What new insights did you gain about how administrators can support teachers to do lesson study?</u>

It is extremely valuable when administrators are part of the lesson study process- not just at the research lesson, but in some of the planning as well. Also, if they can provide time and space for teachers to meet, it allows this process to happen smoothly, as well as places importance on the process. This is a research lesson, and the formative assessment from this lesson shows that students need more work with diagrams, as well as understanding how these diagrams connect to other strategies previously learned. Administrators can continue to help teachers connect across the grades to see where this can be encouraged and strengthened. Was this difficult only for this class? Or this grade? What did it look like in third grade? These are questions that administrators can help deepen by providing common planning time, attending grade-level meetings, co-creating a vision/research theme with the teachers for the school year, providing substitutes for public lessons, encouraging teachers to have an open door policy for public lessons and making lesson study a valued form of professional development. Finally, having administrators who support lesson study, sets the tone for the entire school. Administrators are leaders of their school sites, and their support is needed to effectively and successfully implement a school-wide lesson study program. When administrators value the work, time, and commitment that teachers are putting into their lessons, teachers will feel safe to take more risks in their teaching, and in turn they will be able to deepen their practice to most effectively meet the learning needs of their students.

<u>How does this lesson contribute to our understanding of high impact practices?</u>

"Messy math can create joy in the solution"- expert commentary

It is important to remember that we must be careful to start with student understanding, not our assumption that they know or will use a certain strategy. We need to keep open-ended questions where students can enter in. It is also important to stay with the lesson instead of remain steadfast to the lesson plan. Deviation from the lesson plan is needed many times, which is why it is important to be thoughtful about our “teacher moves” during the lesson. We know where we are headed, but sometimes the road winds in a different direction, and we choose our paths throughout the lesson. There are many roads that will still lead to what we hope the students will understand. But it is really important that the needs and understanding of the students drive the lesson- not the lesson plan. Also, it seems that teachers sometimes make “teacher moves” that avoid the messiness. Although this is understandable, it also cuts off the “deep math” that students can struggle through. Math is messy when we do it right. The “ clean up” comes when students productively arrive at new understanding!

Lesson study is about understanding what students are doing and what they are showing the teacher that they know, or do not know during the lesson. This lesson showed us that the teacher must make a continued effort to look at student work during the lesson, and then shift their instruction accordingly. During this lesson, many students were stuck on the initial expression 60-12=48. Students sat for several minutes and they could not move forward with the problem. This was an opportunity for the teacher to allow students to work with partners, a group, or to go back to the table in order to push their thinking, either by adding the difference row, or creating another table that showed 60-12.

Patterns of participation widely varied between male and female students. In a class of ____, three girls spoke a total of four times and 15 boys spoke a total of 24 times.

Group Questions we had before the lesson:

1. Front Loading - If the teacher puts the tape diagram on the board, does that lead them towards a particular strategy?
2. How are students using tape diagram?
3. What voices being heard? Girls/boys/teacher
4. How is the discussion being facilitated? How are they bringing the diagram/equation into the discussion?
5. Summary/reflection: how does the teacher make sure the goals of the lesson were met and represented in the student work/summary/reflection.

Report of Elementary Mathematics Research Group
Attached Elementary School of Yamanashi University

Theoretical Foundations

YAMAGUCHI, Kuniyuki
NOMURA, Miyoko
OKARI, Hidekazu

1. Research Themes

Devising lessons in which students connect their "friends' questions" and experience "I got it!"

2. About the research theme

(1) Rationale for the theme

The research theme for our school is "students who continue to learn together with their friends: Through lessons in which students feel the value of learning." In this theme, the idea of "together with their friends" is positioned as a strategy to help students "continue to learn." It is one of the characteristics of the whole class instruction that students come to share their "friends'" ideas. Within the team of learners including self and friends, each student can make use of his or her own strengths and deepen the quality of individual learning.

Considering the school's research theme, the mathematics research group has set its own research theme for the first year of the research cycle as devising lessons in which students connect their "friends' questions" and experience "I got it!". The team has investigated learning tasks that can generate the "questions" that can help students "experience 'I got it!'" as well as evaluating individual student's learning through their notebooks and journal entries.

Through our research lessons we have learned that by discussing "friends'" incorrect or incomplete and difficult to understand answers for the "questions" arising from the learning tasks, they generated further "questions" more easily. We have also learned that by trying to interpret and understand each other's ideas, they developed the understanding of the content and experienced "I got it!". We felt that we were able to design lessons that reflected their current understanding and natural ways of reasoning through careful examination of their notebooks and journal entries.

On the other hand, the following points have been identified as future questions to be investigated. First, we need to examine more carefully ways the learning tasks are posed, teacher's key questions, and students noticing all of which trigger students "questions." It is also necessary for us to think carefully about anticipating students' responses so that teachers can flexibly respond to the actual responses in the lesson as well as the roles of teachers in a student-centered lesson.

In a lesson, "questions" students develop vary significantly. However, how those questions are connected depends on teachers and students themselves, and the process is ambiguous. In addition to the learning tasks that generate "questions" which was the focus on the first year, in the second year of the cycle, we decided to investigate how teachers can connect students "questions" and which "question" is the question that must be pursued. Through this examination, we feel that we can actually design lessons in which students connect their "friends' questions" and experience "I got it!".

Therefore, the mathematics research team has decided to continue with the research theme from the first year, "Devising lessons in which students connect their "friends' questions" and experience "I got it!" as the theme for the second year.

(2) About "friends' questions"

The ideal lesson the mathematics research team aims at is the lesson in which students construct mathematics on their own. In such a lesson, students engage in the mathematical task independently and compare and contrast their own ideas with those of their friends'. Through this careful examination, they generate a better approach to the learning task. The engine in such a lesson is the "question," and the "question" represents the pathways of students' reasoning.

There are many different types of "questions" such as the following (Nakamura, 1989):

- Question about prior learning
- Questions about alternative approaches
- Questions about the reason behind an idea
- Questions about commonality and similarity
- Questions about differences
- Questions about generalizability
- Questions about extendability
- Questions about usefulness

Different questions arise depending on the contexts. For example, during the independent problem solving stage of a lesson, students may question, "Can I apply the idea I used previously?" In the whole class discussion stage of the lesson, they may ask questions about rationale, commonality, or generalizability through questions such as "Why can we say that?" "Is there anything in common?" or "Can we use that method always?" The whole class discussion provides the opportunity for students to bring together their own ideas generated during the independent problem solving time and through critical analysis raise the level of their own mathematics. In this way, students' own "questions" become those of the fellow learners.

(3) About connections

In a mathematics lesson which nurture students' mathematical reasoning and disposition, the whole class discussion stage may involve the following 3 phases: (1) interpretation, (2) consolidation, and (3) application (Nakamura, 2015).

Phase (1) is the time in which students interpret their "friends'" ideas and understand the reasoning behind them and their characteristics. Phase (2) is the time where students identify the commonalities among various ideas generated in Phase (1) and consolidate them. It is the time in which students generate the new idea together with their "friends" that didn't come up while working independently. Phase (3) is the time in which the consolidated idea in Phase (2) is applied in other situations.

"Connecting questions" means to help students continuously questions from one phase to the next. For example, students may ask "Why does this idea work?" in one phase, and after interpreting and understanding each other's ideas, they will ask "Is there any commonality among these ideas." They will then ask, "Can we use the idea in other situations?" In this way, students will ask questions continuously toward the resolution of the learning task.

(4) About experiencing "I got it!"

Through our previous research, we have proposed the foundations for student centered mathematics lessons (Figure 1, Research Report, 2004). By clarifying the curricular flow, students' prior learning becomes more explicit, and what needs to be taught explicitly and what students can construct on their own become apparent. Building on this foundation, and considering "questions" as the pillar of a lesson, we utilized our research findings in our lesson designs as well as helping students reach "I got it!" in those lessons.

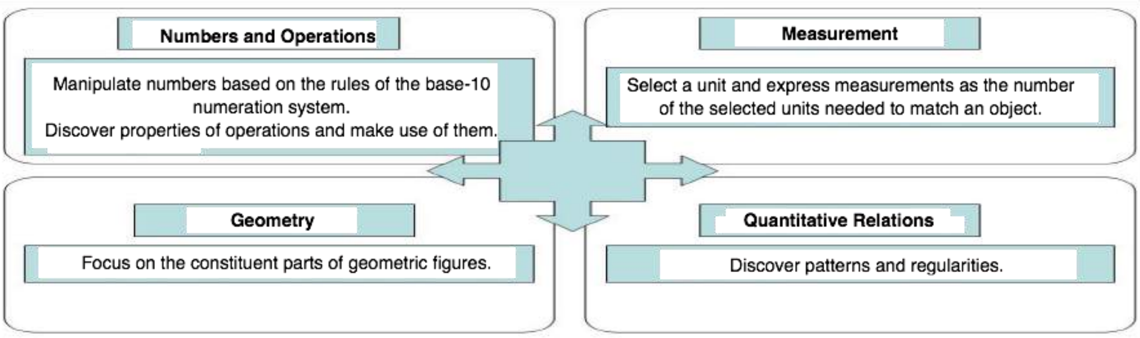


Figure 1 Foundations for mathematics learning

Let us think a little further about the idea of "I got it!". If students were given a procedure (such as the formula to find the speed) and obtained the correct answer using the procedure, that is "I did it" and not "I got it!". When students ask "Why can we do that?" and understand, for example, the meaning of dividing the distance by the time, students have reached the point of "I got it!". Or, when students ask "Is there any commonality or similarity/difference" among the ideas that appear to be very distinct, students will "get" the commonality (similiarty/difference) and discover a new relationship. The mathematics research team has defined "I got it!" to be the understanding of the relationship emerging from various "questions. Moreover, we consider that students experience "I got it!" when "questions" are connected toward the resolution of problems.

3 Relationship to the school research theme

(1) About the concrete image of "lessons in which students feel the value of learning"

In the school research report, the following 3 types of lessons are described as "lessons in which students feel the value of learning."

1. Lessons in which students develop expectations and questions toward learning
2. Lessons in which students are engaged in the task independently, and
3. Lessons in which students realize the sense of achievement and satisfaction.

The mathematics research team consider that students feel "the value of learning" when they experience the usefulness of mathematics. About the usefulness, *Teaching Guide for Elementary School Mathematics* states as follows: "There are usefulnesses that are included in the knowledge and skills related to numbers, quantities and geometric figures. There also usefulness of mathematical reasoning, judgement, and expressions. These usefulness may be characterized as, for example, utility, simplicity, generality, precision, efficiency, expandability, and beauty." To experience the usefulness of mathematics and make it their own means for students to make use of what they have learned previously, to reason based on their prior learning, and to generate new mathematics based on what their prior learning. Students will discover the usefulness of mathematics as they use their own and others' ideas to devise solution approaches, implement the approaches, and reflecting on the results. This in turn will generate new "questions, and motivate further learning. By repeating this cycles, students will experience the usefulness of mathematics and make it their own. The mathematics research team considers "lessons in which students feel the value of learning" are lessons in which students experience "I got it!" (see Figure 2).

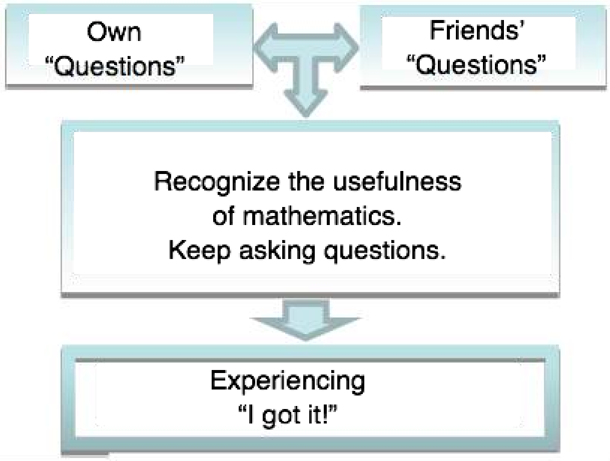


Figure 2 Framework for experiencing "I got it!"

In the past few years, our research has focused on "questions" as we aim at students' "I got it!". We believe that our goal, students' "I got it!" can be nurtured in student centered, teaching through problem solving. In such lessons, "questions" play an important role.

"Lessons in which students feel the value of learning" are possible through continuous questioning. "Questions" in such lessons must be about mathematical reasoning, and they must be appropriate for solving the given task. The mathematics research team considers such "questions" as "questions" that must be asked.

In the school research report, the following 4 dispositions/abilities that are necessary to realize lessons in which students feel the values of learning.

1. Dispositions/abilities that are skill/procedural.
2. Dispositions/abilities that are cognitive.
3. Dispositions/abilities that are affective.
4. Dispositions/abilities that are social.

In order to achieve lessons in which students construct their own mathematics that our research team aims at, we have developed the following framework which incorporate these 4 dispositions/abilities have been matched up with the stages of teaching through problem solving as shown in Figure 3.

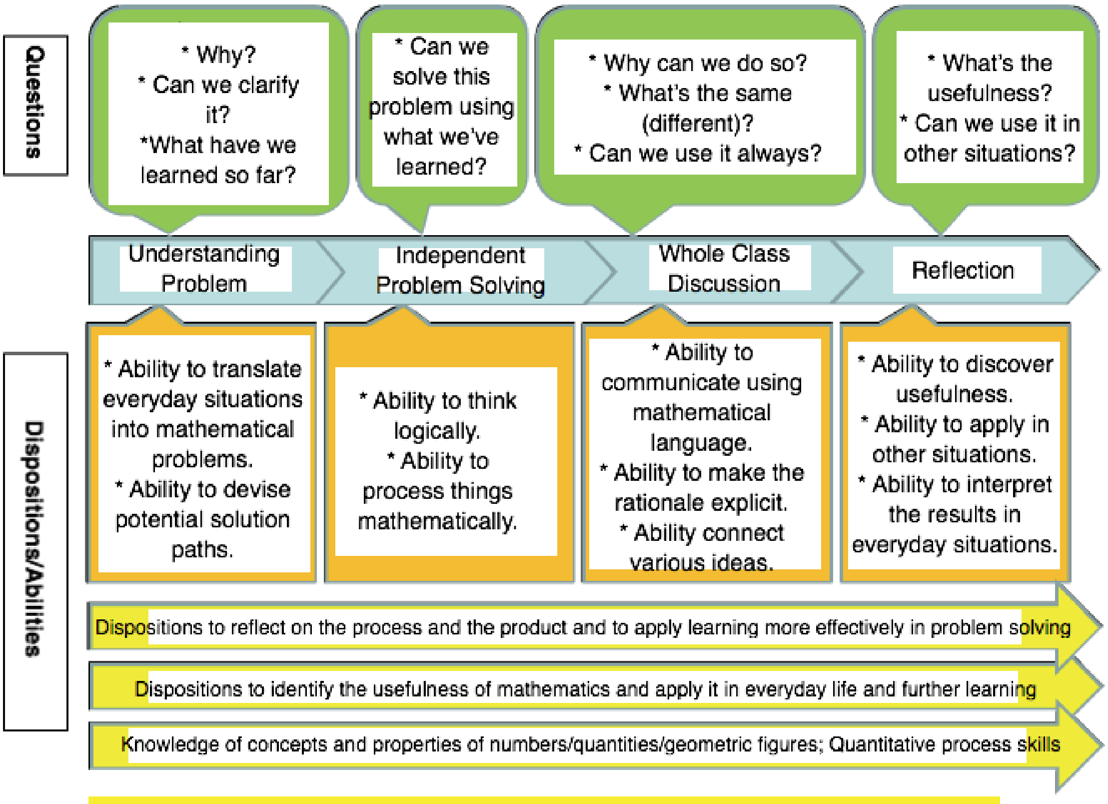


Figure 3 Learning process in lessons in which students can experience the values of learning

These dispositions/abilities do not develop in a single lesson but they are nurtured throughout a unit and throughout the year.

From the learning process depicted in Figure 3, we can see that the relationship between "questions" and the 4 dispositions/abilities are inter-related. In the Understanding Problem stage, students will convert everyday problems into mathematical problems by asking questions such as "Why is it?" and "Can we clarify?" Moreover, by asking "What have we been learning?" students devise a plan of attack and take the first step of problem solving. In the Independent Problem Solving stage, students may ask "Can we use what we learned to solve this problem?" and begin to reason logically. However, it is not always possible to have own approaches, and what is important in this stage is the existence of their "friends." In the Whole Class Discussion stage, students can compare and contrast their own ideas with their "friends'" ideas to generate an even more sophisticated solution strategy and approach the goals of the lesson. At this stage, students may ask about the reason behind an idea, commonality, similiarity or difference, and generalizability. In the Reflection stage, students may ask about the usefulness or extendability of the ideas and apply what they learned in different problems or in everyday situations.

What undergird this learning process are students' "dispositions to reflect on both the process and the product of their learning and apply them to further problem solving, "recognition of usefulness of mathematics and dispositions to apply what they learned in their everyday life," "understanding of concepts and properties of numbers, quantities, and geometric figures," and "skills to process mathematically."

Therefore, we believe that the dispositions/abilities to think mathematically that the team aims to develop in mathematics lessons can be promoted through lessons that generate "questions" continuously.

4 Focus of research (Points of emphasis in Year 2)

(1) Teaching strategies necessary for " lessons in which students feel the value of learning"

a. Devising lessons in which students have questions and the sense of expectations

- Setting up the learning task that generates "questions"

In mathematics, instruction is based on a coherent curriculum flow. Therefore, it is possible to nurture students who can construct new knowledge on their own based on what they have previously learned. In order to devise lessons in which students can construct mathematics on their own, teachers need to examine the values of the instructional materials as well as various ways each task may be solved. Nakamura (1989) suggested that a task/problem needs to have the following characteristics to generate questions:

  - the task/problem can be solved independently using their prior learning
  - the task/problem can generate conflict/harmony and dilemma/resolution
  - the task/problem that generate common challenges
  - the task/problem that leads to a novel challenge
  - the task/problem that can be solved in many different ways or with a variety of solutions

When students encounter such a task, "questions" are generate and lead to additional "questions," and students' dispositions/abilities to think mathematically are nurtured. We believe it is also critical for teachers to think about their students' current understanding along with these characteristics in order to have a productive mathematics lesson.

b. Devising lessons in which students are engaged independently and with each other

- Investigate "questions" that must be asked
- Anticipate how a series of "questions" may be generated

"Questions" each student generates will vary. In a lesson, we must gather those "questions" and think about "friends' questions." What needs to be discussed in the lesson should be "questions" that must be asked. We need to examine the goals of the lesson, what prior understanding students bring to the lesson, and how the topic is related to what students will be learning in the future through *kyozaikenkyu*.

Moreover, as discussed in section 3(1) above, through a series of continuously raised "questions" students can experience "I got it!". As we devise a lesson, we must anticipate how such a series of "questions" may be generated so that we can pull those "questions" out of our students. It is important for teachers to grasp students' current understanding based on their notebooks, reflective journal entries, and the record of blackboard writing to anticipate students "questions." It is inevitable that "questions" a teacher anticipates do not match actual "questions" students may develop. However, by having a good grasp of students' current understanding, the teacher can adjust "questions" and respond flexibly. We believe that when teachers can go alongside of students' own thinking and respond flexibly, students can become more independent learners.

c. Devising lessons in which students can experience the sense of achievement and satisfaction

- Record "questions" on blackboard and reflect on the learning
- Secure time for students to write reflective journal entries

By recording students' "questions," what is the current focus and what must be resolved become clear. Moreover, by recording students' independent and collaborative work toward the solution of the task, students can reflect on the problem solving process and what they learned may also become clear.

In addition, by setting aside time for students to write their reflective journal entries, students can reflect on what they learned in the lesson. By writing down what they thought about during the lesson and how they responded to other students' ideas, students can grasp the state of their own learning and re-examine their past learning. Through reflection, students can see how they have changed and realize the sense of achievement and satisfaction.

(2) About evaluation processes to assess individual students' quality of learning

In order for us to grasp students' understanding and conduct lessons that align with their current thinking, we make use students' notebooks and their reflective journal entries and analyze their each student's strategies and learning.

There are 4 phases in students' reflective journal entries written after a lesson (Nakamura, 2002).

First Phase: Students write about their feeling such as "it was fun" or "I want to study more." There is little specific ideas related to mathematics and it can be written after lessons in other subject areas.

Second Phase: Students begin to write about specific mathematical ideas and what they thought about them. They begin to write more about their own ideas.

Third Phase: Students write about their thought on other people's ideas. Other students' names begin to appear in their entries.

Fourth Phase: Students reflect and re-examine their own ideas. They reframe the tasks, and their attitude to improve their ideas mathematically becomes more evident.

In the prior studies, we identified the inability for teachers to flexibly responding to students' ideas as one of the remaining issues. We hypothesize that this is in great part due to the fact teachers didn't quite grasp students' understanding clearly. By analyzing students' solution strategies and the phases of students' reflective journal entries, teachers can ask, "When this task is given to this student, which prior learning will he/she use to solve it?" or "Based on the history of this student's learning, which strategy will he/she use to tackle this task?" In this way, teachers can grasp students' current understanding more accurately and conduct lessons that can align with their current ways of thinking.

References

Omitted as all of them are in Japanese only.

Grade 3 Mathematics Lesson Plan

June 24
G 3

Date: June 24, 2017
Yamanashi University Attached Elementary School
Grade 3, Classroom 3 (35 students)
Teacher: NOMURA, Miyoko

Mathematics
Division with Remainders

1 About the Unit

"Understanding" and "doing correctly" are critical considerations in the elementary school mathematics teaching and learning. It is important that we teach mathematics in such a way that students can sense they come to undersatnd and become able to carry out procedures correctly. In addition, we we look ahead, another important idea is that students can "use" mathematics. What is needed in today's mathematics education is to nurture the disposition, "I can use mathematics. I am going to use mathematics," in our students.

When we think about the question of how we can teach mathematics so that students will feel that they can "use" mathematics, we realize that one critical idea is for students to be able to understand what they learned in mathematics classrooms mean in the real world. In other words, it is critical that students can interpret mathematical results in the real world. About this process of interpreting mathematical results in the real world situations, Miwa (1983) labeled Mathematical Modeling Process and represented in the diagram shown in Figure 1. The step of interpreting a mathematical result in the real world in this Mathematical Modeling Process is evident in the topic of today's lesson, interpreting the quotient and the remainder and reach the answer that is one more than the quotient. In this lesson, students will be asked the question, "How many boats are needed if 23 students are to ride 4-person boats?" Students will calculate "23 ÷ 4" and obtain the answer of "5 remainder 3." Then, they must interpret the result of the calculation and come up with the answer of "6." This is the situation in which students understand that the results obtained from calculations must be interpreted in the problem contexts and come up with the answer that is different from the calculation results. By tackling this type of situations that are challenging to students and helping them understand it, we want students to feel that they can "use" mathematics.

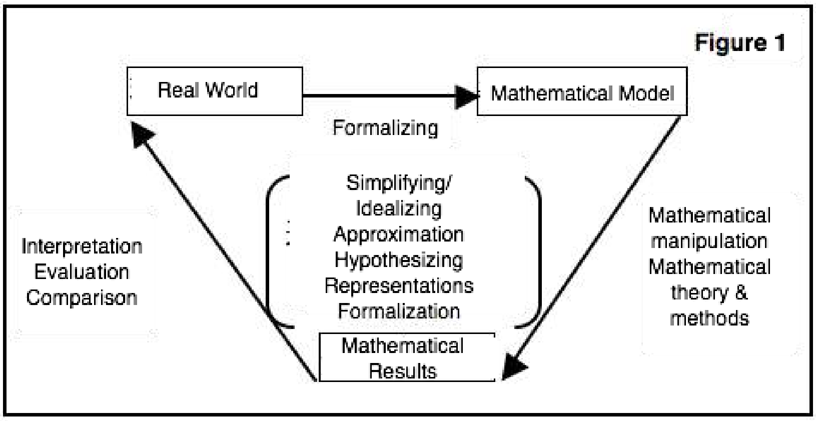


Figure 1

It is anticipated that the topic in this unit, “Division with Remainders,” may be difficult for some students as they get confused which of the numbers obtained in calculation should be considered as the answer. Moreover, a challenge in teaching this topic is how students should interpret the meaning of remainders. In addition, in the 2016 National Assessment at Grade 9, the success rate for the problem, "When a number is divided by 3, the quotient is a and the remainder is 2. Express the number in terms of a," was only 33.6 %. This results shows that many students have difficulty in grasping the relationships among numbers and quantities and expressing them using mathematical equations. These results show that even though students could express the relationship of numbers that is shown as division with a remainder using the inverse operation of multiplication in elementary schools, their success rate declines significantly when variables are used to represent the same relationship. Thus, it is important that, in elementary schools, students develop a deep understanding of the relationship among the dividend, the divisor, the quotient and the remainder.

Students learned about division in Grade 3 for the first time. Up to this point, students' learning of division has been based on their everyday experience of "sharing," and they formalized it as a mathematical idea and learned to represent those situations using calculation expressions. Moreover, they have also learned that there are two types of division situations, fair sharing division and measurement division, and also division might be considered as the inverse operation of multiplication. In teaching division, we have made sure that it went much more than just studying computational methods. Instead, we have intentionally incorporated activities with concrete materials, drawings and diagrams to represent own reasoning processes so that students can deepen the concept of "sharing" mathematically.

The goal of this unit is for students to be able to apply division with remainders in problem situations just as much as division without remainder. The division problems discussed in this unit are those that can be solved by using the basic single-digit multiplication in reverse. Later in Grade 3, students will learn about division with larger dividends only in those cases without remainders. These experiences will be necessary for their study of division using the algorithm in Grade 4, and therefore, it is essential that students muster the necessary skills as well. Therefore, we aim to deepen students' understanding of the meaning of the division operation as well as realizing the meaning of remainders in division situations. We will emphasize the meaning of the dividend, the divisor, the quotient and the remainder in a division equation as numbers in a mathematical expression always have meanings. Moreover, we also want students to think about the results of calculations in the context of the original problems. Our aim is for students to be able to interpret the relationship between the quotient and the remainder obtained from calculation and the problem context correctly.

2 Goals of the Unit

◆ Students will deepen their understanding of the meaning of division by understanding division with remainders, and they can apply their learning in other situations.

- Students try to make sense of the meaning of and method of calculating division with remainders by relating it to division without remainder, multiplication and manipulation of concrete materials. (Interest, Eagerness, and Attitude)
- Students develop a unified understanding of division whether or not there is any remainder, and they can represent its meaning and method of calculation using concrete materials, diagrams, and mathematical expressions. (Mathematical Way of Thinking)
- Students can carry out the calculation of division with remainders and determine the quotients and the remainders correctly. (Mathematical Skills)
- Students understand the meaning of division such as the meaning of remainders and the relationship between the remainder and the divisor. (Knowledge and Understanding)

3 Relationship between the Unit and the Research Theme

(1) About the dispositions/abilities we want to nurture in this unit

In the mathematics group, in order to realize lessons in which students create their own mathematics, we utilize lessons that focus on problem solving (*mondai kaiketsu gakushu*). In the learning processes in problem solving lessons, the four types of dispositions/abilities and "questions" are closely related.

In the stage of grasping the learning task, students will have the question, "What have I learned so far?" and put the problem situations from their daily lives onto the mathematical playing field as the first step of problem solving. In the independent problem solving stage of the lesson, students will ask themselves, "Which of what I have learned may be useful in this problem?" and tackle the problem by comparing it to previously solved problems. However, it is not always possible for students to have their own ideas. Thus, their peers will become an important component of their learning. During the comparison and critical reflection stage, students of compare and contrast their own ideas with those of their peers to generate better solutions, approaching the goals of the lesson. In this stage, students will ask about the rationale, commonality, differences, and generalizability. During the reflection stage, students will ask about the merits and extendability of ideas so that they can use what they learned in other situations. In mathematics, we believe the engine for learning is "question." In the process of learning, when one problem is solved, a new "question" arises. "Questions" are continuously generated. What support this learning process is students disposition to tackle problem solving autonomously.

In this unit, the main question is to think about the meaning of division with remainders. Specific questions such as the following are anticipated: “Even when we cannot divide evenly, can we represent the problem situation using division equation as we learned previously?” and “What does the remainder represent in the problem situation?” Through examining such questions, we want to help students deepen their understanding of the meaning of division. We want to nurture the four types of dispositions/abilities in our students while pulling out questions such as “Why is it?” “How can we solve it?” and “Can we use the method of calculating division we learned previously?” as they encounter situations involving division with remainders.

(2) About strategies for “lessons in which students feel the values of learning”

In the mathematics group, we consider “values of learning” is realized in experiencing the merits of mathematics. The strategies necessary for “lessons in which students feel the values of learning” are as follow:

① Strategies to devise lessons in which students have questions and the sense of expectations
- Set up the learning tasks that generates “questions.”

② Strategies to devise lessons in which students are engaged independently and with each other
- Investigate “questions” that must be asked.
- Anticipate how a series of “questions” may be generated.

③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction
- Record “questions” on blackboard and reflect on the learning.
- Secure time for students to write reflective journal entries.

In this unit, we focus on helping students grasp their everyday experiences of “sharing” as a mathematical idea of “division.” Then, by putting division back in everyday situations, we want students to be able to apply it in their daily life. In order to do so, we need to set up the learning task that is easy for students to relate to as well as interesting enough to hook them to the task. Problem situations that are easy for students to relate to and imagine are easier for them to represent their reasoning using diagrams and words. We want to conclude the task by developing a shared understanding by representing their diagrams and words as mathematical equations.

Moreover, we will pay close attention to the nature of questions so that we will have a series of questions. Throughout the unit, from the initial questions such as "Even when we cannot divide evenly, can we think of it as division?" and "Can we use the method of calculating division we learned previously?" we want to set up tasks so that students may ask questions such as "Can we still use multiplication to check the calculation of division with remainder?" and "How should we deal with the remainder as we answer the problem?" In addition, we want to make sure to record those questions on the blackboard, and we will encourage students to organize their notebooks so that we can look back on the flow of their learning in a lesson as well as the unit as a whole.

As students write reflective journal entries, we will encourage them to write specific ideas. In each lesson, we want students to develop the habit of recording what questions they had, how they resolve the question using their prior learning. By having a record of accumulated changes, we want students to have the sense of achievement. Furthermore, we want students to use their own reflective journal entries from previous lessons as motivation for new "questions" in the following lessons.

<u>(3) About methods for assessing the quality of individual student's learning</u>

As a strategy to assess the quality of individual student's learning, we will make use of their notebooks. We have been encouraging students to make their notebooks align with the process of problem solving lessons, " grasping the learning task → independent problem solving → comparison and critical reflection → reflection." Thus, in the independent problem solving stage, students will write their own ideas. In the comparison and critical reflection stage, they try to record their peers' ideas. When they do so, instead of simply writing down the answers, we have encouraged them to include the steps and process of getting the answers, using words, pictures, diagrams and mathematical expressions. By writing their learning journal entries, students can organize their ideas, reflect on them deeply and make use of the ideas in new problem situations. This way, students can reflect on the development of their ideas. By checking students' entries in independent problem solving/comparison and critical reflection/learning journal against each other, we will know what ideas students initially had, what challenges they faced, and how their ideas evolved. In this way, we want to assess our efforts to increase the quality of individual student's learning.

In this unit, we design lessons so that students will develop the ability to express their ideas related to the meaning of equations and expressions in their notebooks. In addition, we want to assess students' ability to connect problem situations with the meaning of division as well as their ability to effectively use their prior learning. Moreover, we plan to provide in-class support so that students might express changes in their own ideas and also similarities and differences with their friends' ideas. We plan to provide necessary support so that students will understand useful ways of observing problem situations in order to find solutions.

4 Unit Plan and Assessment (Total of 10 lessons)

#	Goals	Learning Activity	Assessment Standards
① Division with remainders (6 lessons)			
1	Students will understand ways to calculate division with remainders in the case of 1-digit divisors and 1-digit quotients.	○ Think about ways to find the answer for 14÷3. ○ Learn that 14÷3=4 rem. 2.	[Interest] Students are trying to figure out ways to calculate division with remainders based on their prior knowledge of division. [Thinking] Students can explain ways to calculate division with remainders based on their prior knowledge of division using concrete materials, diagrams and equations/ expressions.
2		○ Lear the meaning of remainders. ○ Learn that even when the dividend cannot be divided evenly, we can still use the ideas of division.	
3	Students understand the relationship between the divisor and the remainder.	○ Explore the relationship between the divisor and the remainder in the case of 13÷4.	[Knowledge] Students understand that the remainder must be less than the divisor.
4	Students understand that division with remainders can apply in the case of fair sharing division situations.	○ Set up the calculation expression, 16÷3, based on the understanding of the problem context and think about ways to find the answer.	[Thinking] Students can explain ways to calculate division with remainders in the fair sharing situations based on their prior understanding of fair sharing division using concrete materials, diagrams and equations/ expressions.
5 6	Students understand how to check their calculation of division with remainders.	○ Think about ways to check division calculations, including situations with remainders.	[Knowledge] Students understand ways to check division calculation even when there are remainders.

② Problems that require the meaning of remainders (2 lessons)			
7 Today's Lesson	Students will deepen their understanding of how to think about remainders.	○ Based on the understanding of the problem situation, think about the reason why the answer for the problem should be Quotient+1 when the calculation is 24÷3=5 rem.3. ○ Think about other dividends for which the answers are "6 boats" based on the idea of inverse operation.	[Knowledge] Students understand how to deal with remainders based on problem contexts.
8		○ Based on the understanding of the problem situation, think about whether the answer to the problem should be the quotient as it is or Quotient+1 when the calculation is 30÷4=7 rem. 2.	
③ Unit Summary (2 lessons)			
9	Students can create problems while thinking about the meaning of division as well as how remainders should be treated.	○ Create problems that reflect different meanings of division.	[Knowledge] Students can solve problems using what they learned in this unit.
10	Consolidate students' understanding by applying their learning in various situations.	○ Engage in application problems and deepen their understanding.	

5 Today's lesson

(1) Date: June 24, 2017 (9:00 - 9:45)
(2) Location: Yamanashi University Attached Elementary School

(3) Goal of the lesson

- Students will deepen their understanding of remainders, and they can obtain answers to problems by rounding up the remainder, if necessary.

(4) Reason for teaching this lesson

Through the previous lesson, students have been exploring ways to calculate division with remainders, how to calculate, and using the inverse operation to check the calculations. Students understand that in both fair sharing (partitive) and measurement (quotative) situations, we can use division to represent situations when we are making equal groups even if there are remainders. They know that even when there are remainders they can use multiplication to find the quotients and use the inverse operation (multiplication) to check the results of calculation. However, students can easily fall into believing they "understood" division with remainders when they master the calculations. Therefore, by engaging in this lesson, students will think about the meaning of division and what the quotients and remainders represent, which in turn will deepen their understanding of division with remainder.

Today's problem is set in the context of a school excursion. As we think about the boat ride at the amusement park, they will be asked the question, "All 23 of us will ride the boats. Each boat can carry 4 people. How many boats will we need?"

In one of the research lessons in 2015, we presented a research lesson titled, "Let's think about division: Situations to think about remainder (1)" (Ohma). In that lesson, just as in today's lesson, students tackled the problem involving 23 ÷ 4 = 5 rem. 3." They then have to think about the problem context and decide how to answer the original problem, rounding up the remainder if necessary. For the 2015 research lesson, we anticipated that students might have difficulty answering the problem with "6 boats" by rounding up the remainder based on the problem context. Therefore, we hypothesized that by examining the calculation process to check division, students will understand the reason for rounding up the remainder to answer the real world problem. However, in that lesson, it was observed that some students clung to the quotient of "5" they obtained through calculation and had difficulty answering the problem with "6 boats." Although the calculation, 4 × 5 + 3, is the calculation to check the answer, 5 remainder 3, but it was also noticed that some students thought it was the calculation to check the answer to the problem, 6 boats (Seino, 2015).

The goal of this lesson is to help students understand how to deal with quotients and remainders based on the problem context. It is anticipated that those students who use the calculation result as the answer to the problem will respond with "5 boats with the remainder of 3 students." In addition, since the question asks students "how many boats," some might ignore the remainder and simply answer, "5 boats." However, the problem statements says "all 23 students will ride the boats." Therefore, students will realize that they need a boat for the remainder of 3 students, thus affecting how they answer the problem. Through this experience, we hope that students can deal with remainders appropriately.

In today's lesson, students will be asked the question, "What are other possible number of students who can ride with 6 boats?" Based on the fact the situation, 23 ÷ 4 = 5 rem. 3, can be represented using the inverse operation, 4 × 5 + 3, students will think about what are other possible number of students for whom we need 6 boats, and think about when we have to round up the remainder to answer the problem. In today's situation, there are 4 cases when we need 6 boats: ① $4 \times 5 + 1 = 21$, ② $4 \times 5 + 2 = 22$, ③ $4 \times 5 + 3 = 23$, and ④ $4 \times 6 = 24$. Thus, they notice that the cases when we need 6 boats are when the number of students are 21 through 24. In that proces, we will emphasize "$4 \times 5 = 20$" means "4 students in each boat, and 5 boats will make the total of 20 students." Students will then notice that if we have 1, 2, or 3 more students than 20, we will need 6 boats. Thus, by going through the process of interpreting mathematical results in the context of the real world, we want to help students to overcome the challenge of coming up with the answers by rounding up the remainders. We also hope that students will deepen their understanding of division with remainders and consolidate their understanding of how to deal with remainders based on the problem contexts.

(5) Flow of the lesson

min.	Main learning activity/content Anticipated responses	• Points of consideration • Strategies for "lessons in which students feel the values of learning"
5 G R A S P	1. Grasp today's problem We are all going on the boat ride. Each boat can carry 4 people. How many boats do we need? • Grasp the problem situation. ○ It's division. • What if there is a family of 4? ○ They can ride the boat. ○ They need 1 boat. • What if there is a group of 8 people? ○ They can ride on the boat. ○ They can if they have another boat. ○ They can split into 2 groups of 4. So, they need 2 boats.	• Display a picture of 4-passenger boat so that students can grasp the problem context more easily. • By considering the situations with 4 and 8 people, we try to help students imagine the problem situation and develop a plan to tackle the problem. • By considering the situations with 4 and 8 people, make sure that students understand that there are more than 1 boat. **In the process of "lessons in which students feel the values of learning"** ⦿ Strategies • Motivate students to tackle the problem by helping them grasp the problem situation accurately. • Set up the learning tasks that generates "questions." • Write down students' "questions" on the board and help them understand a division equation can represent the problem situation.

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<tr>
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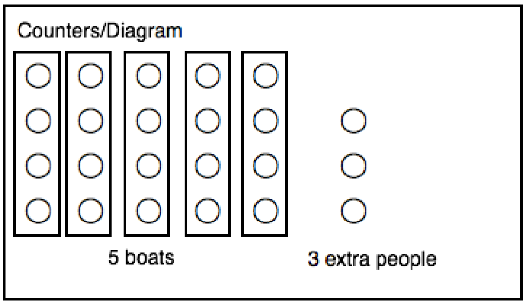
<td>2. Independent problem solving
○ Use their prior learning, students will find the answer.
a) Use counters.
b) Draw diagrams


c) Use multiplication
4 ×☐ = 23
Since 23 is not in the 4's table, we will have a remainder.
23 ÷ 4 = 5 rem. 3
d) Use the quotient and the remainder as the answer to the problem, 5 boats with 3 people remainder.
e) Because there is a remainder, not sure what the answer is.
f) Use only the quotient of 5 and answer "5 boats."
g) Realize that the 3 students left over must be on a boat, so the answer is "6 boats."</td>
<td>In the process of "lessons in which students feel the values of learning"
⦿ Dispositions/abilities we want to nurture in this unit
• Think about whether or not we can use a division to represent the problem situation by clearly understanding the problem situation and thinking logically.

• Observe students carefully so that the teacher has good sense of how students are using the strategies (a) ~ (c) and obtaining the answers (d) ~ (g).</td>
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<td>3. Compare and contrast solutions.

• Share how many boats are needed

e) 5 boats with 3 people remainder.
f) I was able to do the calculation, but I'm not sure what the answer is.
g) "5 boats."
h) "6 boats."</td>
<td>• We will discuss the solutions in the order of (e), (f), (g) and (h).
• Share all the ideas and think about what's different and why.
• Discuss cases where the answer is incomplete, like (f), and clarify what it is that might be creating the challenge.
• Ask students to raise their hands to show which one matches their answers.</td>
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	"Even though the calculation results are all the same, why do we have different answers?" • Comparing the differences in answers, and discuss how many boats are needed. ○ (d) and (g) are similar, but one includes the remainder of 3 and the other one does not. ○ In (h), the answer is "6," and that's different from the calculation result of "5 remainder 3."	**In the process of "lessons in which students feel the values of learning"** • In order to solve the problem, explore the task using mathematical thinking. ⊙ Students who feel the values of learning • Using the prior learning, students can represent the problem situation using diagram, words, and equations.
	"I wonder if we can explain using diagrams or counters?" (a) and (b) ○ It's the same with counters or with diagrams. ○ There are 4 people in each boat, and there are 5 boats.	• Share the idea of the students who used counters as a class.
	"Let's represent in an equation." • Think about how different ideas that have been shared can be represented by an equation. ○ $23 \div 4 = 5$ rem. 3 ○ $4 \times 5 + 3 = 23$	**In the process of "lessons in which students feel the values of learning"** ⊙ Students who feel the values of learning • Students realize that they can explain their ideas clearly to others by using mathematical expressions.
	"Is '5 remainder 3" an answer?" ○ "5 remainder 3" is not an answer. ○ "remainder 3" does not match "how many boats do we need?" "What should we do about the remainder of 3?" ○ We want the 3 remaining students to be on a boat, too. ○ Is it ok to use one boat even though there are only 3 people? ○ If all of us are going to enjoy the ride, we can't leave them out. ○ We need one more boat. ○ I think we need 6 boats.	• Make sure students understand where the 3 remaining students come from and the fact that they are not on a boat. • Because students' ideas might have changed from the beginning of the discussion time, ask students what their answers are and record the numbers.

	• Listen to the explanation of students who are not convinced "6 boats" is the answer, and discuss. ○ The part inside the box (in the diagram) shows 5 boats. ○ I understand we need 5 boats.	• If any students is still clinging on the answer of "5 boats," or if anyone is not convinced "6 boats" is the answer, ask them to explain why they feel that way. Then, discuss how many people can ride on 5 boats.
	"If there are 5 boats, how many people can ride?	• By confirming to what "5 boats" corresponds, help students understand the quotient of 5 corresponds to "5 boats" and how 5 boats are represented in the diagrams or counters. • Help students recognize "4 × 5" in the diagrams or counters. • Make sure students understand that "4×5=20" means that 20 people can ride the boats. Draw out the idea that 3 more students need to get on a boat.
	"How many people can ride on 6 boats?" • Think about the necessity of having 6 boats based on the inverse operation, i.e., multiplication. ○ Since there are 4 people on each boat and there are 6 boats, 4 × 6 = 24, or 24 people. ○ Since we can represent it as ☐ ÷ 4 = 6, ☐ must be 24. ○ 24 people can ride the boats if there are 6 boats.	• Ask students who think the answer is "6 boats" to explain the reason why they think so. Then, discuss the number of people that can ride 6 boats. **In the process of "lessons in which students feel the values of learning"** ⦿ Strategies • Draw out students' own "questions" from the task. "What should we do about the remainder of 3 people?" "Why do we need 6 boats?
	• Based on the fact that 20 people can ride on 5 boats and 24 people on 6 boats, represent the number of people between 20 and 24 using division or the inverse operation. Think about when we need 6 boats. ○ For 20 people, 20 ÷ 4 = 5 5 boats 4 × 5 = 20	• By thinking about other cases where we need 6 boats, verify that we get the answer, "We need 6 boats," from the calculation of 23 ÷ 4. • Sometimes in addition to the calculation results, we must pay

	○ For 21 people, $21 \div 4 = 5$ rem. 1 6 boats $4 \times 5 + 1 = 21$ ○ For 22 people, $22 \div 4 = 5$ rem. 2 6 boats $4 \times 5 + 2 = 22$ ○ For 23 people (today's problem) $23 \div 4 = 5$ rem. 3 6 boats $4 \times 5 + 3 = 23$ ○ For 24 people, $24 \div 4 = 6$ 6 boats $4 \times 5 = 24$ • Think about how many boats are needed if there are 35 students (the number of students in our class). ○ $35 \div 4 = 8$ rem. 3 Therefore, 9 boats. ○ $4 \times 8 + 3 = 35$ Therefore, 9 boats. ○ If there is any remainder, we need one more boat to put everyone on the boats. ○ Even if we have many more people, the remainder means one more boat.	attention to the remainder to answer the problem. • Reflecting on their prior learning of the relationship between the quotient and the remainder, from without remainder, that is "remainder 0," to "remainder 3" are the cases where we need 6 boats.
	5. Reflect on today's lesson ○ Sometimes we must consider the remainder to answer the problem. ○ We cannot ignore remainders. • Write reflective journal entries.	**In the process of "lessons in which students feel the values of learning"** ⦿ Methods for assessing the quality of individual student's learning • Through students' reflective journal entries, assess students' reasoning processes. • From what students wrote in their notebooks, assess how their thinking changed and whether or not they truly understood the reason why the answer is 6 boats.

(6) Points of observation

- Were the strategies to help students feel the values of learning?
  ① Strategies to devise lessons in which students have questions and the sense of expectations
  ② Strategies to devise lessons in which students are engaged independently and with each other
  ③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction bt reflecting on the lesson

(7) References

Omitted

Grade 3 Yamanashi Lesson Report

Report created by: Meghan Smith, Nakachi Kasimu, Sarah Horwitz, Rory Dearlove, David Allyn

Name of Lesson: 3rd Grade, Division with Remainders

Date of Lesson: June 24, 2017

What are the primary lesson goals?

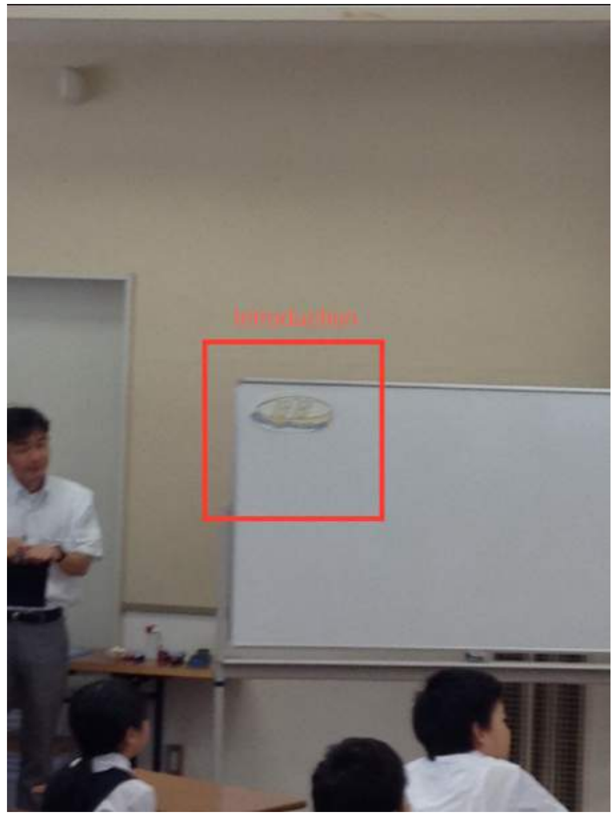
Students will deepen their understanding of how to think about remainders.

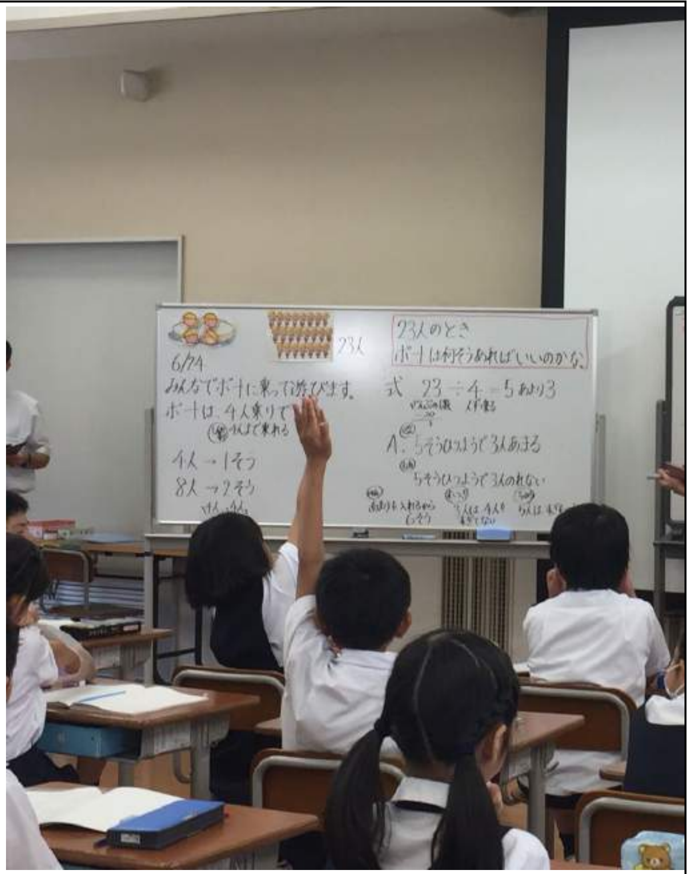
Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

This is Lesson 7 in a unit of 10 lessons. Children have been dividing with remainders in lessons previous to this one in the case of 1-digit divisors with 1-digit quotients. In addition, they have worked to understand the relationship between the divisor and the remainder. Their work has been with partitive division, or fair sharing division, and they have also worked on understanding how to check their calculation of division with remainders.

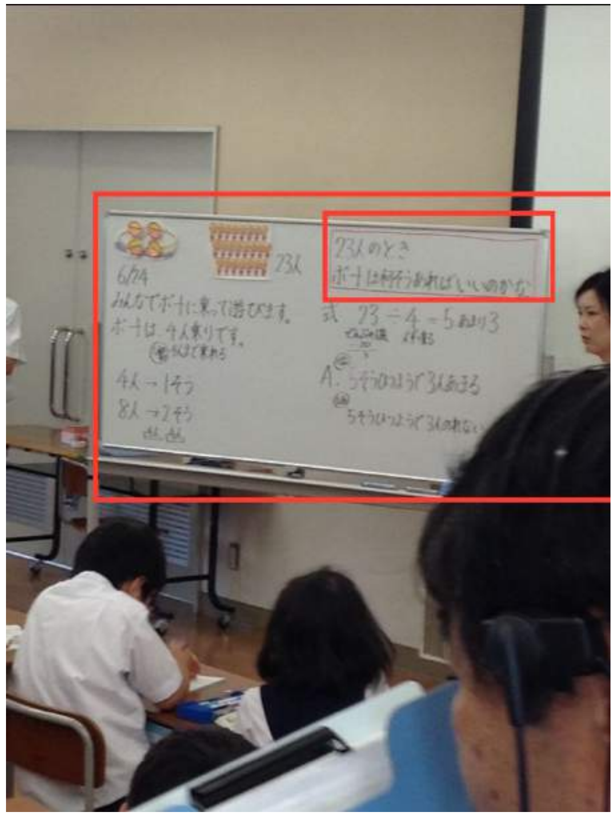
Summary of Lesson

Start & End Time	Lesson Phase	Notes
09:00	**Introduction, Posing Task**	**Strategies to build interest and to connect to prior knowledge** The teacher began the lesson by introducing the story of going to a lake where pleasure boats that seat four people each are available for some fun. This boat was going to be used in today's lesson for math and children were instructed to take out their math notebooks.

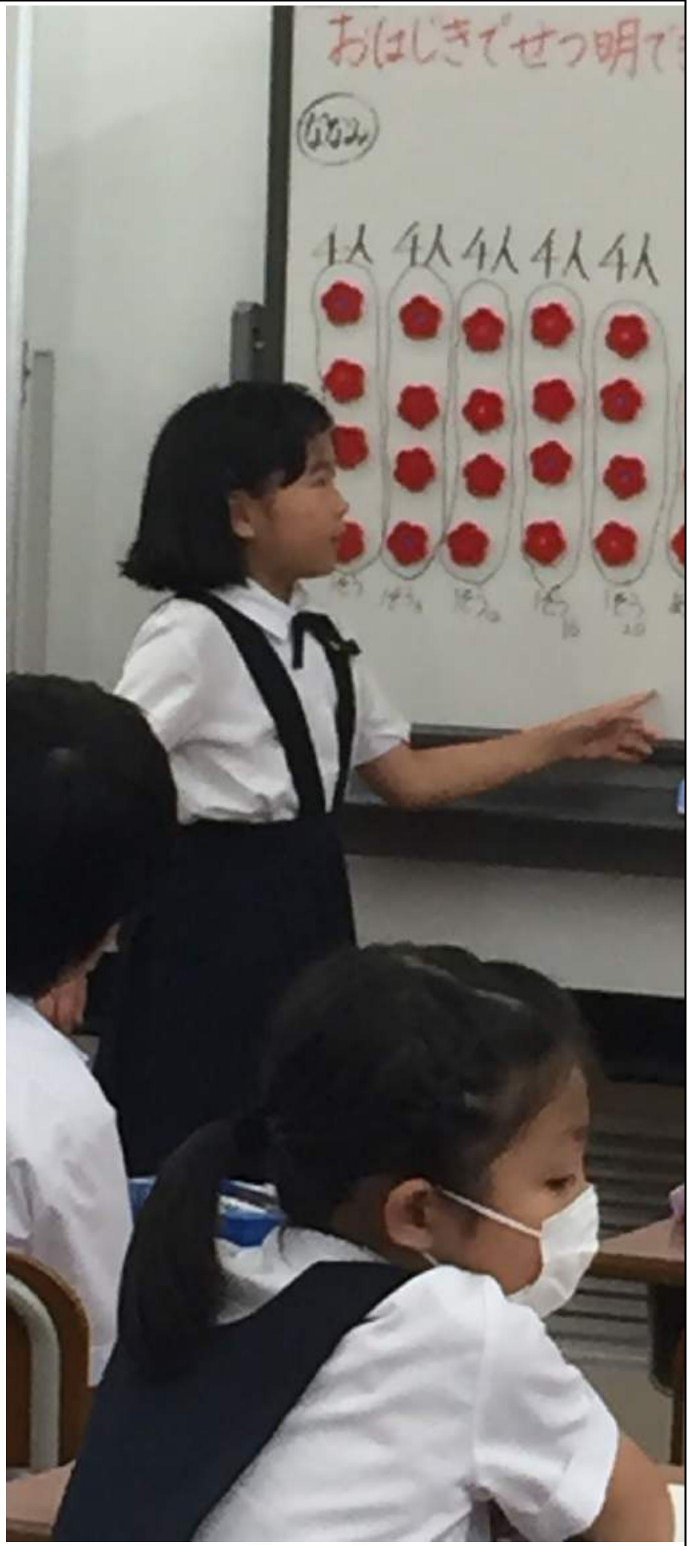
		 Question: What if 8 people came for boating, how many boats would be needed? Student replied 2 boats. Teacher: How do you know? What if 23 kids wanted to ride in boats, how many boats?

23人のとき
6/24
みんなでボートに乗って遊びます。
ボートは、4人乗りで
式 23÷4＝5あまり3
4人→1そう
8人→2そう
A. 5そうひつようで3人あまる

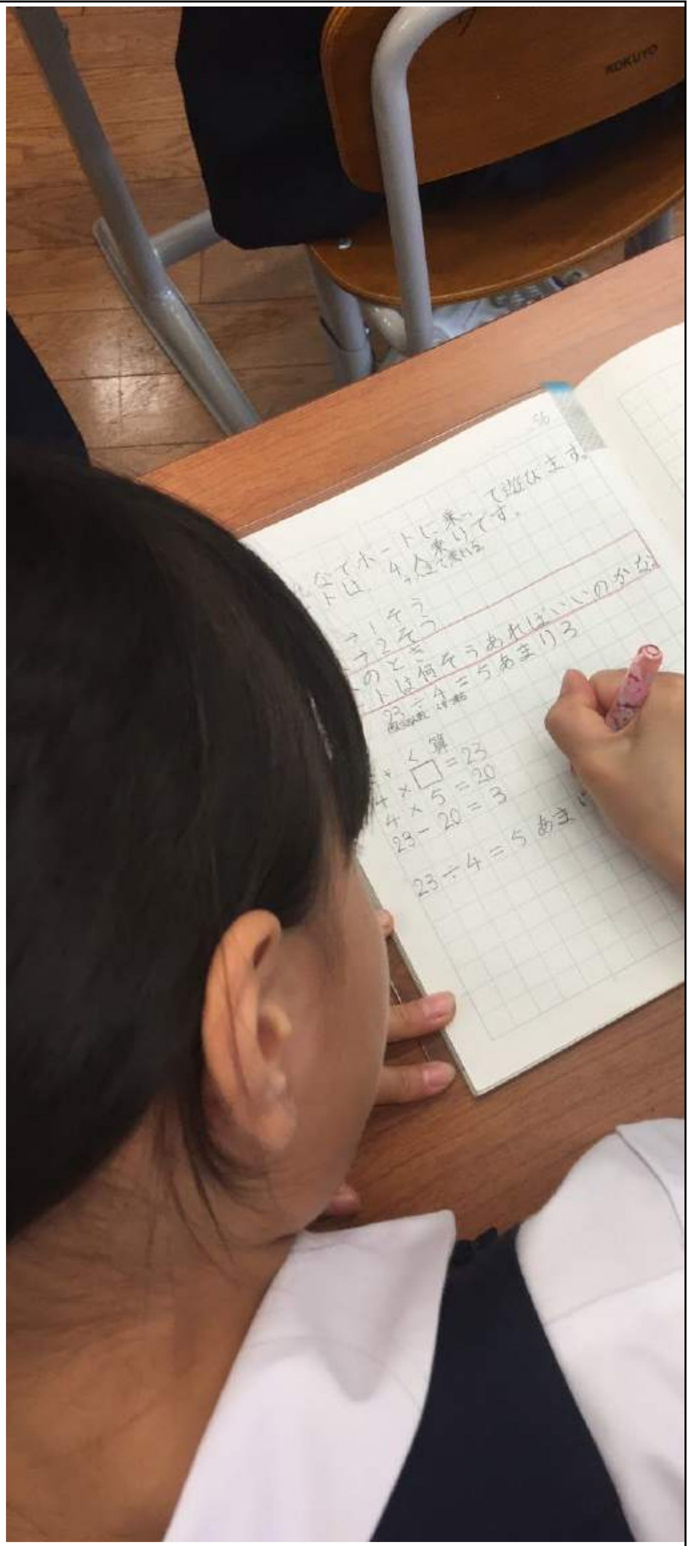
		
09:11 - 09:19	**Independent Problem Solving**	**Individual, pairs, group, or combination of strategies** ● **experience of diverse learners** Students worked entirely independently. Of 35 students present, only one arrived at 6 boats. The majority of students wrote: Calculation: 23/4=5 remainder 3 Many (but fewer) students recorded Answer: 5 boats 7 students utilized manipulatives Several students utilized a drawing of a dot array. ● **teacher's activities** Teacher reminded students whole group and individually to record not only their calculation but also their answer.
09:20-09:49	**Presentation of Students' Thinking, Class Discussion**	**Student Thinking/ Visuals/ Peer Responses/ Teacher Responses** T: What is the answer for the calculation? S1: 5 remainder 3. T: Do others agree? S (Many): Yes T: How did you solve?

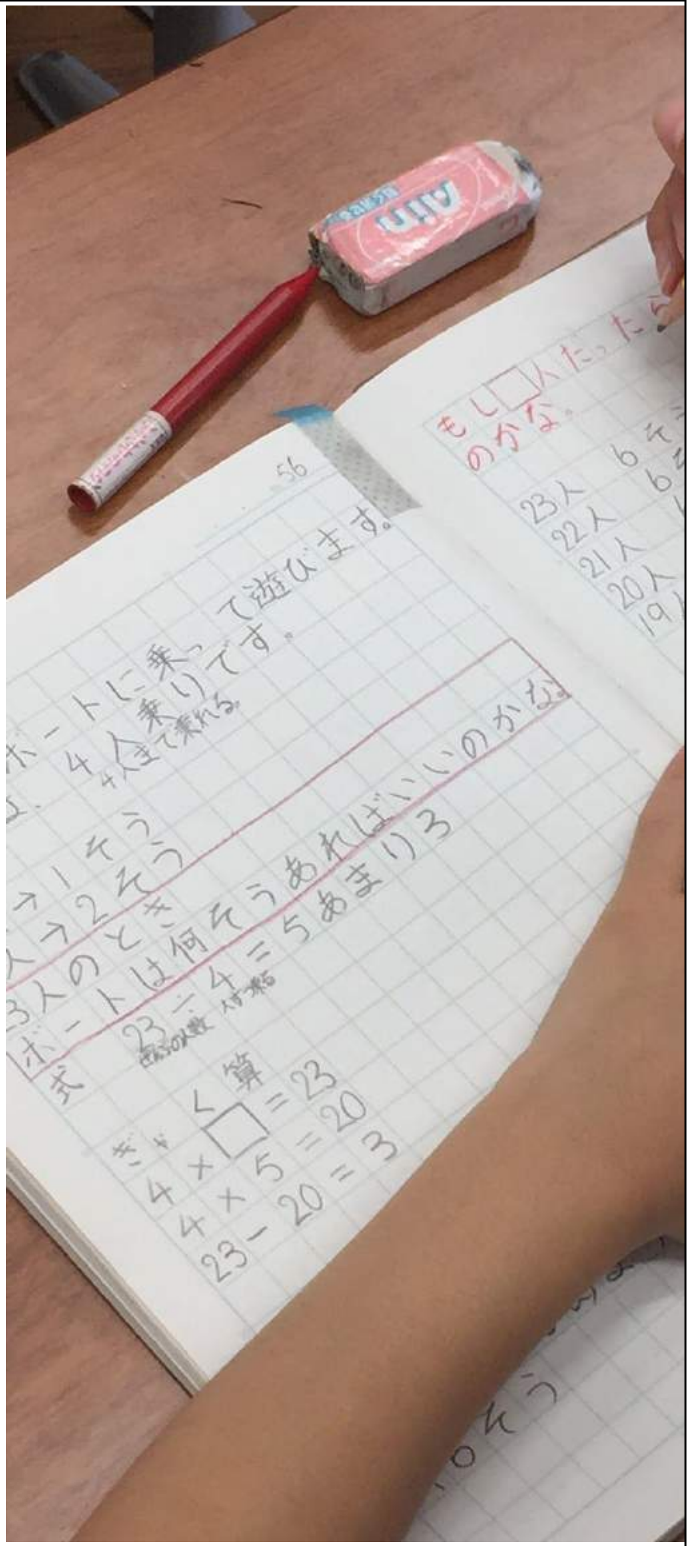
		S1: Multiplication. T: What was your answer? S2: 5 boats, 3 remainder. S3: 3 people remainder. S4: 3 people cannot ride the board. T: Is this ok? S5: It's sad for them. S6: The question asks *How many boats are needed*, not What's the remainder. S5: The question says *A 4 passenger boat*, so 3 people can ride in it. T: Who can say what he said? S7: (Repeats) T: So can five people ride in one boat? S8: No, that's too many. T: Let's make it more clear. Can you explain using counters? (Counts out 23 counters in rows of 10, counting by ones) S9: Moves counters into an array of 4 rows of 5, circles to show 4 per group with three in a sixth row that she also circles, labels the three in a sixth boat. T: Records skip counts up to 23 total below, also labels the circled groups as boats. Do you agree? S10: The question asks "How many boats." Even though there are only 3 people, this boat can carry an extra. T: So they are not really remainders? Who can explain? S11: I agree, we need 6 boats.

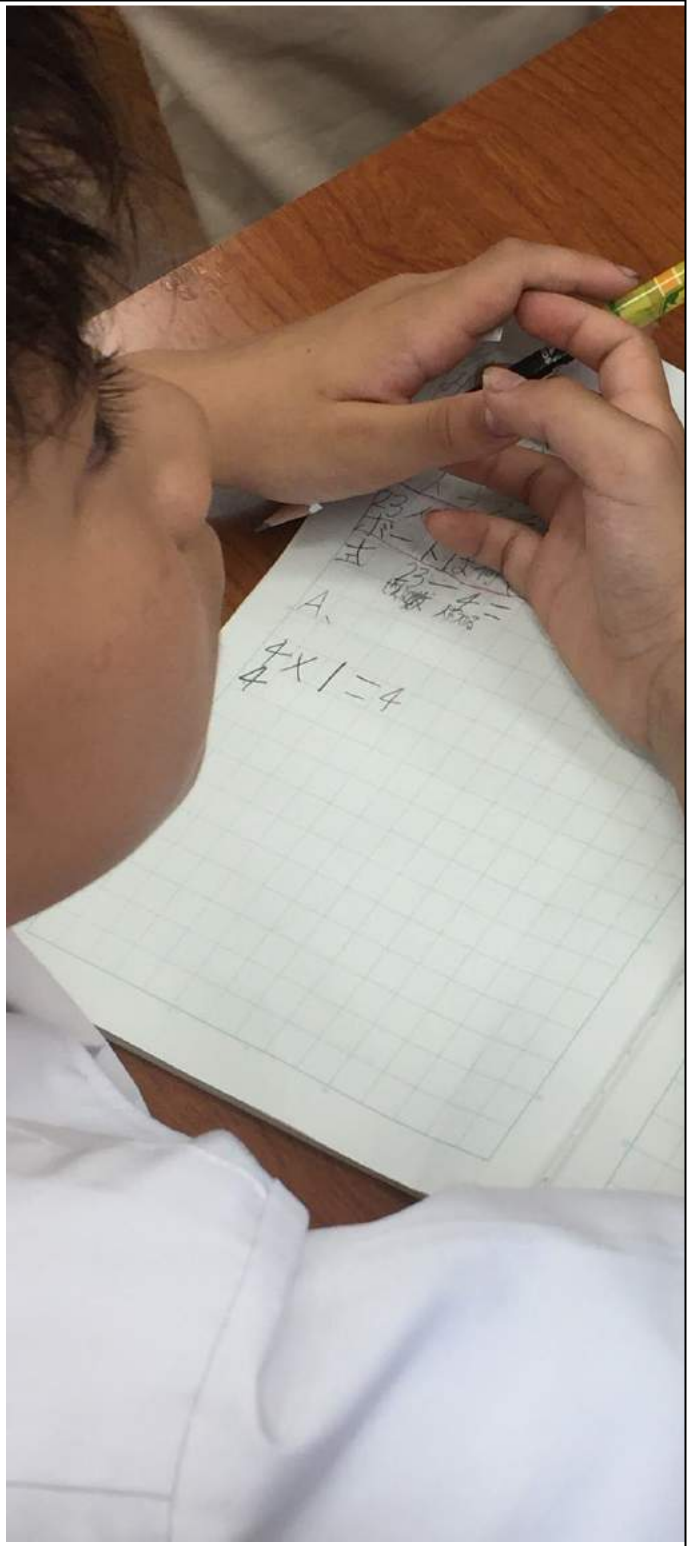
		 S12: Yes. They can fit even though there are only 3 people. T: “Up to 4 people” - Is that what the problem means?

		S14: Yes. 1 person, 2, 3 or 4 can fit in a boat. T: So what do we do? S15: I think we can add one more boat. T: Where are we adding it? S15: Here - (points to the row of three) T: So there is no remainder? S15: No. T: Are you ok with no remainder? What is the answer? S16: Yes, there is a remainder when you calculate, but the **answer** is 6 boats. T: So you are saying that the equation is still 23/4=5 R 3, but that that isn't what the question is asking! You are saying the equation and the problem are different! T: (Writes on board If there were ___ people…) From this point, the teacher leads the students by writing a chart of the number of people (column one) to number of boats needed (column two), and the corresponding equation with remainders (column three). Some students chose to record this process in their journals, but no direction was given. The teacher recorded several student ideas on the far right, but no time is allotted for independent summary or reflection. Teacher then says, "We are out of time, let's end the lesson."
	Summary/Consolidation of Knowledge	**Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals** Teacher made a connection to what that remainder means by asking why? Why must the remainder be smaller than the divisor? **From the plan:** This is the situation in which students understand that the results obtained from calculations must be interpreted in the problem contexts and come up with the answer that is different from the calculation results.

		There is evidence of this in the lesson when many children in the class insisted there needed to be an answer to the problem giving the number of boats regardless of the remainder in the division. Any scenario with a remainder less than 4 will require another boat so that children are not left behind. This continuing questioning also kept students engaged.

		4 × □ = 23 4 × 5 = 20 23 − 20 = 3 23 ÷ 4 = 5

		 This boy seems stuck:

		4×1=4

What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?
Our team gained valuable insight about mathematics and pedagogy from the post lesson discussion as well as the IMPULS participant discussion. One key takeaway we had was the allowing time for children to think and show their understanding independently after the group discussion. In the context of the lesson, student comprehension may have been secured and extended by giving students the opportunity go back to the problem, and/or similar problems, in order to revise or add to their initial thinking and show evidence of their developed understanding of remainders.

Lesson extensions could have included either independent or group work with similar problems with different quantities. Time for this could have been made by allowing children to independently solve the problems that were answered in the chart during the group discussion. A second group discussion after this could have supported children in understanding the pattern that was being illustrated i.e. that for division problems that can only have whole number answers it does not matter what the remainder is as the answer will be the number of times the divisor go into the dividend plus one if their is a remainder.

At times it appeared that not all children were as involved in their learning as they could be so as a group we discussed how engagement could be increased. One idea was to give them greater even ownership of their learning. A small tweak we agreed on was making the initial problem more meaningful by making it about the children themselves, “I was wondering how many boats we would need if we were to go on a boat ride..”. The number could be manipulated to ensure an appropriate remainder by including a number of adults in the group if necessary.

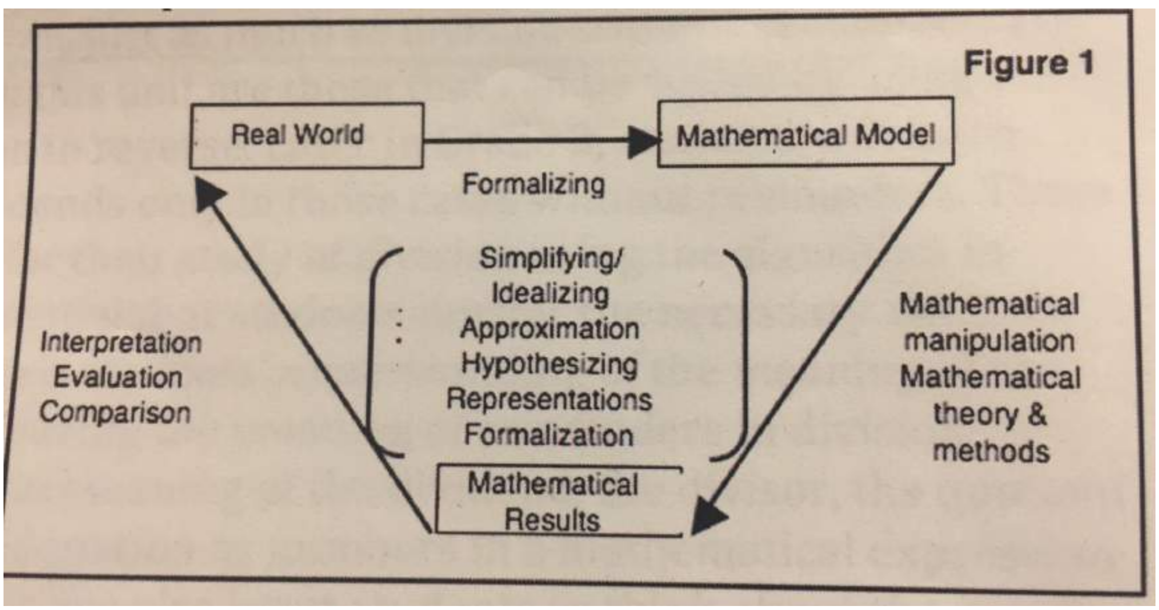
We also agreed that allowing students more time to talk and work independently would have given better insight into what they were understanding and taking away from the lesson and allowed the teacher to have consolidated their learning more meaningfully and concisely. Once the children understood that for some division problems an answer with a remainder is inappropriate it would have been useful to see how they could build on this understanding to make some generalisations (e.g. how you must round up the quotient when the dividend is not in the times table of the divisor) either through independent work or small group work.

Finally our group discussed how it is important to move thoughtfully from concrete to abstract in teaching mathematical concepts we debated if the flower counter diagram in the lesson was necessary and if it was deployed at the appropriate point in the students learning. We decided that flower counters were not concrete enough and that there was a missing step between the for the students. We would have liked to have seen a more concrete diagram prior to this (either using the children themselves or pictures of children being placed into boats) and hypothesize that this would have strengthened children's' understanding.

What new insights did you gain about how administrators can support teachers to do lesson study?

After observing this lesson and the post-lesson discussion we found that having two research lessons back to back is overwhelming for observers and causes the final comments and discussion to be limited. We found that it is also important for the facilitator to be objective and outside of the team in order to promote critical discussion. Administrators are the connecting point between schools so that the work is open and a sharing of knowledge occurs. For participants we found how important it was to read the lesson before coming to the lesson as an observer. This idea could be stressed by administrators as well as the important role that observers play in the lesson.

How does this lesson contribute to our understanding of high impact practices?



We learned a great amount from this lesson in terms of our mathematical understanding as well as high impact practices. One key takeaway we had was the level of questioning from the teacher. In the lesson almost all teaching talk was done through questioning which forced the children to think for themselves and did not allow them to simply accept information because the teacher had told it to them. We also felt, as previously discussed, that student to student discussions are important because they allow students to investigate the learning which allows them to engage with group discussion at a deeper level. Based on observations and final comments we found that it was important to have a meaningful context for students in order for them to be invested in solving the problem in a meaningful way (i.e. not just calculating but actually solving the problem) and coming to an understanding because it directly relates to them.

A final key takeaway we had from this lesson as well as many others was time management and pacing. We felt it was important to maximize the amount of time that students are working and discussing. Our team found that it was vital for teachers to have deep math knowledge in order to anticipate when to allow children to work independently and when to use group discussion to consolidate and advance learning by using children's responses to move towards a generalized property or rule with maximum efficiency.

June 24
G 6

Date: June 24, 2017
Yamanashi University Attached Elementary School
Grade 6, Classroom 2 (32 students)
Teacher: YAMAGUCHI, Kuniyuki

Mathematics
Organizing Data

1 About the Unit

The mathematics working group for the Central Education Council examined possible points of improvement in school education toward nurturing of 3 types of dispositions and abilities in our students. One of the recommendations presented in the summary report is the improvement and enrichment of statistics education. In particular, following four types of instructional activities were suggested as examples.

- After students represent a given data set in a graph, they will examine it and generate a new set of questions. Then, they will revise the graph to match the new purposes.
- Critically examine a graph which has been created for a certain purpose from multiple perspectives.
- Represent two or more sets of data on the same graph (bar graph, broken line graph and/or histogram) to compare them.
- Discuss representative measure of data set other than mean.
- Intentionally make connections to the content in other subject areas.

In addition, in the new National Course of Study released on March 31, 2017, a new domain, D. Use of Data, was added in the elementary mathematics, and terms such as mean, median, mode, and ranks have been shifted from Grade 7 (in lower secondary schools) to Grade 6 (in elementary schools).

As we planned lessons in this unit, these recommendations and changes were always kept in mind. For our unit, we decided to focus on devising rich activities related to problem solving and decision making based on statistical analysis. In addition, we also wanted to focus on critical examination of statistical methods and conclusions. Our overarching goal is set as "Helping students purposefully collect data, organize them in appropriate tables and graphs, and then identify trends in the data set by observing the representative values and the distribution."

Students have learned about using pictures and diagrams to represent ideas in Grade 1. In Grade 2, they learned 1-dimensional tables and pictographs, and in Grade 3, they learned 2-dimensional tables and bar graphs. Broken line graphs were discussed in Grade 4. In Grade 5, they learned about percent, circle graphs and percent bar chart as well as average of measured quantities. In each of these units, students engaged in activities of organizing and interpreting phenomena by representing phenomena in tables and/or graphs and also interpreting tables and graphs.

The goal of the unit is, "Students understand mean as the representative value of a data set, spread of a data set, and frequency distribution, and make use of those ideas purposefully to examine situations statistically and make appropriate representations." Students will collect data purposefully and organize them in a frequency distribution table and also in a histogram. Using those tools, students will interpret characteristics and trends in the data set. Although students have learned about mean previously, in this unit, we will study mean as a type of representative measure of data set. Students will also try to solve problems by making use of data based on what they have learned previously. Students will engage in various learning activities generally sequenced as Identify the Problem → Devise a Plan → Collect Data → Analyze Data → Develop a Conclusion (PPDAC cycle).

In teaching this unit, we wat students to understand that, instead of looking at the data set as collected, it is helpful to grasp trends when the data are organized in a frequency distribution table. Moreover, we also want them to understand that a histogram is also helpful to observe how the data are distributed. We want to make sure students understand the merits of various ways to organize data. As students make use of data to solve problems, we also want them to critically reflect on both the methods and conclusions obtained statistically by employing tools such as frequency distribution tables, histograms, and representative values.

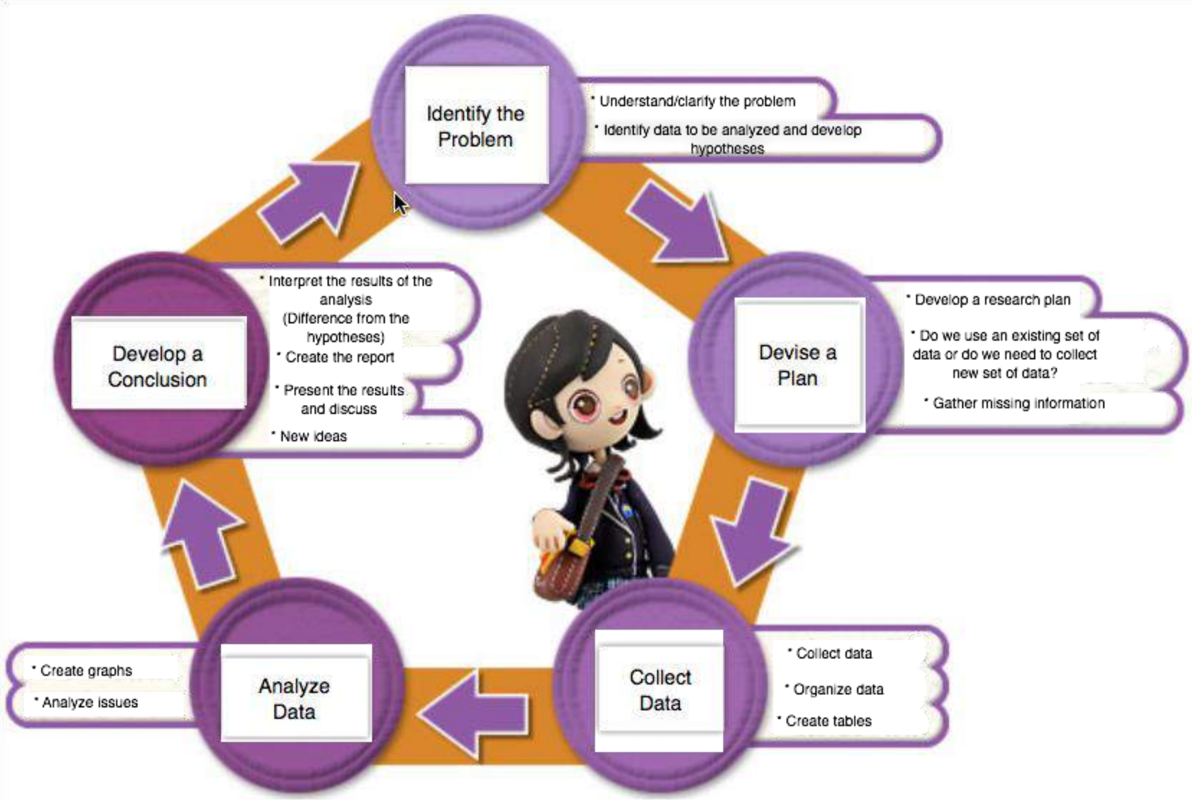


Figure 1 PPDAC cycle

2 Goals of the Unit

◆ Students will understand mean as a representative value, dispersion of data, and frequency distribution. They can use those ideas purposefully to examine and represent phenomena statistically.

- o Students recognize the usefulness of mean as a way to represent characteristics of a data set, and they try to use it purposefully to examine and represent phenomena in their surroundings statistically. [Interest, Eagerness, and Attitude]
- o Students can examine data sets to identify their characteristics using tools such as mean and the way data are scattered. [Mathematical Way of Thinking]
- o Students can represent data sets in frequency distribution tables and histograms as well as interpret given tables and histograms. [Mathematical Skills]
- o Students understand mean as a representative value of a data set. They also understand frequency distribution tables and histograms. [Knowledge and Understanding]

3 Relationship between the Unit and the Research Theme

<u>(1) About the dispositions/abilities we want to nurture in this unit</u>

In the mathematics group, in order to realize lessons in which students create their own mathematics, we utilize lessons that focus on problem solving (*mondai kaiketsu gakushu*). In the learning processes in problem solving lessons, the four dispositions/abilities and "questions" are closely related.

In the stage of grasping the learning task, students will have the question, "What have I learned so far?" and put the problem situations from their daily lives onto the mathematical playing field as the first step of problem solving. In the independent problem solving stage of the lesson, students will ask themselves, "Which of what I have learned may be useful in this problem?" and tackle the problem by comparing it to previously solved problems. However, it is not always possible for students to have their own ideas. Thus, their peers will become an important component of their learning. During the comparison and critical reflection stage, students of compare and contrast their own ideas with those of their peers to generate better solutions, approaching the goals of the lesson. In this stage, students will ask about the rationale, commonality, differences, and generalizability. During the reflection stage, students will ask about the merits and extendability of ideas so that they can use what they learned in other situations. In mathematics, we believe the engine for learning is "question." In the process of learning, when one problem is solved, a new "question" arises. "Questions" are continuously generated. What support this learning process is students disposition to tackle problem solving autonomously.

In this unit, questions will be generated as students represent the data set in tables and graphs. Students will hold various questions as they go through the learning activity sequence. In the step of Identify the Problem, they will ask themselves, “What is the issue in this situation?” As they Devise a Plan, they will ask, “What information is needed to solve this problem?” They will ask, “What method should we use to collect the data?” in the Data Collection stage. As they engage in data analysis, the question may be “What can we tell from the data?” Finally, as they try to generate a conclusion, they will be asking “What can we conclude from our analysis?” As students engage in problem solving through these steps, there will be a question that is the question that must be asked. In other words, those are the questions, students want to answer as they identify the problem. By collecting, organizing, and analyzing data, students will then generate the solution for the problem. We hope students will actively engage in this problem solving process.

(2) About strategies for "lessons in which students feel the values of learning"

In the mathematics group, we consider "values of learning" is realized in experiencing the merits of mathematics. The strategies necessary for "lessons in which students feel the values of learning" are as follow:

① Strategies to devise lessons in which students have questions and the sense of expectations
  - Set up the learning tasks that generates "questions."

② Strategies to devise lessons in which students are engaged independently and with each other
  - Investigate "questions" that must be asked.
  - Anticipate how a series of "questions" may be generated.

③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction
  - Record "questions" on blackboard and reflect on the learning.
  - Secure time for students to write reflective journal entries.

In this unit, we will set up the learning tasks that are easy for our students to relate to by picking situations from their everyday life. Thus, it is hoped that students will feel that they are dealing with problems that are relevant to them. As they progress through their problem solving processes, they will generate their own questions and make own decisions. We will try to anticipate what questions students might generate and how they may be connected by carefully observing them during the lessons and reading their reflective journal entries.

(3) About methods for assessing the quality of individual student's learning

As a strategy to assess the quality of individual student's learning, we will make use of their notebooks. We have been encouraging students to make their notebooks align with the process of problem solving lessons, "grasping the learning task → independent problem solving → comparison and critical reflection → reflection." Thus, in the independent problem solving stage, students will write their own ideas. In the comparison and critical reflection stage, they try to record their peers' ideas. When they do so, instead of simply writing down the answers, we have encouraged them to include the steps and process of getting the answers, using words, pictures, diagrams and mathematical expressions. By writing their learning journal entries, students can organize their ideas, reflect on them deeply and make use of the ideas in new problem situations. This way, students can reflect on the development of their ideas. By checking students' entries in independent problem solving/comparison and critical reflection/learning journal against each other, we will know what ideas students initially had, what challenges they faced, and how their ideas evolved. In this way, we want to assess our efforts to increase the quality of individual student's learning.

In this unit, students will be solving a common problem in collaboration instead of each one solving his or her own problems. As students devise a plan, figure out the method of data collection, and chose ways to analyze the collected data, we will try to grasp each student's ideas from their notebooks and reflective journal entries. We want to make use of students' ideas in the following lessons. Moreover, we want to carefully observe what each student thinks about to reach his or her conclusion as students discuss their ideas with each other so that we can assess individual student's learning.

4 Unit Plan and Assessment (Total of 10 lessons)

#	Goals	Learning Activity	Assessment Standards
① How to analyze data (6 lessons)			
1	○ Students understand mean as a representative value.	• From a table, summarize the data by identifying features such as the greatest value, the least value, and the total. • Think about the usefulness of using means to compare two more more sets of data.	[Interest] Students recognize the usefulness of using means to compare sets of data. [Knowledge] Students understand that we sometimes use means to analyze characteristics of data sets.
2	○ Students can observe and think about the spread of data.	• Students investigate how data are spread from a table. • Students note that if they investigate how data are spread then it will be easier to see characteristics of a data set.	[Thinking] Students think about th eneeds for investigating the spread of data and analyze a data set statistically.
3	○ Students understand the methods to organize a data set into a frequency distribution table, and they can also interpret a frequency distribution table.	• Students will organize the spread of data in a table. • Students will identify the frequencies of data in various classes and also calculate the relative frequency of each class to the total number of data.	[Skills] Students can organize a data set in a frequency distribution table. They can interpret a frequency distribution table. [Knowledge] Students understand that if we investigate the spread of data, we can more easily see characteristics of a data set.
4	○ Students understand how to draw and interpret bar graphs.	• Students identify characteristics that can be observed from a graph.	[Skills] Students can draw and interpret bar graphs. [Knowledge] Students understand that if we organize data in a bar graph, we can more easily see characteristics of a data set.
5	○ Students will deepen their understanding of methods of analyzing data by summarizing what they observed using different statistical methods.	• Discuss what they noticed about various ways of comparisons and conclusions obtained.	[Thinking] Students can identify and explain trends in data using tools such as representative values and histograms.

② Use of data (5 lessons)			
1	○ Students can identify a problem and clarify the purposes for collecting data.	• Look for and identify a problem and clarify the purposes for collecting data.	[Interest] Students try to identify a problem and examining the purposes.
2	○ Students can devise a plan for conducting a study.	• Devise a plan for data collection. • Identify various conditions for data collection.	[Interest] Students try to devise a plan to collect data purposefully.
3	○ Students can collect data purposefully.	• Collect data purposefully and organize them in a frequency distribution table.	[Interest] Students try to represent data in a frequency distribution table. [Skills] Students can represent data in a frequency distribution table.
4	○ Students can analyze the collected data (measurements) and think about ways to compare data sets that match the purposes they have identified.	• Analyze the collected data (measurements). • Based on the purposes they have identified, generate their own interpretations of data.	[Thinking] Students can identify trends in data sets by using representative values and bar graphs, and they can explain their reasoning.
5 Today's Lesson	○ Students can organize data and identify trends so that they can solve the identified problem. They can explain their reasoning.	• Identify trends in data and explain their own interpretation.	[Interest] Students become interested in making use of frequency distribution tables, histograms, relative values, etc., to solve problems, and they are trying to apply them in other problems. [Thinking] Students can identify and explain trends in data using representative values and histograms.

5 Today's Lesson

(1) Date: Saturday, June 24, 2017

(2) Location: Yamanashi University Attached Elementary School

(3) Goal of the lesson

○ Students can organize data to solve problems and explain their reasoning.

(4) Reason for teaching this lesson

Problem B[5] in the 2016 National Assessment asked students to explain mathematically the reason why they made a certain judgment based on data. The report of the assessment recommends that "it is important to help students make appropriate judgments based on trends in data through activities in which they determine representative values of data or examining the spread of data." In today's lesson, students will be presented with a problem situation that is familiar in their everyday life. It is hoped that students will be able to discuss the issues mathematically using what

In order to greet as many people as possible, from what time to what time should we be standing?

they observed in data as the rationale.

This lesson is the second of a 2-lesson sequence in the second sub-unit of the unit. Students have been trying to find the answer to the question, "" They have collected on what time students tend to come to school, and they have determined representative values and summarized the dat in a frequency distribution table and a histogram. Based on the data, they have reached their own conclusions.

In today's lesson, students will share the conclusions they have reached about what time they should be coming to school. First, we will focus on various representative values for the data set, and discuss what we can observe from those.

First, we will discuss the idea of mean. Students have learned about mean in Grade 5. In addition, in the sub-unit 1, students used means while comparing data from multiple groups. The word "mean" is used in our daily life frequently, therefore, we expect it is easier for students to think of mean as the representative vale that other measures. On the other hand, if we draw a conclusion on what time to be standing based only on the mean, it is not necessarily the case that the data will be around the mean. Thus, we need to question the validity of using the mean to make a conclusion. We also need to consider mode, the frequency districution table and/or the histogram.

Next we will turn our attention to those who considered mode. This discussion may help those who only used the mean to make decisions realize that looking at the mean alone will not be sufficient. Moreover, in addition to considering the mode, if we look at the histogram, we can see the time frames that are second or third most frequent as well. Based on students' own experiences, they may hypothesize that those peak times may relate to the bus schedule. Based on this hypothesis, some students may want to create a histogram for those students who are coming to school by buses alone. This way, we will create 2 different histograms, one for those students who come to school by buses and one for those who walk. The reason for creating the 2 histograms is to give students an experience that by doing so they can actually make new interpretations. This will, in turn, develop the ability to more critically examine results obtained from the analysis instead of simply creating a graph. They will have the ability to reflect on the data and make better decisions. Because the experience of creating 2 different histograms is critical, if that idea does not come from the students, the teacher will make this suggestion.

Based on the histogram for bus riders, the class will discuss what they can observe from the graph and make suggestions about what time they should be standing to greet others. Some students might notice that the suggestions based on the histogram for bus riders only will focus on narrower range of time than the suggestions made from the histogram for all students. We want to emphasize the values of making conclusions by comparing 2 different graphs.

After discussing the histogram for bus riders, some students may wonder about the distribution of times for those students who come to school on foot. Based on that wondering, we will examine the histogram for walkers, and try to interpret the graph. Finally, based on the 3 histograms, total, bus riders and walkers, the class will come up with the conclusion when they should be standing to greet arriving students.

(5) Flow of the lesson

<table>
<tr><th>min.</th><th>Main learning activity/content
Anticipated responses</th><th>• Points of consideration
• Strategies for “lessons in which students feel the values of learning”</th></tr>
<tr><td>5

G
R
A
S
P</td><td>1. Look back on the previous lesson
• Make sure students understand the time for the "meet & greet"
○ We were standing between 8:05 and 8:15.
○ It was for 10 minutes.
• Make sure students understand the issues for the "meet & greet"
○ Not many students came.
○ It is hard for the members of the students council to be at school so early.

2. Understand the task and devise a plan

In order to greet as many people as possible, from what time to what time should we be standing?</td><td>In the process of “lessons in which students feel the values of learning”
⦿ Strategies
• Motivate students to tackle the task with the understanding of the problem situation.
• Set up the learning tasks that generates “questions.”

• Make sure students understand the issues in the "meet & greet"

• Instruct students to make recommendations and the reason whey</td></tr>
<tr><td>35

E
X
P
L
O
R
E</td><td>3. Develop recommendations based on representative value and histograms

"I wonder what time is the best if we use the mean arrival time?"
• Thinking about the mean arrival time, 7:53.
○ 7:48 - 7:58 --- ①
5 minutes before and after 7:53.
Because this is the mean, I think we get a lot of students coming around that time.
○ 7:50 - 8:00 --- ②
I included the mean arrival time and surrounding time with a lot of students.
○ Both cincludes the mean arrival time of 7:53.</td><td>• From the journal entries from the previous lesson, bring up the idea of the mean.

In the process of “lessons in which students feel the values of learning”
⦿ Students who feel the values of learning
• Expaling the conclusions they reached, listen to their friends' explanations, and look for similarities and differences.
• Through data analysis, grasp trends in the data set.

• Ask students to identify commonalities. Make sure students see that all ideas include the mean arrival time.</td></tr>
</table>

	• Thinking about other times ○ Many of bus riders arrive at different times. ○ There are a lot at 7:45. ○ There are a lot at 7:55, too. ○ I walk to school, and I'm usually at school earlier. • Thinking about the mode, 7:57 ○ 7:52 - 8:02 --- ③ 5 minutes before and after 7:57. Because many students arrive then. ○ 7:55 - 8:05 --- ④ I used the interval with the most students and the interval to the right. ○ 7:50 - 8:00 --- ⑤ I used the interval with the most students and the interval to the left. ○ ⑤ includes the mean arrival time, too. **"I wonder what time is the best if we use the graph?"** • Thinking about a histogram. ○ With ①, it includes an interval with small number of students, so we can't greet that many students. ○ I don't think we can tell which interval will have a lot of students. ○ I think either one of the intervals based on the histogram is the best, 7:50 - 8:00 or 7:55 - 8:05. • Think about the second most frequent interval. ○ I think 7:45 - 7:50 is because of bus riders. ○ I arrive at that time, but I walk to school. ○ I think it would be helpful to have a histogram for bus riders only. 4. Make recommendations using the histogram for bus riders **"I wonder what time is the best if we use the histogram for bus riders?"** • Share what they observed from the histogram	**In the process of "lessons in which students feel the values of learning"** ⦿ Students who feel the values of learning • Expaling the conclusions they reached, listen to their friends' explanations, and look for similarities and differences. • Through data analysis, grasp trends in the data set. • Help students realize that if they use other tools in addition to the mean, they can make a better decision. **In the process of "lessons in which students feel the values of learning"** ⦿ Strategies • Students generate their own "questions" from the task. "I wonder what time is the best if we use the mean arrival time?" "I wonder what time is the best if we use the graph?" "I wonder what time is the best if we use the histogram for bus riders?" "I wonder what time is the best based among all of the recommendations?" • Make sure that the mean arrival time is clear in the histogram.

	for bus riders. ○ There are a lot of students arriving 7:55 - 8:00. ○ 7:45 - 7:50 is the next. ○ There are very few before 7:35. ○ I thought many arriving 7:45 - 7:50 are bus riders, but many of them are actually walkers. • Make a recommendation for the best time. ○ 7:50 - 8:00 --- ⑥ Because this interval has the most students in the histogram for bus riders. It's the same for the whole student body. • Share what they observe in the histogram for walkers. ○ 8:00 - 8:05 has the most students. ○ 7:55 - 8:00 also has many students. ○ Unlike for bus riders, the studeans are spread out more widely. • Make a recommendation for the best time. ○ 7:55 - 8:05 --- ⑦ There are most walkers arriving in this time interval. **"I wonder what time is the best based among all of the recommendations?"** • Make recommendations using the mean and the histograms. ○ 7:50 - 8:00 Because it came up in ②, ⑤, and ⑥. ○ 7:55 - 8:05 Because the most popular intervals are different for bus riders and walkers. ○ We split ourselves to 2 groups, one for 7:50 - 8:00 to greet bus riders and the other for 7:55 - 8:05 for walkers.	• Using a spreadsheet softwared, display histograms for bus riders and walkers. • Acknowledge the value of comparing the 2 histograms, one for bus riders and one for the entire students. • Help students realize that the data are more wide spread compared to the data for bus riders. • Even though the total interval becomes 15 minutes, accept as a recommendation.

	5. Look back on today's lesson • First, reflect on the main task. ○ Not many students came at the time we tried ➔ We learned from the graph that there aren't many students arriving in 8:05 - 8:15. ○ Early morning is difficult for the council members ➔ There are many students who arrive early, too. • Develop a class recommendation. • Write reflective journal entries. ○ I was glad that we were able to make a recommendation for the "meet and greet." ○ By separately looking at bus riders and walkers, we were able to make a better recommendation.	**In the process of "lessons in which students feel the values of learning"** ⦿ Methods for assessing the quality of individual student's learning • Through students' reflective journal entries, assess if lesson goal was met. • From students' notebooks, assess how students ideas developed across the time. • From reflective journal entries, assess the effectiveness of the lesson and students' learning.

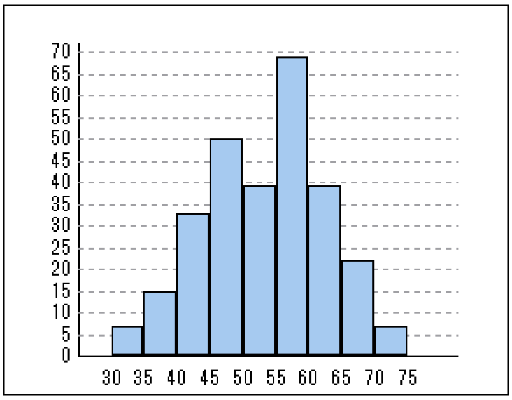
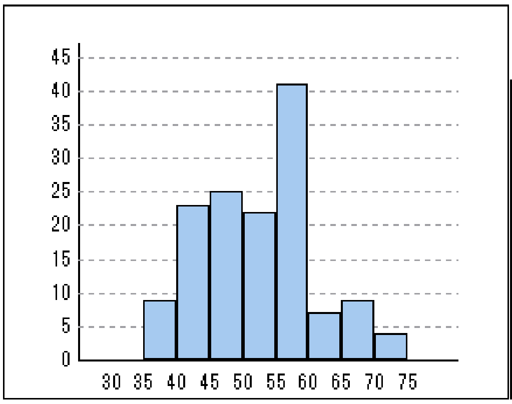


Figure 2 Histogram for the whole student body



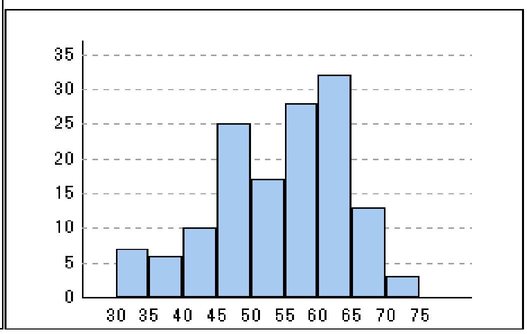


Figure 3 Histogram for bus riders

Figure 4 Histogram for walkers

(6) Points of observation

- Were the strategies to help students feel the values of learning?
  ① Strategies to devise lessons in which students have questions and the sense of expectations
  ② Strategies to devise lessons in which students are engaged independently and with each other
  ③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction

(7) References

Omitted

Lesson Report

(Annotate with pictures, quotes, student work examples, board work etc.)

Report created by: Laura Schmidt-Nojima, Lauren Goss, Nora Houseman
Name of Lesson: Grade 6 Mathematics: Organizing Data
Date of Lesson: June 24, 2017

What are the primary lesson goals?

Research Focus: "Helping students purposefully collect data, organize them in appropriate tables and graphs, and then identify trends in the data set by observing the representative values and the distribution."

Unit Goal: "Students understand mean as the representative value of a data set, spread of a data set, and frequency distribution, and make use of those ideas purposefully to examine situations statistically and make appropriate representations."

Lesson Goal:

- Goals:
  - Students can organize data and identify trends so that they can solve the identified problem. They can explain their reasoning.
    - "In order to greet as many people as possible, from what time to what time should we be standing?"
- Learning Activity:
  - Identify trends in data and explain their own interpretation
- Assessment Standard:
  - *Interest:* Students become interested in making use of frequency distribution tables, histograms, relative values, etc. to solve problems, and they are trying to apply them in other problems.
  - *Thinking:* Students can identify and explain trends in data using representative values and histograms.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

This lesson occurs 10th in a series of 10. The topic of the first series of 5 lessons is "how to analyze data." The second series of 5 lessons is focused on "use of data." This lesson is the second of a 2-lesson sequence in the second sub-unit of the unit. Students have been trying to find the answer to the question, "In order to greet as many people as possible, from what time to what time should we be standing?" They have collected data on what time students tend to come to school, and they have determined representative values and summarized the data in a frequency distribution table and a histogram. Based on the data, they have begun to reach their own conclusions as answers to the question.

Summary of Anticipated Strategies

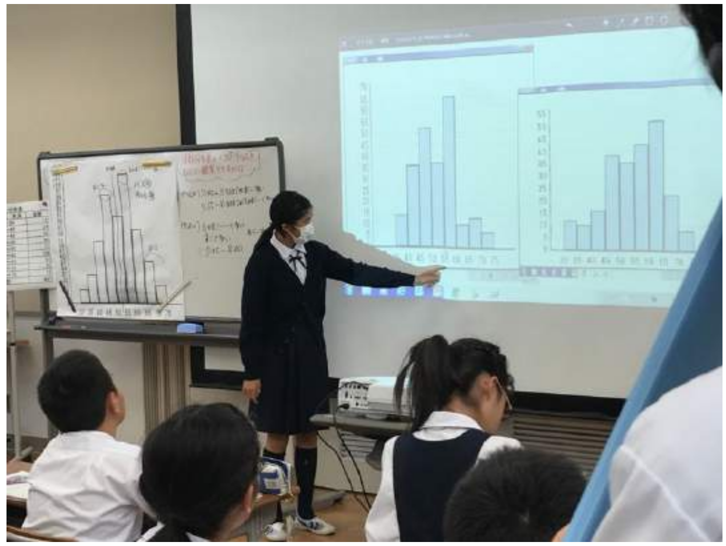
- Use of the mean to determine time for the "meet & greet" (identify mean - 7:53 - and select 5 minutes on either side of the mean for the 10 minute time frame)
- Use of the mode to determine time for the "meet & greet" (identify mode - 7:57 - and select 5 minutes on either side of the mode for the 10 minute time frame)
- Selecting the 5 minute interval with the most students (7:55-8:00) and the interval to its right or left (they are identical)
- Selecting the 5 minute interval with the most students (7:55-8:00) and then using the frequency tables broken down by walkers and bike-riders to select between the intervals to the right or left (based on representation of students by walkers vs. riders)

Summary of Lesson

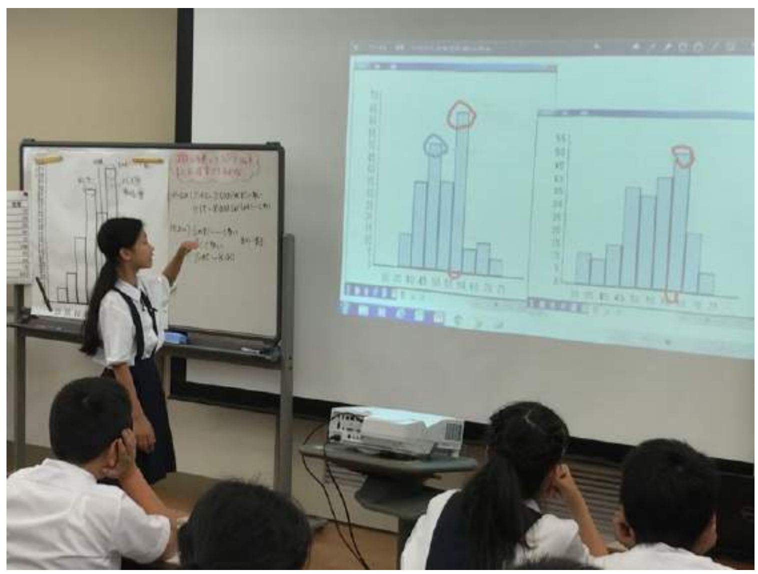
Start & End Time	Lesson Phase	Notes
10-10:20	**Introduction, Posing Task**	**Strategies to build interest and to connect to prior knowledge** Teacher reviewed the task from the day before, including a 10 min time frame for greeting. Students noted that they used the mean and or the mode to figure out a time recommendation. Teacher noted that other factors could affect when students arrive. Students called out, volunteering: rain, late train/bus, oversleeping etc. At 20 min, students said that they would need to use histograms of walkers and bus riders to make other recommendations.
10:20-10:42	**Independent Problem Solving**	**Individual, pairs, group, or combination of strategies** ▪ **experience of diverse learners** ▪ **teacher's activities** Teacher had a student pull up the histograms for the walkers and bus riders. The discussion continued whole group. Teacher asked repeatedly, based on the data, what summary can we make? How do we know? Are we sure? Where is that in the data? What do you notice?

		Students noted that they would need to use the data from both the walker and bus rider histograms. 13 different students spoke. 3 of the 13 spoke several times. When they looked at 2 charts of arrival times, the class noted that the scaling is different and the teacher adjusted the size of the charts to better compare them. Teacher asked students, “What can you recommend?” At 42 minutes, the students had group work in pairs for 2 min. Teacher circulated around the room, checked in with one group. Teacher asks whole group for recommendations. 4 students came to the front to explain to the class and the work is noted below. 

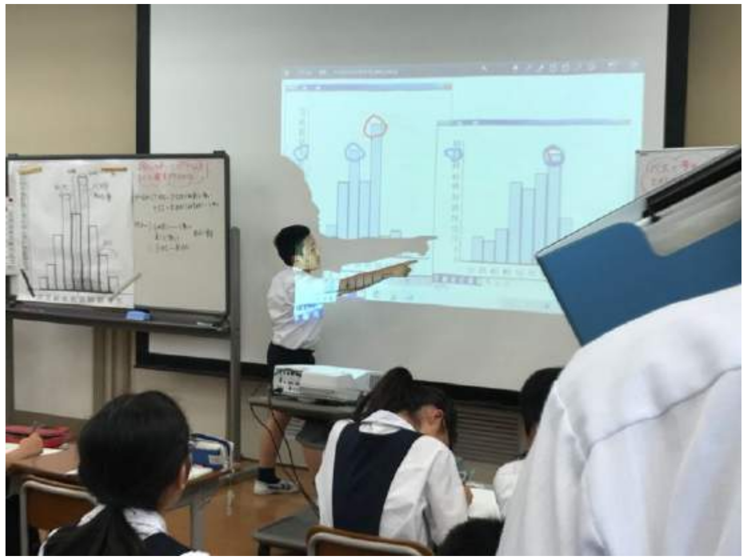
		
	Presentation of Students' Thinking, Class Discussion	**Student Thinking/ Visuals/ Peer Responses/ Teacher Responses** **Student 2 indicates where on the histogram indicates the most students arrive**  **Student 31 compares the time frame on the 2 histograms**



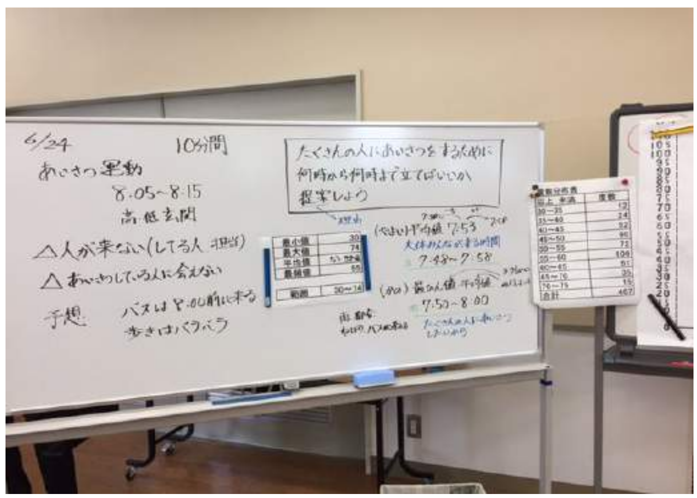
Student 23 makes a recommendation based on the totals from both histograms.

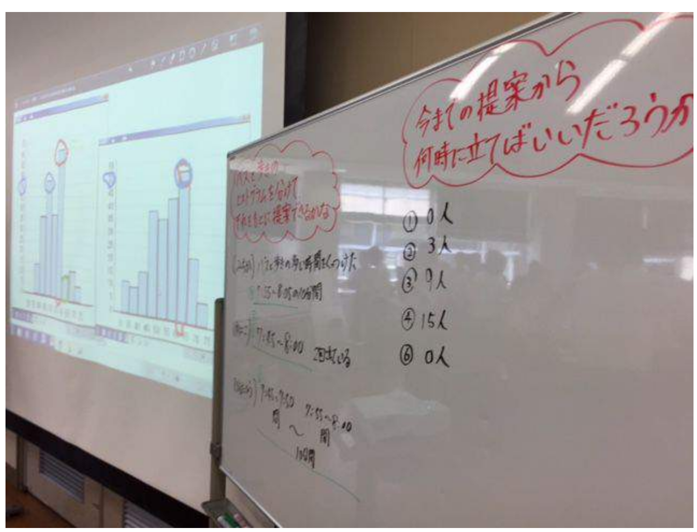


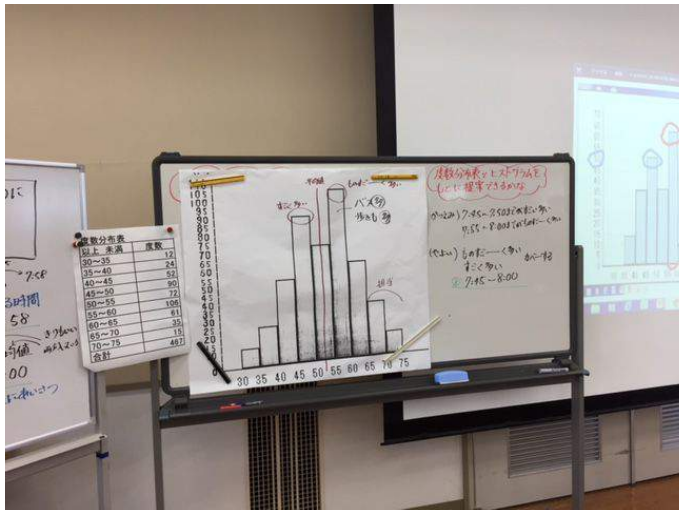
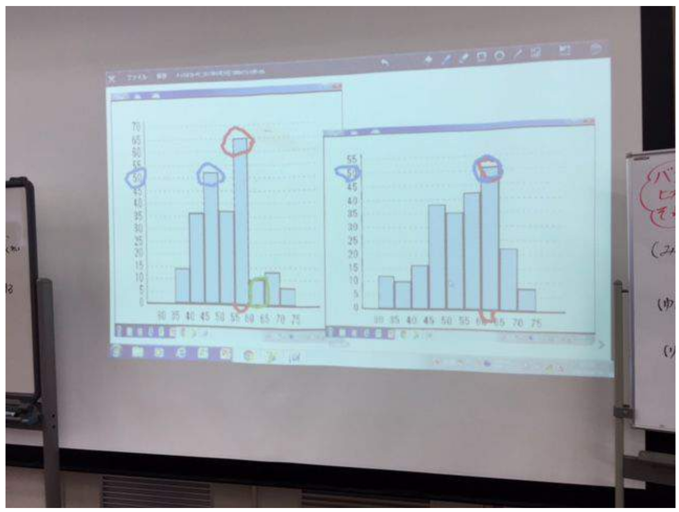
Student 18 notes that the number of students is the same if the walkers and riders are considered as totals for 2 timeframes and so there is no clear recommendation.

		
10:50-10 :55	**Summary/ Consolidation of Knowledge**	**Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals** Teacher states that we have many recommendations. Teacher asked students to vote for which proposal was best #1- 0 votes (time frame: 7:48-7:58) #2- 3 votes (time frame: 7:50-8:00) #3- 9 votes (time frame: 7:45-8:00) #4-15 votes (time frame: 7:55-8:05) #5- 0 votes (time frame: 7:45-7:50 & 7:55-8:00) Teacher told students to think about it at home and to bring a reflection on Monday.

Board Organization at end of class





What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?

Choice of task/problem for the lesson:

It is critical to have a strong rationale for the choice of the math lesson that illuminates the need to use math in an authentic context.

- What was the point of this lesson? This lesson did not provide a rationale for using statistics - thus students did not learn the value of math for problem-solving.
    - There was not a strong purpose for presenting the information on walkers vs riders as the questions was not “when is the best time to meet/greet” but rather “in order to greet as many people as possible, what time should we be standing”. As a result, there was not a mathematical purpose for investigating the additional histograms, as the goal was not to consider how to be

representative of walkers and riders, but simply to determine a time frame for when the most people will arrive at school.
    - Further, the time-based arrival data from one day was not necessarily consistent beyond that day, thus the data may not have even been useful for solving the problem generally - this may have reinforced mathematical misconceptions about the use of data.

A genuine TTP lesson must allow students to authentically grapple with the math, reach an AHA moment (climax), and consolidate their understanding through discussion and summary

- This was not really a problem-solving lesson. It was whole group, teacher-guided conversation for 90% of the lesson. Students had minimal opportunities to think independently or grapple independently or collectively. Students were unable to truly consolidate their understanding as they were not asked to provide a rationale for the time frame chosen that accounted for the pros and cons of that choice over others.

A genuine TTP lesson builds deep student investment in solving the problem by building curiosity (hooking students), creating an authentic need to solve (relevancy, high interest), and illuminating the need to use math to solve.

- Students were not all invested in solving the problem - there did not appear to be a strong desire to resolve the problem, thus the lesson lacked genuine authenticity and relevance. It was noted that most students were interested in the problem context of the Student Council needing to greet the maximum number of students in a given time frame.

Teacher choices within the lesson:

- *Graphs:* Choice of 5 minute intervals - was this the right choice? Should students have investigated how their recommendation might change by analyzing 1 minute intervals, 10 minute intervals, etc.? Students may have benefited from examination of other intervals as the data presented different patterns when organized by other intervals - an important concept to understand about data in this context and data generally (bias in how data organized and presented).

- *Pacing and Independent Work:* Students needed more time to investigate the new histograms (when they were introduced into the lesson) and make their own recommendations based on the additional information (histograms of walkers vs. bus riders). The teacher guided this portion thus students did not have time to grapple with the data and make meaning themselves - and as a result it was not deeply internalized or understood by all.

- *Class Discussion:* When making recommendations about time intervals, students needed to be probed to assess and articulate the strengths and weaknesses of each recommendation. In order to internalize the complexities of the scenario (and the math applications re: data and statistics - specifically the benefits and drawbacks of the mean vs mode vs frequency tables), students needed to articulate the pros/cons of each approach during the class conversation.

Other

Student voice:
- Speaking stats from the lesson:
  - male students spoke 25 times
  - female students spoke 21 times
  - the teacher spoke 38 times (comprising 33 minutes of the roughly 55 minute lesson)

  The gender breakdown was relatively balanced, which we were relieved to see as we noticed most classrooms visited during IMPULS were either very balanced or dramatically unbalanced (for example, in one room male students spoke 28 times and female students spoke twice).
- We were concerned, however, in the imbalance of teacher talk time vs. student talk time. As noted above, the lesson was teacher-driven and most thinking/talking was done by the teacher, not the students. As a result, students were unable to truly grapple, arrive at independent ideas and conclusions, and consolidate understanding.

Student engagement:
- In tracking student participation/engagement (on task vs off task observable behavior), students were on task for the entirety of the lesson and very few students engaged in visibly off-task behaviors. However, the 3 girls in the back row and a few on the sides were occasionally distracted (giggling and chatting about non-math topics, looking around the room, etc.). Given the high number of the students in the class, it would be important to consider how to hold the attention of students on the periphery of the seating arrangement.

What new insights did you gain about how administrators can support teachers to do lesson study?

Supporting with coordinating and implementing a Research Steering Committee
- Organizing a process to nominate/elect teachers to committee from each lesson study team

- Coordinating meeting schedules/timelines for all teams - including meeting times and research lesson dates for each lesson study team
- Facilitating or coordinating process of choosing and writing a research theme for school
- Facilitating or coordinating process of reflecting on learnings from lesson study and writing research report

Supporting with identifying relevant input in relation to the research theme (books, articles, videos, collaborators, etc.)
- importance of bringing in new knowledge & new strategies to the LS process (to build knowledge base and tool-kit of participating educators)
- Importance of building from research-based practices

Supporting with identifying and inviting knowledgeable others/expert commentators (from local universities, from district departments, from other school sites, from external/grant partners)
- importance of an outsider perspective
- importance of bringing in new knowledge/perspective to the LS process

Supporting teacher to be reflective and open to feedback in the post-lesson discussion (as opposed to summarizing the lesson or defending his/her own teacher moves)
- Importance of building a culture of self-reflection and vulnerability

Supporting all teachers to build comfort giving critical feedback
- Importance of building an adult culture of peer-to-peer critique and open classroom practice

How does this lesson contribute to our understanding of high impact practices?

Teaching *with the textbook* versus *teaching the textbook:*
- This teacher, like all we have observed, did not teach directly from the textbook. He adapted and used the textbook as a base. We strive to utilize this framework for textbook (in)dependence moving forward.
- However, this teacher also created an alternative plan to the textbook that was not necessarily stronger than the textbook (in fact, it was arguably worse). It is a tricky balance to determine when to draw from the textbook and when to stray from the textbook - a decision hopefully carefully debated by the lesson study team.

Teacher responsiveness to students during the public lesson (or any lesson):

- This teacher followed his anticipated lesson plan with little deviation from the plan. This is a pattern we have seen across classrooms.
- It is important not to create one pathway in a lesson plan, but rather multiple, flexible pathways so the teacher is prepared to be responsive to the students in the room and the direction in which the lesson/discussion authentically moves.
- The teacher should begin with his/her lesson plan then throw it out (literally put it down and away) so s/he can be fully responsive to students during the lesson and truly hear and build from student thinking/analysis.

Effective pacing:

- This teacher did not create time for students to engage independently, apply learnings beyond the problem context, or make meaning of their learnings (summary) from the various graphs presented.
- Effective pacing is critical, so students can reach the AHA moment (authentic climax) of the lesson, then have time to grapple deeper, then have time to consolidate learning through the summary and reflection. The math content is not fully accessed until this full cycle has occurred, and without effective, strategic pacing students can be left with greater misunderstandings, gaps in understanding, or in frustrated confusion. A teacher must carefully monitor time to ensure opportunities for students to apply new learning (try problem/rule in a new context and/or with new numbers) and consolidate understanding of that new learning (generate summary).

Independent work:

- This teacher did not create opportunities for independent grappling or productive struggle beyond the one partner talk. There was almost no independent work time as the lesson was primarily teacher-led.
- American teachers are taught to minimize student confusion; however, this can easily eliminate opportunities for students to engage in productive struggle. Teachers therefore must make careful choices about whether to provide additional support as students grapple with a problem/task. Giving “hints” narrows student thinking and student reasoning options, thus generally, the teacher should not provide additional support or hints. However, it is the responsibility of the teacher to:
    - formatively assess independent work to identify patterns/outliers and strategically choose students work to present
    - Support students that can’t tackle the problem and are stuck

  Thus the teacher DOES need to intervene when a student cannot find an entry point into the problem, through “hints” or small group support. Generally, however, the teacher should allow the student to grapple independently, unless they have reached/exceeded the frustration point.

Contrived vs authentic partnering and group work:

- This teacher created space for a partner talk but otherwise led the class through a lengthened student discussion (with no other partner/group work and no independent work time).
- It is critical to have a rationale for why partner work or group work is included in the lesson. Ideally, these structures should allow for deeper learning, not simply be included for the sake of working in a group. A teacher should consider whether working with others, as opposed to individually, will foster greater student thinking. If groupwork is selected, strategic choices should be made about group size, group roles, the assigned task of the group, and how the group will manage and maximize time. Groupwork is a tool like any other, requiring strategic planning, and enough front-loading and coaching so students are able to utilize the time to extend and push their own thinking beyond what they can accomplish working alone.
- However, we wonder about the lack of oral participation in many Japanese classrooms when opportunities for pair-sharing and group work are rare. Articulating concepts orally, explaining or defending to others, and engaging through language is another way to build deeper understanding. Without opportunities to do so, are all students truly maximizing their learning? While we appreciate the focus on authentic partner and group opportunities, we also wonder about the lack of oral participation and the inequities of voice (who is/n't speaking) in many classrooms. What would authentic engagement and equitable participation look like in a Japanese classroom?

Student-centered learning:

- This teacher generally paced his lesson with the students that chose to engage with him in the whole class discussion. Often this was the students who shouted out or raised their hands first.
- What is a student-centered lesson? If a teacher is building off student responses but it is the responses of the quickest students or students with the most understanding, is this truly student-centered? This teacher, like many we have seen, led his class discussion through those students that chose to engage and were ready first with a response. Rather than build from the first students to respond, we would love to see this teacher survey the class (during independent work time), identify patterns and outliers in student work/thinking, then strategically craft the narrative for the class conversation based on whole class data, not the student(s) shouting out or finishing first.

Instructional emphases at Kiyose Lower Secondary School No. 4

At our school, we aim to develop students who can think for themselves and express their ideas effectively. To do so, we are continuously trying to improve and enrich communication activities in teaching of all academic subject matters, moral education, extracurricular activities and other school events. In addition, we want our students to develop important human characteristics for successful self-expression such as their ability to interact with others and various social groups, disposition to collaborate, and strong will. For that end, we are trying to enrich various experiential learning opportunities such as extracurricular activities and integrated-study periods.

In the mathematics department, we also teach our everyday lessons aiming at "increasing students' motivation for learning, " "students' mastery' of knowledge and skills," and "developing students' abilities to reason, judge, and express." Furthermore, we aim at providing better problem solving situations so that students can "enhance their communication activities," "improve their ability to apply their knowledge and skills," and "develop their ability to learn collaboratively."

June 26

Grade 9 Mathematics Lesson Plan

Date: June 26, 2016
Period # 6
Kiyose Lower Secondary School No. 4
Grade 9 Classrooms A & B
(Standard course, 24 students)
Teacher: GINNAN Yuzo

I. About the Unit

1. Name of the Unit: Square Roots [Grade 9 textbook by Tokyo Shoseki, Ch. 2]

2. Unit Plan

Sub-Unit	Section	Goals	#
1. Square Roots	① Square Roots	• Students understand the meaning of square roots, and they can find the square roots of given numbers. • Students can represent the size relationships of square roots using inequality symbols. • Students understand the meaning of rational numbers and irrational numbers, and they can sort the numbers they have learned into the appropriate types.	4
	② Prime Factorization	• Students understand the meaning of prime factorization, and they can factorize the given natural numbers. In addition, students can determine the square roots of natural numbers by using prime factorization.	1
	• Basic Problems		1
2. Calculations of expressions with square roots	① Mult. & division of expressions with square roots	• Students can calculate expressions involving multiplication and division of square roots. • Students can manipulate expressions with square roots. They can also transform the given expression to an equivalent expression to determine the approximate value of the expression. • Students can rationalize the denominator. • Students can devise ways to calculate expressions with square roots.	4
	② Addition and subtraction of expressions with square roots	• Students can simplify expressions with square roots by combining like square roots. • Students can calculate expressions with unlike square roots by transforming square roots.	2
	③ Calculations of various expressions with square roots	• Students can calculate and determine the value of expressions with square roots by making use of the distributive property and the multiplication formulae.	1
	④ Application of square roots	• Students can identify situations involving square roots in their surroundings.	1
	• Basic Problems		1
Chapter Problems			2

3. Goals of instruction

The main goal of this unit is to expand the numbers to irrational numbers as we expanded the numbers to negative numbers in Grade 7. Students will learn that we can perform the arithmetic operations with irrational numbers as well as properties of operations such as commutativity, associativity and the distributive property remain true with irrational numbers. We want students to truly understand how the world of numbers are expanding; therefore, instead of simply telling students how to perform the arithmetic operations, we want students to investigate and discover ways to calculate with irrational numbers while imagining the actual quantities irrational numbers represent.

In the introduction of the unit, students tackled the problem of finding the dimensions of the square with the area of 50 cm^2. In that lesson, students actually folded a 100 cm^2 square paper to create a square with the area of 50 cm^2 so that they can have a concrete image of that length. Students can also experience that the number we are considering is not a part of the numbers they have learned previously by actually trying to find the number that equals 50 when it is squared using a calculator. In addition, when comparing the sizes of square roots, they tried to estimate their sizes by making use of special cases such as square roots of 4 and 9. They can also compare the lengths of sides of squares of area 2 and 5 to compare $\sqrt{2}$ and $\sqrt{5}$. We believe these experiences help students as they think about ways to calculate.

As they continue their study of square roots, some students may wonder why irrational numbers can be represented by fractions. In this unit, we will touch upon the fact that an irrational number cannot be expressed as a fraction, $\frac{a}{b}$, as well as how repeating decimals can be converted to fractions. However, the purpose of the discussion is not to develop the expertise in proof by contradiction or converting repeating decimals into fractions. A more in-depth treatment of those topics will be done in the 10^{th} grade mathematics, and the discussion in this unit is simply to serve as a bridge to the Upper Secondary School mathematics.

4. Assessment Standards

Interest, Eagerness, and Attitude Toward Mathematics
• Students will be interested in thinking mathematically by using square roots to make sense of various phenomena and examine relationships among them. Students are eagerly seeking ways to make use of square roots to solve mathematical problems.
Mathematical Way of Thinking
• Students will master mathematical ways of observing and thinking such as applying the basic knowledge and skills about square roots of numbers to think logically and represent mathematically.
Mathematical Skills
• Students calculate expressions including square roots of numbers. They also master the skills to represent and process situations using expressions involving square roots of numbers.
Knowledge and Understanding of Numbers, Quantities, and Geometric Figures
• Students understand the meaning and the need for square roots of numbers.

II. Instruction of the Unit

1. Topic

Chapter 2, Square Roots, Sub-Unit, 2. Calculations of expressions with square roots, 4, Applications of Square Roots

2. Goal

Students can identify square roots of numbers in their surroundings.

3. About the students

For the mathematics instruction at our school, we have been splitting students into 2 groups (Basic course and Standard course) based on their mastery of prior materials. The research lesson is with the students in the Standard course, and many students are willing to participate in mathematics lessons actively and they have, in general, mastered the basic skills and procedures. They do not hesitate to share their ideas during the class, and they eagerly tackle problems that require students to think differently or explain their reasoning.

4 Key points of the lesson

In this lesson, students are introduced to the fact that irrational numbers are being used in many situations in our everyday life. In the recent lessons, we have focused on calculations with irrational numbers, and students may have developed the image that irrational numbers were created to keep the calculations consistent. Thus, we want students to realize that irrational numbers are being used in various places in our surroundings.

Students will be shown several books, and we will try to sort them based on their shapes. By doing so, we want students to pay attention to the ratios of the short and the long sides of the books. Then, each student will receive a piece of B-5 paper which is used to make the mathematics textbook. When they are asked to compare the lengths of the short and the long sides, many will fold the paper and notice that the long side is about 1.5 times as long as the short side. After students unfold the paper, they are asked to pay attention to the crease line, and some may notice that the length of the crease line is equal to the length of the long side of the paper. Students will then be asked to determine the length of the long side when the length of the short side is considered as 1, but this might be too difficult for some. Thus, we will have students work in small groups, and through the group investigation, we hope that students will realize that the length of the long side is the side of a square with the area of 2 square units (using the short side of the paper as the unit length), or $\sqrt{2}$ units.

Moreover, when students compare the ratio of sides of the rectangle obtained by folding the B5 paper in half, they notice that the ratio is $1:\sqrt{2}$. Thus, the shape of the paper remains unchanged. Students may be impressed by the wisdom of people who devised these size of papers.

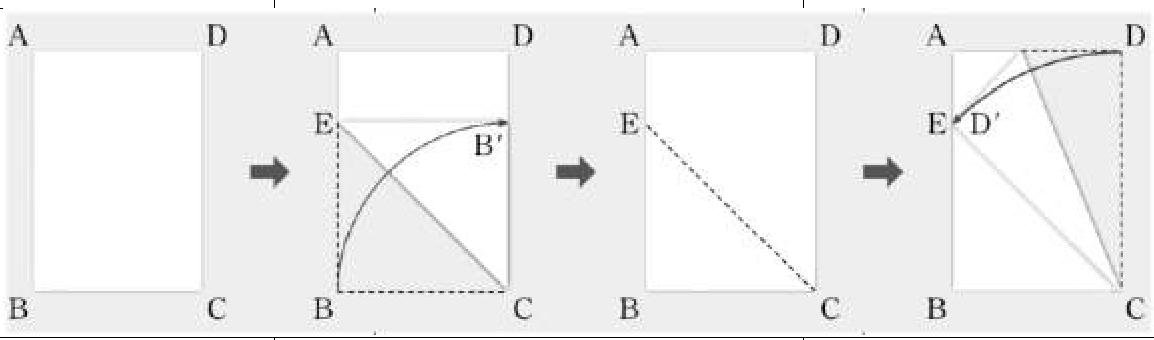
5. Assessment Standards

[Interest] Students will be interested in comparing the ratio of the lengths of the short and the long sides of the standard paper, and they are trying to apply what they have learned about square roots.
[Way of Thinking] Students can explain what the length of the long side is $\sqrt{2}$ if the short side is considered as 1 by using diagrams.
[Skills] Students can determine the ratio of the lengths of the short and the long sides of the rectangle obtained by folding a B5 paper in half is $1{:}\sqrt{2}$.
[Knowledge] Students understand the characteristics of the standard sizes of papers.

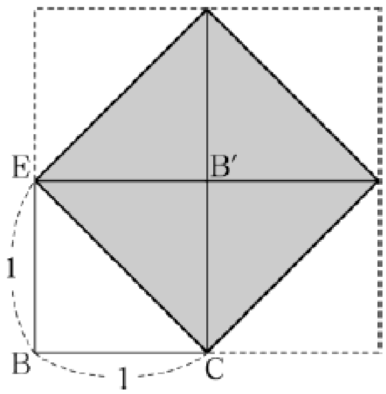
6. Flow of the Lesson

Teacher Moves	Learning Activity & Anticipated Responses	Instructional Points of Consideration
(Show several books of different sizes and shapes to the students.)	● Although the books were all rectangular in shapes, there are many different shapes. Students will realize, however, that some books are the same shape as the shape of their mathematics textbook.	
• Let's sort these books into groups. • What do you mean by the shape of rectangles being the same? • Let's compare the ratio of the lengths of the short and the long side of the paper that is the same size as our mathematics textbook.	S: What do we need to focus on to sort them? S1: Their angles are congruent. S2: Their short and long sides are equal. S3: The lengths of their short and long sides are in the same proportion. S4: The ratios of the lengths of the short and the long side are equal.	• Call on students and have them sort the books. • If students focus on the contents or topics of the books, remind them that we are in a mathematics lesson, and they should focus on the shapes of the books. • If necessary, ask additional questions to help students realize that the ratios are equal.

(Distribute B5 sheets of paper to the students.) • Let's compare the lengths of sides using this sheet of paper. • Can someone share what you noticed about the crease line?	● Determine the ratio of the lengths of the short and the long sides of a B5 sheet of paper. • Many students will fold the paper to match the short side and the long side. S1: The long side is about 1.5 times as long as the short side. S2: I think it is a little shorter than 1.5 times. • Color the crease line and look at the students. S: I think it is the same length as the long side.	 • If no student colors the crease line, suggest them to do so. • Have students fold the paper to verify their observation.

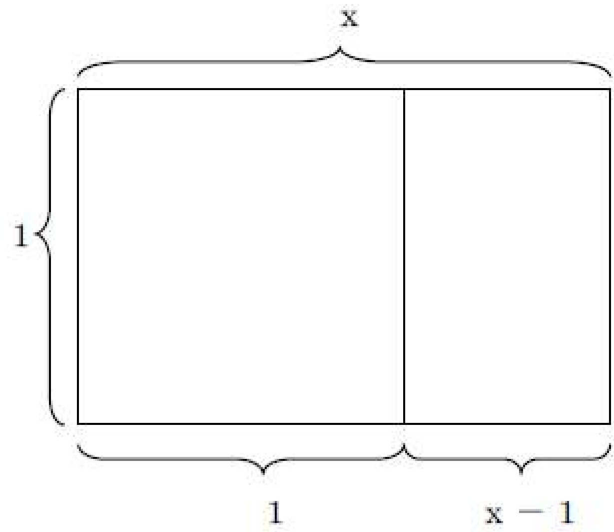


• Please find the lengths of the long side if we consider the length of the short side as 1. • Please share the ratios you determined.	• In groups of 4, students will figure out how to determine the length of the long side when the length of the short side is considered as 1. ● Groups will present that the ratio of the length of the short side to the length of the long side is $1{:}\sqrt{2}$. S: As you can see in the diagram, if we put together 4 copies of the right isosceles triangles together, we can make a square with the area of 2. Therefore, the length of the side must be $\sqrt{2}$.	• Instruct students to use the papers each has folded. • Observe the groups, and have those groups that determined the length of the long side to be ready to present their answers to the whole class. • If others used different approaches, have them share their ideas, too.

• I wonder why they use the ratio of 1:$\sqrt{2}$. Fold the paper in half and share what you notice about the new rectangle.	 ● Students discover that the ratio of the lengths of the short and the long sides of the rectangle obtained by folding a B5 sheet in half is also 1:$\sqrt{2}$. • Students will determine the ration individually. S1: The ratio of the length of the short side to the length of the long side is $\frac{\sqrt{2}}{2}$: 1. S2: The ratio is $\sqrt{2}$: 1. S3: The ratio is 1:$\sqrt{2}$.	• Have students actually fold the sheet and help them observe that the shape of the smaller rectangle is the same as the shape of the original rectangle. Have them verify that the ratio of the length of the short side to the length of the long side is 1:$\sqrt{2}$. • Have someone explain the ratios $\frac{\sqrt{2}}{2}$: 1 and 1:$\sqrt{2}$ are equivalent by manipulating expressions.
• Let's look back on what we did in today's lesson.	● Students will know that the paper size standards such as B5 make use of irrational numbers.	• Have students verify that the size of a common pocket-size books are obtained by cutting an A4 sheet of paper into 4 equal parts. However, the shape of another popular series of pocket-size books is different from those, and help students see that the ratio of the lengths of sides of those books is not in 1:$\sqrt{2}$. This should pique students' interest to want to find the ratio of those sides in the next lesson. (*)

(*) Extension in the next lesson

- In the 2nd type of pocket-size book, the paper is in the shape of rectangle such that the rectangle obtained from the original rectangle by removing the square portion of it is in the same shape as the original rectangle. After students learn this relationship, have them think about ways to determine the ratio of the lengths of the short and the long sides of this rectangle.



From the proportion, $1 : x = (x - 1) : 1$, students will obtain the equation, $x^2 - x = 1$, as they try to determine the value of x.

This will serve as the motivation for the topic of the next chapter, quadratic equations.

June 27
Revised

Grade 4 Mathematics Lesson Plan

Date: June 27, 2017
Grade 4 Classroom 1 (33 students)
Teacher: KIMIJIMA, Kazuto

1 Name of the Unit Let's investigate quadrilaterals

2 Goals of the Unit

Through activities of observing and constructing spatial relationships of lines and quadrilaterals, students understand perpendicular and parallel lines, trapezoids, parallelograms, and rhombi. Students will enrich their spatial sense and ways of observing geometric figures.

3 Assessment Standards for the Unit

Interest, Eagerness, and Attitude	Mathematical Way of Thinking	Mathematical Skills	Knowledge and understanding
Students try to find perpendicular and parallel lines, trapezoids, parallelograms, and rhombi in their surroundings, and they try to think about how those figures are being used. Students try to discuss with their friends to solve problems based on their prior learning.	Students can identify and represent characteristics of different types of quadrilaterals based on the relationships of sides and other constituent parts. Students can grasp the characteristics of diagonals in various quadrilaterals.	Students can construct perpendicular and parallel lines, trapezoids, parallelograms, and rhombi.	Students understand the meanings and characteristics of perpendicular and parallel lines, trapezoids, parallelograms, and rhombi. Students have rich spatial sense.

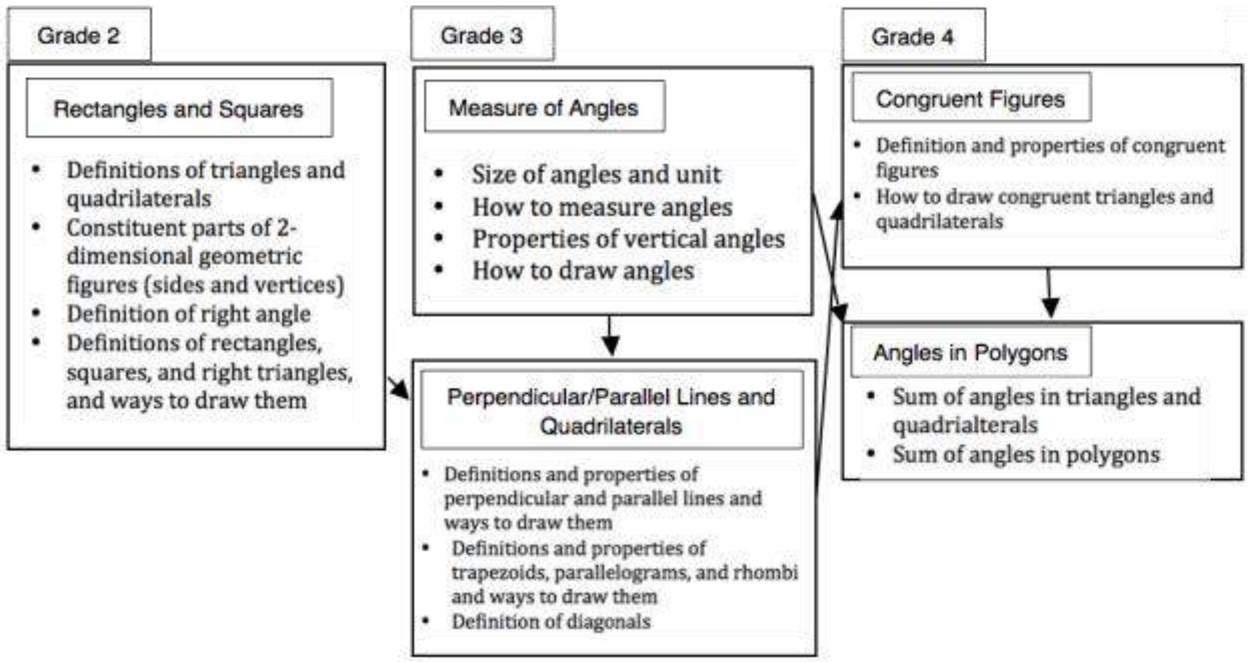
4 About the Unit

The National Course of Study states the following about the content of this unit.

Grade 4 Domain C: Geometric Figures
(1) Students will deepen their understanding of geometric figures by paying attention to the elements that compose them and their positional relationships, through observing and composing geometric figures.
a. To understand relationships such as parallelism and perpendicularity of lines.
b. To recognize parallelograms, trapezoids and rhombuses.

In the domain of Geometric Figures, students learned about "rectangles, squares and right triangles" in Grade 2 and "isosceles and equilateral triangles" in Grade 3. Up to this point, students have used the lenses of "number of sides or vertices," "right angles", and "sizes of angles" to observe geometric figures. In this unit, the new lenses of "perpendicular," "parallel," and "the lengths of diagonals and spatial relationships of diagonals" are introduced.

(1) Scope and sequence of topics related to this unit



(2) About mathematics in this unit

As stated above, up to this point, students have used the lenses of "number of sides or vertices," "right angles", and "sizes of angles" to observe geometric figures. By the end of this unit, students will add new lenses of "perpendicular," "parallel," and "length of diagonals and how they intersect."

First, we will define perpendicular lines through activities of sorting various quadrilaterals into those with right angles and those without. Next, through activities to observe various quadrilaterals, help students notice that there are different ways sides of quadrilaterals are positioned.

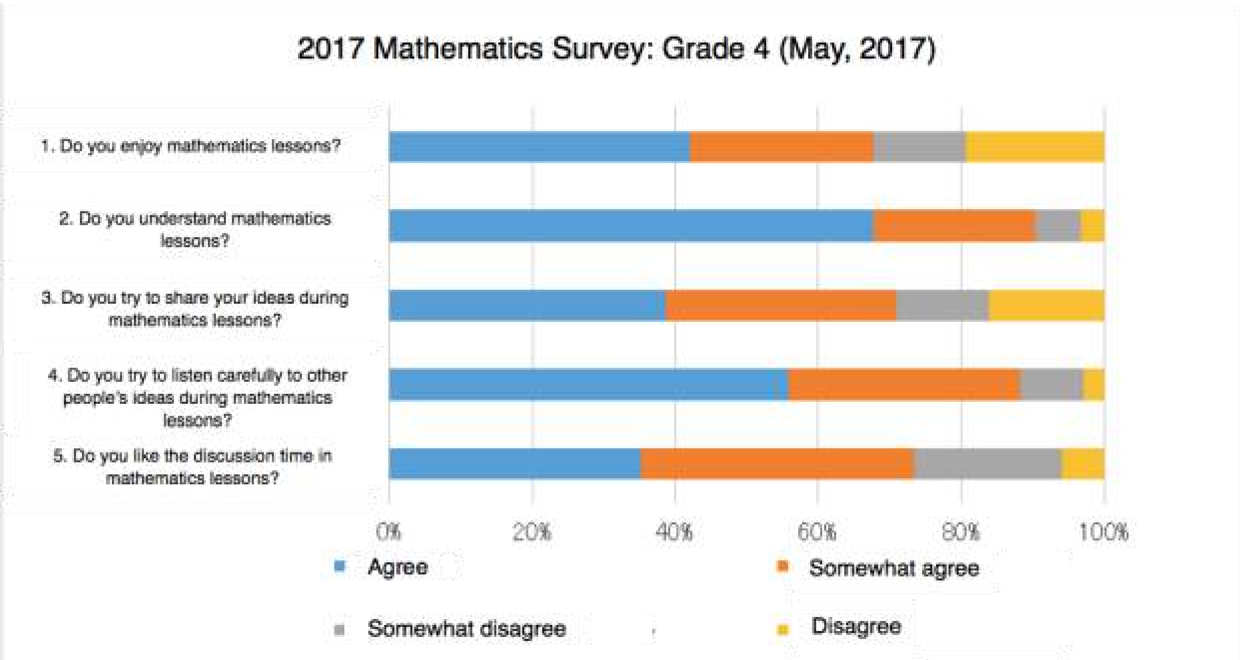
In order to create this flow of learning, we will set up learning activities intentionally so that they will promote students' own noticing. By engaging in sorting activities based on their prior learning, we want to help students recognize new ways of viewing geometric figures.

Then, by using the new lenses of "perpendicular" and "parallel," students will sort quadrilaterals and define trapezoids, parallelograms, and rhombi. After students develop the understanding of the definitions and properties of each type of quadrilateral, students will learn to draw those figures based on their prior learning.

In this way, students will repeatedly engage in sorting activities and activities of solving novel problems by making use of prior learning throughout the unit. Through these activities, we hope to develop students' abilities to identify similarities and differences, as well as their abilities to apply their prior learning based on solid understanding.

5 About the Students (Grade 4 Classroom 1)

(1) From a survey



① Although almost 90% of students responded positively to "2. Do you understand mathematics lessons?" more than 30% of students responded negatively to "1. Do you enjoy mathematics lessons?"

② Although almost 90% of students responded positively to "4. Do you try to listen carefully to other people's ideas during mathematics lessons? about 30% of students responded negatively to both "3. Do you try to share your ideas during mathematics lessons?" and "5. Do you like the discussion time in mathematics lessons?"

From observation ①, we can say that a fairly significant part of the students "do not always enjoy mathematics lessons, but they are understanding the materials discussed in the lessons." From observation ②, we can say that about the same proportion of the students "listen carefully to the teacher and other students in the class, but they do not often volunteer to share their ideas. Moreover, they do not enjoy discussion mathematics lessons, and they probably do not volunteer their ideas even in the group discussion."

This tendency can be sensed during everyday mathematics lessons, and only a certain group of students seem to raise their hands to share their ideas. In addition, a good number of students will preface their ideas by saying, "I might be wrogn, but..." These observations also suggest that some of the students find it difficult to share their ideas and participate in discussion.

To deal with these issues, we have been implementing several strategies. Those strategies include: try to pose questions in such a way that what students must think about and address is clear; participate in class discussion indirectly by using hand signals; and creating a student-centered discussion atmosphere by having students call on next speakers. In addition, when students make good points in their own notebooks, we try to share it with the whole class as a model of examplary ideas and value their contribution to our learning.

In teaching of this unit, in order to continue to address these issues, we will emphasize the use of small group (3-4 students) discussion time. This is because we feel students are more willing to share their ideas with a smaller number of people. Moreover, instead of just simply presenting their ideas, we want students to come up with a group consensus. This additional task will, hopefully, result in more active discussion in the group.

(2) Current state of the students (from the readiness test)

Content	All correct	Partialy collect	All incorrect
① Students can construct isosceles triangles using compass. (Skills)	53 %		47%
② Students can know the size of a right angle, and express other angles in terms of the number of right angles. (Knowledge)	83%	14%	3%
③ Students can identify geometric shapes with their correct names. (Knoweldge)	50%	43%	7%
④ Students can think about properties of shapes made up of pieces of set squares. (Thinking)	20%	40%	40%
⑤ Students understand the property of vertical angles. (Knowledge)	53%	33%	14%
⑥ Students understand perpendicular and parallel lines. (Content of this unit)	17%	20%	63%

As we planned the unit, what we focused on were the shaded items in the table. From item ⑥, we can tell that the students in our class, with the exception of a very few, have no prior knowledge of the content of this unit. As for their prior learning, their success rates are not exceptionally high, but the number of students with very limited mastery is rather small. However, the results of item ④ suggests that many students may have difficulty reasoning with their prior knowledge.

In addition, from item ①, we can also see that many students have trouble with drawing/constructing figures. A more detailed results on drawing/construction items are as follows.

Students can construct isosceles triangles using compass. (Skills)	53%
Students understand the methods of drawing, but their skills are lacking.	7%
Students do not understand the methods of drawing.	37%
No response	3%

These results also support the observation that many of the students have difficulty applying their prior knowledge.

Based on these observations, we want to treat the basic/foundational ideas very carefully. Moreover, in teaching skills such as construction, we want students to fully master the use of tools such as set squares and compass. To do so, we want to figure out ways to incorporate many individual instruction opportunity with multiple teachers. Furthermore, in order to consolidate students' understanding, it is important that they will be asked to express their ideas in an orderly manner using their own words. This will help them reflect on and organize their ideas. To do so, we want to emphasize discussion in our instruction.

6 "Students who can feel, think, and extend" in this unit

(1) Students who can feel

- Students who feel the problem is their own and try to solve the problem independently
  → Students are tackling problems using the lenses of parallel lines, perpendicular lines, lengths of the diagonals and the way the diagonals intersect, building on their prior knowledge of the number of sides and vertices, right angles, lengths of sides, and measure of angles.

(2) Students who can think

- Students who can think about way to solve problems by making use of their prior knowledge.
- Students who can acknowledge other people's ideas and incorporate them into their own thinking.
  → Students can solve problems using a variety of reasoning, applying ideas such as the definitions of perpendicular lines, parallel lines, and various quadrilaterals.

(3) Students who can extend

- Students who can explain their solutions to others by actually drawing figures and using their own words instead of simply solving problems.
- Students who can make use of their prior learning in mathematics in various situations in their daily life and also in problems in other subject areas.
  → Students can demonstrate how to construct figures and explain their ideas using their own words so that others can understand them.

7 Situations in this unit where discussion will be incorporated

- Situations in which students learn the definitions of perpendicular and parallel lines and identify which cases are which.

- Situations in which student think about ways to draw perpendicular or parallel lines using set squares.
- Situations in which students sort quadrilaterals by focusing on the number of pairs of parallel sides.
- Situations in which students investigate properties of parallelograms.
- Situations in which students think about ways to draw parallelograms using their properties.
- Situations in which students think about ways to draw rhombi using their properties.
- Situations in which students make various quadrilaterals using diagonals.

8 Unit Plan

#	Goal	Learning Activity	Main Assessment Standard
(1) Ways lines intersect (2 lessons)			
1	○ Through activities to investigate ways 2 lines intersect, students will learn the definition of perpendicular lines, and they can distinguish perpendicular lines from those that are not.	• Investigate ways 2 lines intersect. • Learn the definition of "perpendicular" lines.	[Interest] Students try to investigate ways lines intersect by focusing on the angles formed by the lines. [Knowledge] Students understand the definition of perpendicular lines.
2	○ Students will learn to draw perpendicular lines using set squares.	• Think about ways to draw perpendicular lines using set squares. • Student draw perpendicular lines.	[Thinking] Student think about and explain ways to draw perpendicular lines using set squares by focusing on the right angles in set squares. [Skills] Students can draw perpendicular lines using set squares.
(2) Ways lines are arranged (4 lessons)			
3	○ Through activities to investigate ways 2 lines are arranged, students will learn the definition of parallel lines, and they can distinguish parallel lines from those that are not.	• Investigate how lines are arranged. • Learn the definition of parallel lines. • Investigate sides of rectangles that are perpendicular and parallel to each other.	[Skills] Students can distinguish parallel lines and those that are not. [Knowledge] Students understand the definition of parallel lines.

4	○ Students will understand properties of parallel lines such as they will intersect another line at the same angle and the distance between them is constant.	• Investigate angles formed by parallel lines and another line. • Summarize the property that the parallel lines will intersect with another line at the same angle. • Investigate the width between parallel lines. • Summarize the property that the width between parallel lines is constant. • Learn that we only use the term parallel when the geometric figures are lines and deepen their understanding of parallel lines. • Understand the origin of the Chinese characters used for the term "parallel" and deepen their understanding of parallel lines.	[Knowledge] Students understand properties of parallel lines such as they will intersect another line at the same angle and the distance between them is constant.
5	○ Students will learn to draw parallel lines using set squares.	• Think about ways to draw parallel lines using set squares. • Draw parallel lines.	[Thinking] Students can think about and explain ways to draw parallel lines by focusing on the property that corresponding angles are congruent. [Skills] Students can draw parallel lines.
6	○ Students understand perpendicular and parallel lines on a grid paper.	• Think about ways to draw perpendicular and parallel lines using the lines on a grid paper.	[Knowledge] Students can use the lines on a grid paper to distinguish perpendicular lines and those that are not and also parallel lines and those that are not.

(3) Various quadrilaterals (6 lessons)			
7	○ Through activities of sorting quadrilaterals, students will understand the definitions of trapezoids and parallelograms.	• Sort given quadrilaterals. • Learn the definitions of trapezoids and parallelograms. • By using the lines on a grid paper or given parallel lines, draw trapezoids and parallelograms.	[Interest] Students try to sort quadrilaterals by focusing on the number of pairs of parallel sides. [Knowledge] Students understand the definitions of trapezoids and parallelograms.
8	○ Students will understand the properties of parallelograms.	• Investigate the lengths of sides and measures of angles to identify properties of parallelograms. • Summarize the properties of parallelograms. • Investigate the common characteristics between rectangles and parallelograms to deepen their understanding of parallelograms.	[Thinking] Students can identify and explaining the properties of parallelograms by considering the relationships of sides, lengths of sides and measurements of angles. [Knowledge] Students understand the properties of parallelograms.
9	○ Students can construct parallelograms.	• Think about ways to construct parallelograms. • Construct parallelograms by making use of the definitions and properties of parallelograms.	[Thinking] Students can think about and explain ways to draw parallelograms by considering the definition and properties of parallelograms. [Skills] Students can construct parallelograms.
10		• Engage in application problems.	
11	○ Students will learn the definition and properties of rhombi and can construct them.	• Learn the definition of rhombi. • Summarize the properties of rhombi. • Construct rhombi. • Investigate the common characteristics of squares and rhombi to deepen their understanding of rhombi.	[Thinking] Students can identify and explaining the properties of rhombi by considering the relationships of sides, lengths of sides and measurements of angles. [Skills] Students can construct rhombi.
12	○ Through mathematical activities, students will deepen their understanding of the unit contents and	• Make tessellations using quadrilaterals. • Look for quadrilaterals in their surroundings. • Learn about isosceles	[Interest] Students try to make use of what they learned as they engage in various activities.

	increase their interest in quadrilaterals.	trapezoids and kites.	
(4) Characteristics of diagonals of quadrilaterals (2 lessons)			
13	○ Students will understand the definition and properties of diagonals of quadrilaterals.	• Connect vertices of various quadrilaterals and investigate their characteristics. • Learn the definition of "diagonal." • Summarize properties of diagonals of various quadrilaterals. • Investigate diagonals in isosceles trapezoids and kites to deepen their understanding of diagonals.	[Thinking] Students grasp the relationships of quadrilaterals by considering the characteristics of diagonals. [Knowledge] Students understand the definition and properties of diagonals in quadrilaterals.
14	○ Students will learn that triangles obtained by partitioning rectangles, parallelograms and rhombi are congruent, and they will use those triangles to form various quadrilaterals.	• Investigate 2 triangles obtained by partitioning rectangles, parallelograms and rhombi. • Compose various quadrilaterals using these triangles.	[Skills] Students can make various quadrilaterals using 2 congruent triangles. [Knowledge] Students understand that 2 triangles obtained by partitioning certain quadrilaterals are congruent.
Unit Summary (2 lessons)			
15	○ Students will solve problems applying what they learned in the unit.	• Work on problems in the problem set in the textbook.	[Skills] Students can solve problems applying what they learned in the unit.
16	○ Students will solidify their understanding of contents in the unit.	• Work on problems in the end-of-unit problem set.	[Knowledge] Students have mastered the content of the unit.

9 About the lesson

(1) "Feel, think, and extend" in this lesson

[Feel] Students who devise a plan to construct parallelograms by making use of their prior knowledge and persist in tacklying the task.

[Think] Students who can think about ways to construct parallelograms using the definition and properties of parallelograms.

[Extend] Students who can explain what property (or properties) they used to construct parallelograms.

(2) Discussion times in this lesson and strategies to support them

① Discussion to clearly understand the learning task (whole class)

In order for students to engage in a learning task independently, it is important to motivate them. It is important that the teacher presents the task in a way to pique students' interest so that they will say, "I want to try it" or "I want to solve it." It is also important that students feel the need for solving the task.

Therefore, we will propose a problem situation that connects students' mathematics learning and their school life. The proposal is, "Let's make a display full of various parallelograms to welcome - and surprise - our friends from Kanayama Elementary School who will visiting us next month." (This task is coordinated with the activity in the Art curriculum.) We want to have many different color/shapes/sizes of parallelograms all over our classroom walls to welcome our friends. This way, the need to create various parallelograms efficiently will be created.

After this proposal is made, the teacher will demonstrate how to draw a parallelogram by first drawing side BC, then angle B and finally side AB. Then, the teacher will complete the parallelogram by eye-balling the remaining sides, making intentionally close but also obviously not a parallelogram (so that students might think "That's close"). As the teacher tries to draw a parallelogram, we want to draw out students' own problem such as "I want to draw true and beautiful parallelograms," "I want to think about ways to draw parallelograms," and "I think I have an idea on how to draw parallelograms."

② Strategies to have active discussion times

After students clearly understood the task, students will engage in independent problem solving. We will have a worksheet in which there is a space for students to record the sequence of their steps in words. We hope that by recording each step as students construct their parallelograms, they can share their methods in an orderly manner so that others can understand more easily. Furthermore, in order for students to make use of their prior learning, various posters summarizing what they have been studying will be placed on the classroom walls so that students can constantly consider them.

③ Discussion with the teacher

For students who are having difficulty constructing parallelograms on their own, the teacher will gather them together and have a discussion time to look back on the definition and properties of parallelograms and how to draw parallel lines. If the number of students is large, we will first suggest to use the definition, "there are 2 pairs of parallel sides," to construct parallelograms. This is because students have studied methods of drawing parallel lines in 2 lessons prior to this research lesson. In addition, we believe it is easier for students to imagine parallel lines from the name, "parallelograms." Since students have learned to construct isosceles triangles and equilateral triangles using a compass, depending on students' understanding, we may discuss the method utilizing the property, "opposite sides are equal in length."

Because this discussion is conducted with students who were having difficulty drawing parallelograms on their own, there is a risk that this session will turn in to a teacher-centered teaching by telling. In order to avoid this risk, the teacher will try to pose questions such as the followings at key moments so that it will become truely a discussion time: "What do you think we should do next?" "What property do you think we should use?" "What tool do you think will help us?"

④ Discussion to find commonalities (in small group)

After the independent problem solving time, students will be sharing their ideas in small groups of 3-4 students. [Students do many school related activities, both academic and non-academic, in the same groups.] Students will explain their methods to other members of the group, and they will identify commonalities in their methods. Then, they will try to identify and verify which property (or properties) of parallelograms that justify the commonalities. In addition, they will discuss to make sure each one's method is indeed a valid approach.

⑤ Discussion to seek better ideas among many ideas (in small group)

After discussion ④, students will remain in the groups and continue their discussion. At this point, the focus of the discussion is to identify the methods that is "fast & simple" from various ideas that have been presented. For example, students might consider the number of steps involved in construction as a criterion for a "fast" method. For a "simple" method, they might consider the number of tools used in construction. Some students might propose "accuracy" as a criterion, but since we want to focus today's discussion on the definition and properties of paralelograms, we will try not to get too deeply into this idea. In some cases, students may combine parts of several methods to create a new method.

This discussion should start from students' sharing what they wrote in the worksheet. After all students shared their ideas, the groups are instructed to come up with one idea to be shared to the whole clas and write it on the whiteboards that have been distributed.

⑥ Discussion to seek better ideas among many ideas (whole class)

After each group shared their groups' methods, we will have a whole class discussion to decide which idea (or ideas) is "fast and simple." Through this discussion, we want to focus on the following 3 methods.

i. Using 2 pieces of set squares, draw a pair of parallel lines to construct parallel sides. (about 2 steps using 2 pieces of set squares)
ii. Using a compass, construct a pair of opposite sides that are equal in length. (about 2 steps using a compass)
iii. Using a protractor, draw construct opposite sides by making use of the congruent angles property. (about 2 steps with a protractor)

(3) Goals of the lesson

- Students can think about ways to construct parallelograms.
- Students try to discuss with your friends to solve problems based on their prior learning.

(4) Assessment standards

- Students think about and explain ways to construct parallelograms using the definition and properties of parallelograms. (Mathematical Way of Thinking)
- Students can construct parallelograms. (Mathematical Skills)

(5) Flow of the lesson

	Main questions and anticipated students' responses	Discussion Activities	○ Instructional points of consideration ● Assessment
Introduction/Look back	1. Look back on what was discussed in the prior lesson T: (Posting a parallelogram on the blackboard) What is this shape? S: It's a parallelogram. T: Can you tell us how you know it is a parallelogram? S: Because both pairs of opposite sides are parallel. S: Because both pairs of opposite sides are equal in th length. S: Because both pairs of opposite angles are congruent. T: Thos were the characteristics of parallelograms, weren't they. Let's keep those characteristics in mind as we tackle today's lesson.	Discussion to look back on prior learning that will be the foundation for today's lesson. (Whole class)	○ Make use of posters showing students' prior learning so that they may be able to utlilize them in today's learning. ○ Re-affirm the definition and properties of parallelograms that will be used to justify various steps in today's lesson.

Identify the Mathematical Task	2 Understand' today's task T: What do you think about the idea of making parallelogram displays by using what we've been learning to surprise our friends from Kanayama Elementary School who will visiting us next month? S: Yes, we want to do it. T: Then, let's think about ways to draw parallelograms today. Problem: Let's think about ways to construct parallelograms. T: OK, I will demonstrate how to draw a parallelogram. S: That figure looks like a parallelogram, but it is not a parallelogram. T: How can we construct a parallelogram?	① Discussion to clearly understand the learning task (whole class)	○ Using a ruler only, draw a quadrilateral that is close to but not quite a parallelogram (so that students might think, "That's very close, but...").
Devise a plan	3. Share plans to tackle the task Task: What do we need to do to complete the construction? T: Let's think about strategies to tackle this task. Which property should we use? What tool should we use (and how)? S: To make the opposite sides parallel, we can use 2 pieces of set squares. S: To make the length of opposite sides equal, we can use a compass. S: To make the opposite angles equal, we can use a protractor.		○ Draw sides BC, angle B, and side AB, then ask students to think about how to complete the construction.

Independent Problem Solving	4. Based on own strategies, construct parallelograms and record the steps in the worksheet	③ Discussion with the teacher (small group)	○ Have students turn their desks so that they are facing the side wall where the posters showing their prior learning are displayed. ○ For those students having difficulty constructing on their own, gather them in a separate location in the room to have a discussion with the teacher and obtain hints. • Students can construct parallelograms. (Skills) [Worksheet]
Sharing	5. Share their ideas in small groups T: Please share your ideas in your usual groups. As you listen to each other's idea, think about the similarities in your methods. S: What's similar is that we were all trying to use the property, opposite sides are equal in length, but some of us used a compass while others used set squares. 5. In small groups, develop a better method T: Now, as a group, try to come up with a better method to construct parallelograms.	④ Discussion to find commonalities (in small group) ⑤ Discussion to seek better ideas among many ideas (in small group)	○ Have students use their worksheet to share the steps of their construction. ○ Suggest students to actively engage in discussion by adding on to other's ideas or using hand signals to express their reactions, if appropriate.
	6. Identify better ideas from many T: We are going to have each group share what they came up with. T: Which idea do you think is a better one? Can you please tell us why you think it is better.	⑥ Discussion to seek better ideas among many ideas (whole class)	

	S: I think Group #'s idea is very good because it is fast and simple. S: I also think that idea is good because we need only a few steps. S: I think it is a good idea because it uses only a few tools. S: If we use a compass, we don't need a ruler to measure the lengths of sides.		○ Suggest, if necessary, that "fast" means the number of steps is small, and "simple" means the number of tools is small. ○ Since we are focusing on finding out the location of vertex D, we will not count the ruler as a tool nor drawing of a side as a construction step.
Summary and Reflective Journal Writing	7. Summarize today's lesson T: What is necessary when you are constructing a parallelogram? S: To make use of the characteristics of parallelograms. Summary: We can construct parallelograms by making use of their characteristics such as parallel sides, lengths of sides and measures of angles. 8. Write journal entries and share S: In today's lesson, what we have been learning previously were very useful. I want to make use of my prior learning in future learning.		○ Connect to the idea that "making use of prior learning" is important for problem solving • Students think about and explain ways to construct parallelograms by making use of the definition and properties of parallelogram. (Thinking) [Worksheet/Sharing]

Lesson 7 Report

Report created by: Karen Wilding, Shelby Halela, and Evanne Ushman
Name of Lesson: Let's Investigate Quadrilaterals, Lesson 9
Date of Lesson: June 27, 2017

What are the primary lesson goals?
- Students can construct parallelograms.
- Students can discuss with classmates to solve problems based on their prior learning.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?
- <u>Prior Lesson Topics:</u>
  - Students have defined and investigated the properties of parallel and perpendicular lines.
  - Students have used the set squares tool to draw parallel and perpendicular lines.
  - Students have investigated the width between parallel lines and concluded that the width is constant.
  - Students have defined the properties/characteristics of trapezoids.
  - Students have defined the properties/characteristics of parallelograms.
- <u>Future Lesson Topics:</u>
  - Students will define the properties/characteristics of rhombi.
  - Students will make tessellations using quadrilaterals.
  - Students will define the properties/characteristics of isosceles trapezoids and kites.
  - Students will define "diagonal" and investigate diagonals in various quadrilaterals.
  - Students will partition rectangles, parallelograms, and rhombi into triangles.
  - Students will compose quadrilaterals using these triangles.

Summary of Lesson

Start & End Time	**Lesson Phase**	**Notes**
00:00 - 22:38	**Introduction, Posing Task** *Class discussion about parallelogram properties:*	**Strategies to build interest and to connect to prior knowledge:** Initially the teacher displayed a parallelogram on the board and asked students "What is this shape?" Students correctly determined that it was a parallelogram. The teacher then asked students to explain why it was a parallelogram. Students defined the three properties: 1) Pairs of opposite sides are parallel 2) Opposite sides are the same length 3) When you add the parallel angles together they measure 180 degrees. After establishing what makes a parallelogram, the teacher asked students "How can we be sure that these conditions are met?" Students responded that you can use set squares, a ruler, and/or a compass to check the lengths/angle measurements. After reviewing the properties, the teacher asked students to brainstorm examples of parallelograms in daily life. This proved challenging, so the teacher told

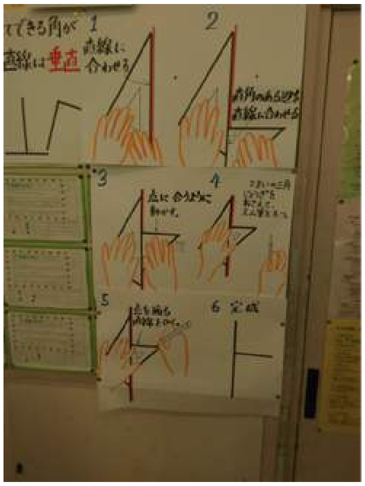
Introduction of the task:

Teacher constructing an incorrect parallelogram:

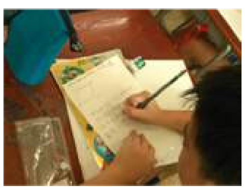
students that they will be making a poster to show some different types of parallelograms. The students seemed very excited about the prospect of creating and further exploring parallelograms and sharing their examples with another class.

The teacher then stated that to do this project they would need to draw parallelograms. He then drew a “parallelogram” on the board and asked students whether or not it looked like a parallelogram. Students seemed to disagree, some saying that it kind of looked like it, while others said that the opposite sides didn’t seem the same. The teacher then asked students to come back to their proofs and think about strategies that they could use to know for sure. The students were very engaged in correcting their teacher’s mistake and were eager to prove that he was wrong. This activity built enthusiasm about the task as well as drawing their attention to the need for precision in their construction.

	Students evaluating whether or not this parallelogram is correct: 	
22:38-35:12	**Independent Problem Solving** *Children's clear enthusiasm in initial stages of lesson:*  *Teacher modelling a 'close to but not quite' parallelogram:*  *Posters used/referenced by students throughout the lesson:*	**Individual, pairs, group, or combination of strategies** **Teacher's Activities:** The teacher began at the front of the room and engaged children through the review of prior learning and establishing a new focus for their skills application: "The creation of a display full of parallelograms to welcome and surprise our friends from Kanayama Elementary School next month" (coordinated to link to the art curriculum). The teacher shared a large poster with the children to ensure the children all understood this proposal. High levels of motivation and engagement from the pupils was observed immediately (photo) and clear evidence of listening and willingness to participate from most. The teacher modeled very precise language use and encouraged children to consider as many ways as they could to prove that the shape displayed was a parallelogram. This included using set squares and pairs of compasses. Children were encouraged to come up to the board to demonstrate. The children were encouraged to cite where they had seen parallelograms in real life. (Interestingly, they all cited only rectangles although the shape shown on the board was a trapezium). The teacher shared that he had thought hard about this and only come up with a cable car. It was interesting to compare the pupils' perspective that a parallelogram



Examples of students' responses to first worksheet task:



Teacher supporting on a 1:1 basis during first independent activity

Children given choice for extra small group support at beginning of first task:

was a rectangle and the teacher's mental image of a trapezium. (I would have like to have known if the students could justify whether both examples fit the definition and why.)

When the teacher (see section 1) had modeled a parallelogram (trapezium) on the board the children began to laugh. The teacher played along and asked "Why are you laughing?" and this elected many responses which matched the anticipated student responses very accurately: "It's close but not a parallelogram." This then led to the statement:

"To construct a parallelogram we have been saying 'draw' but mathematically we say 'construct'."

Displays on the classroom wall were clearly evident modelling prior learning and specific techniques pertinent to this lesson and unit of learning. The children were guided to use these prompts (including turning their desks to face them directly) during the whole class teaching.

In the post-lesson feedback, the teacher commented very early on that he had found not being able to get to his students, due to the number of people in the classroom, very frustrating (he was very polite in the way he shared this!). He reflected that a key priority for him was listening to pupils and responding to their needs.

The presence of observers in the room did not seem to have the same impact on these children as we'd seen in some classes. This may have been due to their age but could have also been a reflection on his relationship with them.

The teacher also assumed prior knowledge from the previous grade and understanding from earlier lessons in the unit The range of evidence in this part of the lesson suggested that, despite the prompts and the previous learnings from the unit, most children were not secure with their skills and needed a great deal more purposeful practice in using the set squares and other geometry equipment effectively.

Experiences of Diverse Learners

Whilst teacher showed prompts and questioned children, many took out their set squares and rulers from their pencil cases and some half up their hands mimicking the displays on the wall.

Teacher listening and responding to students during small group task:

Child in group of five who remained detached from task due to positioning and lack of her, the group or teacher action:

Group work using whiteboards:

Group boards from collaborative work:

The children were then all asked to use a worksheet to construct a parallelogram from incomplete pieces and write their instructions explaining how to do it as they worked. Chairs turned to face wall with displays showing hands using set squares to remind children of previous learning.

A wide variety of responses were observed; from children copying the posters on the wall and repeatedly drawing round their set squares, others using both set squares and working on trying to apply the steps and some using set squares proficiently and writing steps wither as they worked or retrospectively.

Many children appeared to make a considerable and sustained effort to use the two set squares as modelled by the teacher and as experienced in previous lessons.

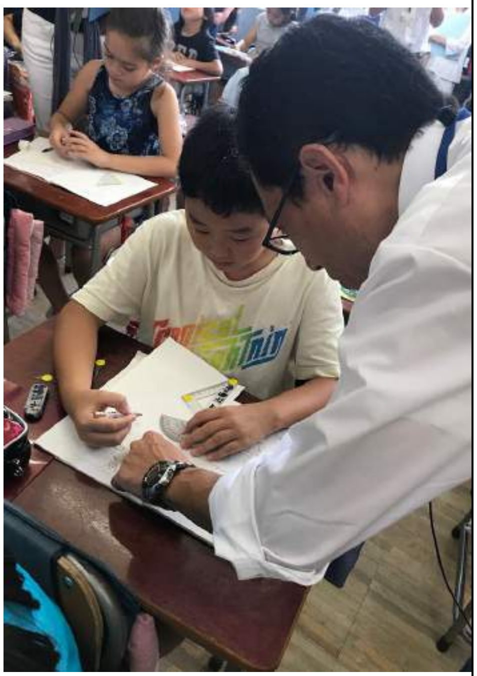
During this independent time, the teacher initially gave children a choice as to whether they'd like to come and work in a smaller group with him if they weren't sure in how to proceed. A small number of children did this and then moved away visibly happier and more relaxed.

The teacher then proceeded to support individuals during this task. (I was unable to understand the exchanges made.)

Next the lesson moved the children into collaborating in groups of four and five. It was noticeable that where there was a five, the seating arrangement meant one child was left out of the group and this was not resolved by the child, the group or the teacher so this particular pupil remained disengaged for the entirety of the group session. There were a number of other children who demonstrated little, if any, progress but made themselves look 'busy' throughout the session. I was interested to know more about whether these children are known to the teacher and what his usual strategies for supporting their involvement and progress are.

Within the groups, the children were asked to discuss their strategies and select one to improve. They were given an A3 sized whiteboard to record their ideas. This request aimed very high in terms of expectations regarding interaction, reflection and application of mathematical knowledge. Although I was unable to understand the language, it appeared that the children who took the broad recreated their idea and there were very varying levels of effective interaction within the groups regarding the mathematical discussion and sharing of methods. One particular group became very

		engaged together and the teacher spent time with them discussing their work. Other groups appeared to find it difficult to get going and share their ideas which then led to some disengagement. The group work, on the whole, led to most children sharing and discussing ideas, and in many cases, modelling their drawing strategies to others. This meant that the assessment for learning opportunities were very rich and the children's learning, misconceptions and need to improve particular skills were far more visible. This was a specific goal for the school using lesson study. Interestingly, many children whilst drawing the parallelograms, then resorted to the squares on the board or freehand drawing rather than using what they'd be learning in this lesson and previous lessons. The teacher ran over on his time (which naturally we, as observers, have all done ourselves many times). This meant that the whole class discussion and sharing of ideas - neriage - did not take place. The students' boards were collected up and displayed on the main board.
35:12- 54:21	**Presentation of Students' Thinking, Class Discussion** *Teacher working with students one on one to support independent work and groupwork:* 	**Student Thinking/ Visuals/ Peer Responses/ Teacher Responses** Unfortunately, the teacher ran out of time before the class discussion and summary. However, we can get a good picture of student thinking during independent work and group work: **Student 1:** During independent work time before the teacher had them form groups with their desks, one student self-identified her confusion and went to the teacher for some help. She wasn't sure how to use the set squares so she asked, "Where should I put the set squares? If it's like this they will hit each other." The teacher made a suggestion and she responded, "Yes! So if I put it like this…" and went back to her seat seeming as though it was a helpful tip to get her started. **Student 2:** After independent work, the teacher asked the students to form small groups of 4 students. One boy in a small group stated, "mine is not a parallelogram because the opposite sides are not the



Students working in small groups:

same length," demonstrating that he understood this characteristic of a parallelogram, but not yet how to construct it.

Student 3: In the same group mentioned above, another boy called on a girl to share her thinking. She responded, "first I put my big set square on BC then the other one next to it so I can slide it so it's parallel to BC." She effectively used the set squares to make the opposite sides parallel during independent work time. However when trying to explain to her peers, she was unable to demonstrate how she had used the set squares. This left the group confused by the end of the lesson. Some of the students in the group continued to persevere, erasing and trying again and again. Some became disengaged.

Student 4: After the small group work, the teacher requested that groups in which everyone had finished sharing could come get whiteboards. He asked groups to choose a leader at each table to represent one method on the whiteboard. He told the students that the chosen method could be one person's idea or a combination of multiple people's ideas.

In one of the groups, the students began pointing at one another to be the leader. After a few minutes of pointing, the girl who eventually decided to be the leader began representing her work on the whiteboard. She began attempting to use the set squares but reverted to freehanding when it became difficult. She did not measure the angles of the parallelogram.

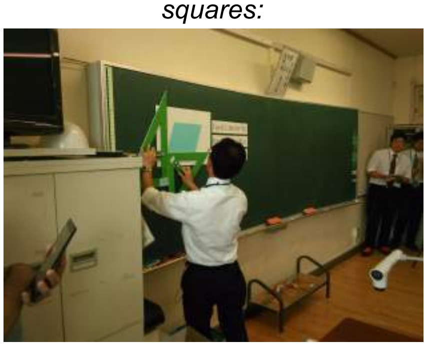
Student 5: A boy in the group began writing directions for how to construct a parallelogram by himself. The other boy in the group was writing for a minute or so and then began playing with his set squares. The team members were not talking to one another.

Many children reverted to freehand drawing rather than using their tools to accurately construct a parallelogram. The amount of individual work and disengagement we observed leads me to believe that the students did not benefit from working in groups for this lesson. Perhaps if the students had roles in the groups, the collaboration would have worked more effectively.

	  	
Teacher ran out of time, we did not get to this portion in the lesson.	**Summary/Consolidation of Knowledge** *Teacher reviewing the properties of parallelograms:*	**Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals:** Before the lesson, the teacher reviewed the properties of parallelograms and recorded the student-generated responses on the board for students to reference. He also reviewed the different tools (compass, protractor, and set squares) and the ways to use them to construct parallel/perpendicular lines and angles, modeling (either through teacher modeling or inviting students to the board to model) their useage on the board. In addition to the teacher's review on the blackboard, there were several posters displayed around the room that highlighted these things as well. Students worked independently in journals to record their thinking and test out their strategies. They had

Students raising hands to share ideas during review:

Teacher reviewing how to use set squares:



Teacher and student reviewing how to use a compass to draw angles:

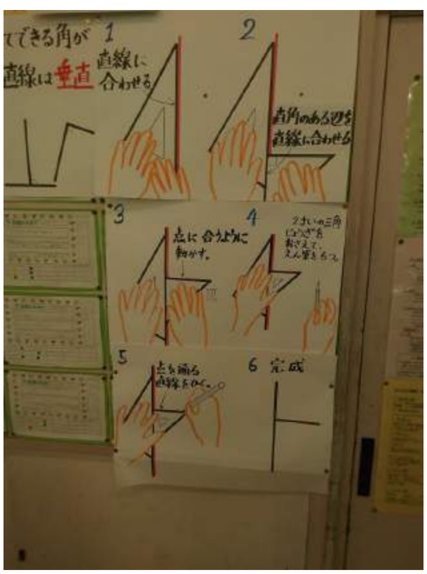
Teacher and student reviewing how to use a protractor to draw angles:

access to rulers, protractors, set squares, and compasses (though no students appeared to use the compass tool) and used these tools to construct parallelograms in their journals and record the steps that they took.

After completing their journal work, students moved into small groups of 4-5 students to share their ideas. After the students discussed and came to a shared understanding (though some groups never actually were able to reach a consensus), they were given a mini whiteboard to record their strategy.

The teacher intended to have students present their solution strategies and use the information that came out of this discussion to make a final summary, but the class did not reach the sharing and summary steps. Instead, the teacher displayed the different student group solutions on the board for teachers to view and discuss during the Post-Lesson Discussion.

Reference posters displayed around the classroom:

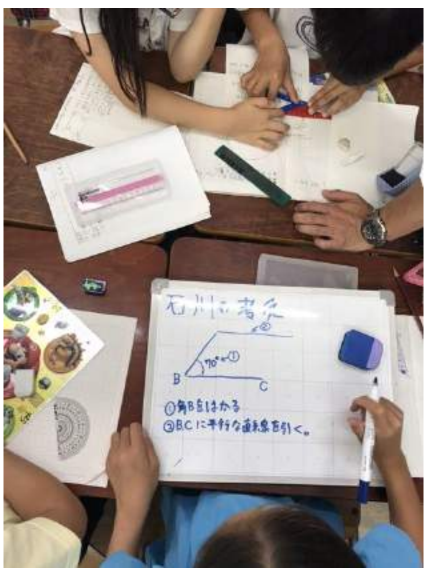


Students working independently in journals:

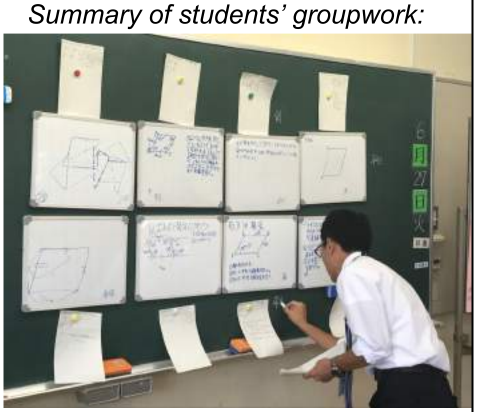
	    *Small group discussions:*	

Recording group work on whiteboards:



Summary of students' groupwork:



What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?

It is essential for students to both understand a task and have motivation to solve it. Overall, students seemed very motivated at the beginning of the lesson. The students were invested in correcting their teacher's incorrect parallelogram example and were eager to share their ideas. They were also enthusiastic about creating their own parallelograms using their tools. This motivation waned as the lesson progressed though, because it became clear that most of the students did not actually understand the task. Though they understood the properties of a parallelogram and they knew the tools that could be used to construct it, they had trouble consolidating this knowledge and using it in their independent application. There is a significant difference between understanding the properties and applying these properties in a new context. Once the students began to struggle, their motivation decreased significantly and many of them became frustrated or disengaged from the task entirely. When designing a task, it is important for a teacher to anticipate student struggles and strike a balance between giving them enough information to solve the task while also maintaining the challenge piece and giving them opportunities for discovery/new learnings. If the task is too easy or too difficult, student motivation is diminished, and then the learning is no longer productive. Students need not only the understanding and motivation to solve a problem, but they also need the confidence and the persistence to be able to accomplish this.

Prior knowledge is critical for students to be able to access a task. Teachers need to be intentional about their presentation/review of prior knowledge though. They need to get students to purposefully apply their prior knowledge to a problem. In order to ensure this, students need to have a solid grasp of concepts that have been previously been taught. If they only have a superficial understanding of a concept, they will not be able to meaningfully access this prior knowledge to use in a new context. It's not only foundational conceptual knowledge that needs to be solid, it is also knowledge and experience with the different mathematical tools. If students have not gotten enough repeated exposure to the tools in multiple contexts, then these tools will not serve to support student's learning because they do not have a clear picture of their purpose and uses.

Timing/pacing are things that a teacher really needs to factor in when planning a lesson. If a teacher anticipates that students will struggle with a portion of the lesson, then he/she needs to compensate for this and allow enough time for students to engage in a productive struggle without shortchanging other critical components of the lesson. In this particular lesson, the bulk of the time was spent on accessing prior knowledge and independent solution time. Because students had a lot of misconceptions, these two portions of the lessons took a lot longer than expected and they were unable to actually get to the most meaningful parts of the lesson - the discussion and summary. Though it is impossible to always anticipate how a lesson will go, the teacher needs to be aware of student struggles in the moment and redirect as needed, improving or adding on to his/her instruction in order to help students access the task. Otherwise, both student and teacher time is wasted and no learning can take place.

In order to gauge student understanding and misconceptions, teachers need to have a way to assess students, both during and after the lesson. The assessment piece was lacking in this particular lesson, as the class was unable to get through the bulk of the lesson. Though the teacher was able to gain some valuable insights from circulating around the room and looking through students' independent journal work, he did not appear to use this information to inform/adapt his instruction. Because of this, many students struggled futilely to understand and solve this problem. True understanding means that the students can solve a problem, explain how they solved it, and apply this learning to a new context. Students were struggling to solve this particular problem and consequently could not explain their solution methods and would likely be unable to apply it, showing that there was a significant disconnect in understanding. Teachers need to use this information to support students. If the bulk of the class is having trouble, then he/she needs to go back and reteach/offer students more scaffolds to support their learning. Assessments aren't very effective if a teacher is not using them to inform his/her instruction in real time.

One of the primary goals of this lesson was to improve student discussion, as students have a lot of difficulty explaining their thinking to their peers. Though there were a lot of opportunities for students to be involved in discussion and communicate with one another, these did not seem like they were always productive interactions. In order to facilitate productive student discussions, they need repeated practice and structures in place to support their conversations. Sentence frames, group roles, and talking chips are just a few ways to ensure students feel ownership in sharing their ideas as well as responsibility to contribute to group discussions. There needs to be some form of accountability to ensure that students are participating equally and in a high-quality way.

The teacher adapted this lesson from a problem in the textbook. In the textbook though, more information was given (including angles and side lengths) to support student access. By choosing to leave out these measurements, the lesson was made less effective/accessible for students. The original problem was more open-ended, offering multiple approaches and a variety of answers. The change that the teacher made constrained students to one solution method. It is important, when adapting curriculum, to be very intentional about what you choose to include/exclude. Sometimes omitting information can provide an exciting challenge for students, but other times it can hinder students because it does not offer a complete picture or narrows their points of access. Just as teachers need to be very intentional with their pacing, they need to be thoughtful about the information that they are presenting in order to provide all students with a "way in" to the mathematical task.

What new insights did you gain about how administrators can support teachers to do lesson study?

<u>Administrator's Role</u>
The role of the school administrator (or 'Senior Leader' in the U.K.) is paramount to the success of lesson study.

In the settings we visited, we saw leaders introducing the sessions and making it clear how much the approach was valued. The administrators facilitated the time for all staff to attend and contribute. The interpretation of accountability in Japan is very different to the UK and US. This has enabled school leaders to avoid judging and monitoring teachers' performance but instead play a significant role in enabling professional learning, risk-taking and deep personal reflection to be the focus.

<u>To support lesson study fully in the UK and US, school leaders need too:</u>
-Ensure they take advice and guidance from individuals trained in the authentic form of Lesson Study (as practised for over a century in Japan)
-Give teachers ownership from the beginning by learning about lesson study together, listening to teacher concerns and working effectively to share and develop skills
-Believe in the process and maintain a constant focus upon developing teachers as great learners
-Fully acknowledge the time, energy and commitment worthwhile change takes to achieve and not be tempted to look for 'Quick fixes' regardless of current climate around accountability
-Seek out and form connections with high-quality provision in secondary and higher education establishments (and where this is not available locally, use social media and existing organisations to achieve this)
-Allocate both financial and time resources over a sustained period and protect these opportunities
-Work hard to challenge the current culture of observation and monitoring to one of trust, collaboration and long term investment in teachers' professional development.

How does this lesson contribute to our understanding of high impact practices?

We understand that students are more engaged in the learning when they are presented with a mathematical problem that they are motivated to solve. In the beginning of this lesson, students seemed excited about constructing parallelogram displays by using what they had been learning to surprise their friends from Kanayama Elementary school. However, this problem requires no real mathematical need to draw the parallelograms accurately. Constructing accurate parallelograms efficiently in this case would be for the purpose of pleasing their friends, but not for a mathematical purpose or need. As the lesson went on, many students expressed understandings of the characteristics of a parallelogram and what tools to use to prove that a parallelogram is indeed a parallelogram. However, many students struggled to actually use their tools to do so and explain their thinking to each other, which led to disengagement in the math and in the collaboration with their peers. Perhaps if there was a real mathematical need or situational need to make the parallelograms accurate, students would be more willing to persevere in constructing them.

Through lesson study, we have also learned that independent work time plays a key role in the teacher's understanding of the students' conceptions and misconceptions. In this lesson, the independent work time was about 5 minutes. Then, the teacher had the students get into small groups of 4 to share their work and consolidate their thinking onto one whiteboard per group. The teacher stated that during the next lesson, they would discuss what the leaders had drawn on their whiteboards. Due to the fact that the students chose the leaders in their groups, the work that will be discussed in the next lesson was not selected by the teacher with the end goal in mind. However, that doesn't necessarily mean that it will not be effective. It may prove to be unclear and cause more confusion. On the other hand, it may provide a more real picture of the students' understandings and misconceptions in order to guide them toward the end goals.

A high impact practice that came up many times when observing teaching-through-problem-solving lessons is listening to students. Teachers often feel married to their plan due to all the research and hard work that went into it. It is key to anticipate many student responses, understandings, and misconceptions in order to guide them toward the end goals. However, when it comes time to teach the lesson to students, it is very important that teachers are willing to observe and listen to what the students are doing, and if it differs from what they anticipated, be willing to diverge from the lesson plan to meet the students where they are at. In this particular lesson, the teacher engaged the students in critiquing his mistakes when drawing a parallelogram incorrectly. He asked them questions like, "Is that true?" and "How can we be sure?" These are essential questions in order to put the cognitive load on the students and foster a culture of justifying their thinking with mathematical proof as well as fostering mathematicians who are willing to tackle problem without the guidance of their teacher. However he quickly answered his own question by saying "let's take a look at these set squares. If we want to see the two sides are parallel, how should I use them? Show with your hands." The more teachers answer their own questions and tell students what to think and the less we listen to students, the less opportunities they are given to construct their learning and grow as problem solvers. These are all important practices in order to get a real picture of the students' conceptions of the math.

2017 Ohata Ward Research Group
June Elementary Mathematics Research Lesson
Proposal by the Kamata District Team

Research Theme

The Instruction and Assessment that Activate Mathematical View and Reasoning, and Deepen Learning ~ Focused on Mathematical Activities ~

Grade 6 Division of Fractions

<With respect to the research theme>
The Kamata District team considered "mathematical view and reasoning" in this unit and in this lesson as follows.

[Mathematical view and reasoning in this unit]

- Students can think about ways to calculate division when the divisors are fractions based on the property of division, proportional relationships, etc. and also represent them using number lines and mathematical expressions and equations.

[Mathematical view and reasoning in this lesson]

- Through activities of thinking and explaining about Fraction ÷ Fraction, students can make connections among various ideas and think about a unified approach.

o Image of students

- Students explain ways to calculate Fraction ÷ Fraction using the property of division and/or area diagrams. As they critically compare and discuss various ideas, students realize the commonality of multiplying the dividend by the reciprocal of the divisor.

<Strategies to approach the research theme>

1. What does it mean to "activate mathematical view and reasoning"?

> During the independent problem solving stage
1) Think about ways to use prior knowledge
- Can we make the divisor into a whole number?
- Can we explain using the property of division?
- Can we use the area diagram that we used when we studied multiplication of fractions?

> During the whole classroom discussion stage
2) Explain other students' ideas and make connections among them

- The ideas of multiplying both the dividend and the divisor by 4 and multiplying them by the reciprocal of the divisor to make the divisor 1 are using the same property of division.
- If you look at the area diagram, we can tell that it (the quotient) is 4 the quantity obtained by partitionoing 2/5 into 3 equal parts.

> During the whole classroom discussion stage/summarizing stages

3) Identify commonalities among ideas and think about ways to unify them
   - In every methods, we see 5×3 and 2×4!
   - We can calculate by multiplying the reciprocal of the divisor.
   - That's the reason we can calculate by multiplying the the reciprocal of the divisor.

2 How should instruction and assessment to deepen learning look like?

What is "deep learning"?
1) By making connections among different ideas and develop a deeper understanding. Examining multiple information and identify issues and generate ways to address them.
2) Develop deeper understanding of knowledge and skills that have been learned previously by connecting them to new knowledge and skills. Realize knowledge and skills they learned are something that can be utilized in the society and actually apply them in everyday situations.

☐ Instruction to deepen learning

To deepen students' learning,
1) made sure that each student has his/her own idea prior to the critical reflection stage of the lesson so that students can engage in the whole class discussion effectively,
2) set up the time where students will explain their own idea for calculating Fraction ÷ Fraction so that they can reason through their conversation with each other,
3) when looking at other students' ideas during the whole class discussion, we include opportunities for students to interpret each other's mathematical expressions/equations in pairs, to complete someone else's ideas, and to extend friends' and own ideas by incorporating other people's idea,
4) ask questions that may help students recall prior learning, and
5) organize board writing so that students can more easily connect different ideas.

☐ How to conduct assessment

Assessment in today's lesson

Were students able to connect various ideas and develop a unified idea?

- What we want to value while assessing students
  - Focus on "finding commonalities"
  - Situations to assess students
    1) When students are trying to identify commonalities (through listening/observing students during the sharing time)
    2) During the summarizing stage (through students' refelctive journal entries)

June 28

Grade 6, Mathematics Lesson Plan

Date & Time: 5th period (13:35 – 14:20), Wednesday, June 28, 2017
Students: Grade 6 Students at Ohta Ward Kojiya Elementary School (62 students)
Teacher's Name: Hiroki Shibata, head teacher (Advance Class, 30 students)
Yukiko Kobayashi, head teacher (Regular Course: 24 students)
Koichi Hashimoto, head teacher (Basic Course: 8 students)
Place: Gymnasium, Grade 6 Class 3 classroom, and No. 3 study room

2017 Research Theme of Ohta Ward Mathematics Education Research Group
"The Instruction and Assessment that Activate Mathematical View
and Reasoning, and Deepen Learning"
~ Focused on Mathematical Activities ~

1. **Name of the Unit:** Let's Think about How to Divide by Fractions (Division of Fractions)

2. **Goals of the Unit:**
   - Students understand the meaning and calculation process of division of fractions when the divisors are fractions, and develop an ability to apply their knowledge.

3. **Assessment Criterion of the Unit:**

	A. Interest, Disposition, Motivation	B. Mathematical Reasoning	C. Skills and Procedures	D. Knowledge and Understanding
Assessment Criterion of the Unit	Students show interest in the meaning and calculation processes of division of fractions when the divisors are fractions. They try their best to figure out solutions by connecting division with fractions to whole number division calculations and properties of calculations they learned previously.	Students are reason about ideas or strategies for how to solve division when divisors are fractions. Their reasoning is based on the property of division and proportional relationships. They express and describe the calculation process using the double number line model and math sentences (expressions)	Students demonstrate they can successfully calculate division problems when the divisor is a fraction and know how to apply strategies.	Students understand the meaning of division with fractions, specifically division when the divisor is a fraction.

Assessment Criterion that Corresponds To Learning Activities	① Students are interested in the meaning and calculation processes of division of fractions when the divisor is a fraction, and they show their best effort to solve these problems by making connections to the calculations and properties of calculations they learned before. ② Students notice the merit of reducing fractions in the process of calculations, realizing that reducing fractions allows them to calculate simply and easily.	① Students are able to use a number line and words to describe, represent, and understand why the expression is fraction ÷ fraction. ② Students are able to explain the process of calculation when problems are fraction ÷ fraction, using the property of division and proportional relationships. They accomplish this by using double number lines, diagrams, and expressions. ③ Students are able to recognize the merit of converting all numbers to fractions when multiplication and division involve calculations with a combination of fractions, decimals and whole numbers; students can also describe why this calculation method is better. ④ Students are able to explain -- using a double number line and words -- the reason why they need to establish a division expression that corresponds to a specific problem situation.	① Students are able calculate fraction ÷ fraction problems that do not involve reduction of fractions. ② Students are able to simplify calculations by reducing fractions in the process of calculating. ③ Students are able to do division of fraction calculations involving mixed numbers. ④ Students are able to do multiplication and division of fractions involving three numbers. ⑤ Students are able to do multiplication and division calculations involving a combination of fractions, decimals, and whole numbers. ⑥ Students are able to find “times as much” (rate) by division, even when the comparison quantity and the base quantity are fractions. ⑦ Students are able to find the comparison quantity from the base quantity and “times as much” (rate) when “times as much” is a fraction. ⑧ Students are able to represent the relationship between quantities with expressions that include division using x; they are able to find the value of the base quantity.	① Students understand the meaning of fraction ÷ fraction problems. ② Students understand the calculation process of fraction ÷ fraction. ③ Students understand that reducing fractions in the process of calculating fraction ÷ fraction helps simplify the calculations. ④ Students understand that the properties of calculations that work for whole number calculations also apply to calculations for division of fractions. ⑤ Students acquire basic knowledge and skills related to division of fractions.

4. Rationale of the Unit:

(1) About the Unit

The Course of Study (Notification) describing the content related to this unit:

> The Content of Grade 6: A.) Numbers and Calculations
>
> (1) Help students acquire the following by providing mathematical activities related to multiplication and division of fractions:
>
> A. Students acquire these knowledge and skills:
>
> (A) Students understand meanings of multiplication and division of fractions including cases when the multipliers and divisors are fractions.
>
> (B) Students are able to calculate multiplication and division of fractions.
>
> (C) Students understand that the same multiplicative relationships and properties for whole numbers apply to multiplication and division of fractions.
>
> B. Students acquire the following thinking (reasoning), judging, and expressing ability.
>
> (A) Students pay attention to the meaning of numbers and expressions, the properties of calculations, and are able to think about the process of calculation from multiple points of view.

In this unit, students will think about how to calculate division of fractions when the divisors are fractions; and they are able to successfully complete the calculation. Based on decimal division they learned in Grade 5, students understand that the meaning of division as the inverse operation of multiplication. Lastly, teachers help students to understand that division has two meanings: division that solves to find "how many times as many (as much)" (quotitive) and division that solves to find "what is the unit" (partitive).

In addition, teachers help students examine the properties of division applied to fraction calculations, i.e., (a) the identity property, or "when multiplying or dividing the dividend and the divisor by the same number, the quotient remains the same;" (b) the commutative property; (c) the associative property; and (d) the distribute property of fraction calculations. Examining the properties of division applied to fraction calculations leads students to see how the whole number calculations and properties that students learned in the past work also with fraction calculations.

In this lesson, we will help students think about and understand the calculation processes and meanings of division with fractions in multiple ways. Students will engage in mathematical activities that help them understand the process of calculation of division with fractions by applying the properties of calculations, and support students understanding of division by fraction visually using area models. By connecting the expressions and diagrams, helping students to use mathematical view and thinking, develop deeper understanding of calculation process of division with fractions instead of just establishing a shallow understanding such as "multiply the inverse of the divisor."

(2) About the Students

We administer a readiness test to the students and decide what course the students need to talk. We usually administer the test at the beginning of every unit, however, we decided to keep the same group of students in each course and provide continuous instruction for the previous unit.

Regarding the advanced course: There are many students who grapple with mathematics problems willingly relatively to others, however, there are only a few students who could express his/her own thinking with basis and present his/her ideas in classroom. Because only a few students presenting ideas regularly, we incorporate pair learning and small group learning and provide more opportunities for the students to practice describing their ideas to others intentionally. From the analysis of the readiness test we found out that 36% of the students (11 students out of 30 students) already know how to find the quotients of the division of fractions. In the previous unit, "multiplication of fractions," we found out that there are 76% of students (23 students out of 30 students) who knew how to find the products of multiplication of fractions, there are only few students who understand the meaning and able to describe the calculation processes with understanding. Therefore, we believe that there are almost none of the students who understand the meaning of division of fractions.

Regarding the regular course: the students are using notebooks actively, and studying willingly. However, there is a large gap in mathematical knowledge and skills among the students in this course. Because of this circumstance, we use a cooperative learning approach of learning that the students who understand fast help others, or asking the students who do not understand gather near that front of the room where the teacher provide small group instruction or support.

Regarding the basic course: Although there are many students who could come up with expressions from the problem situations, many of the students cannot solve the problem completely because of the lack of basic calculation understanding and skills. However, these students often ask questions that could be the center of the learning, and there are many students who could admit they do not know and be able to say "I don't know" frankly. Because of this circumstance, we identify the stumbling blocks of their learning and provide appropriate supports. In order to help the students we also use ICT equipment in this course so that we could support students understanding.

5. Scope and Sequence of Related Topics:

Grade 5

Unit 4: Multiplication of Decimals
- Meaning of decimal x decimal and the calculation
- Application of the properties of calculation to decimal calculations
- Times as much (rate) with decimals. (No.1 & No.2)*

Unit 5: Division of Decimals
- Meaning of decimal ÷ decimal and the calculation
- Time as much (rate) with decimals. (No.1 & No.3)*

Unit 8: Relationship between fractions, decimals, and whole numbers.
- Times as much (rate) with fraction (No. 1)*

Unit 15: Multiplication and Division of Fractions:
- Meaning of decimal ÷ decimal and the calculation
- Times as much (rate) (No. 1 & No. 3)*

Grade 6

Unit 4: Multiplication of Fractions
- Meaning of fraction x fraction
- Calculation of fraction x fraction
- Relationship between product and multiplicand
- Use of fractions in formulae of area and volume
- Application of properties of calculation to fractions calculations.
- Multiplication of multiple fractions
- Reciprocal number

Unit 5: Division of Fractions
- Meaning of division ÷ division
- Calculations of division ÷ division
- Relationship between quotient and dividend
- Calculation of three fractions mixed with multiplication and division.
- Calculations mixed with fraction, decimal and whole numbers
- Multiplication of multiple fractions
- Times as much (rate) with fraction (No. 1, No. 2, No. 3)*

Lower Secondary School

- Multiplication and division of positive and negative numbers (integers)

* Three Types of Ratio (Rate) Situations:

Ratio/Rate – p
Quantity being compared – A
Base quantity – B

No. 1: A ÷ B = p
(Finding ratio)
No. 2: B x p = A
(Finding quantity being compared)
No. 3: A÷ p = A
(Finding base quantity)

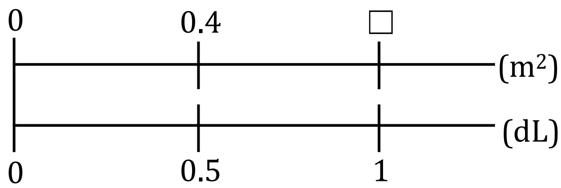
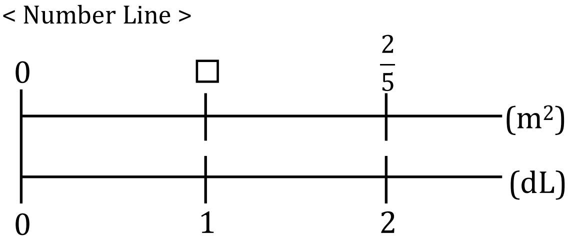
7. **The Overview of the Previous Lesson** (The 1st lesson of the 11 lessons)*

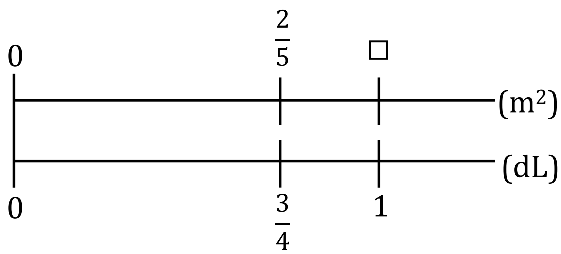
* Differentiate instruction courses: Advanced, regular, and basic courses are indicated in the parenthesis in the text below.

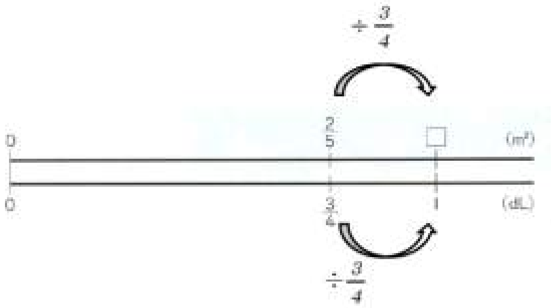
(1) The Goals of the Previous Lesson

- Students understand the meaning of “dividing by fractions” and are able to explain why the expression of the problem is fraction ÷ fraction.

(2) The Flow of the Previous Lesson

	○ **Learning Activities**	**Instructional Points to Remember [Evaluation], ○ Support**
Grasping Problem	○ Reviewing Division (Textbook p. 58) <u>Review Problem 1</u>: Please write a math sentence and find the answer. With 0.5 dL of paint, we could paint 0.4 m^2 of boards. What is the area of boards that we can paint with 1 dL of this paint?	(Basic & Regular Courses) Provide the number line and help students to recall what they learned in the unit “Division of Decimals” in grade 5. (Advanced Course) Student do not solve this problem.
	< Number Line >  < Math Sentence> 0.4 ÷ 0.5 = 0.8 Answer: 0.8 m^2	• By covering the numbers in the problem, help student to make transition from whole numbers to fractions.
	<u>Review Problem 2</u>: Please fill in △ with a whole number in, write down the math sentence, and find the answer. With △ dL of paint, we could paint $\frac{2}{5}$ m^2 of boards. What is the area of boards that we can paint with 1 dL of this paint? < Number Line > 	(Basic & Regular Courses) By modeling the number line of Problem 1, draw number line with students in order to help them find the relationships between quantities. Help students to recall that “fraction ÷ whole number” which they learned in grade 5. (Advance Course) Pose only the problem, and see whether students propose using the number line. The should propose and use the number line as a tool to determine the operation. Once the number line is presented by students, it will be shared with other students as part of discussion. • The value for the △ is a fraction that is less than 1. So confirm with the students that the relations/location of the numbers on the number line changed.

	< Math Sentence> $\frac{2}{5} \div 2 = \frac{1}{5}$ Answer: $\frac{1}{5}$ m^2 ○ Students represent the relationships of quantities on the number line and think about the math sentence that finds the value in the □. Problem: Write a math sentence when we put $\frac{2}{3}$ (fraction) in △ and find the answer. < Number Line >  < Math Sentence> $\frac{2}{5} \div \frac{3}{4}$ I don't know how to do this calculation, because this is the first time I've divided a fraction by fraction. Let's explain why the math sentence is fraction ÷ fraction.	(Basic & Regular Courses) The divisor changed from a whole number to a fraction. Draw the number line together with the students. (Advanced Course) Establish the math sentence by asking students to construct the number line and discuss the differences between this problem and the Review Problem 2. [Students are interested in the meaning and calculation process of fraction ÷ fraction and they are reasoning by trying to connect the new learning with previous learning about properties of calculations and the property of division.] (Observation, speaking/presentation)
Independent Problem Solving	○ Students think about why the math sentence is $\frac{2}{5} \div \frac{3}{4}$. (1) The value of △ changed from a whole number to a fraction but the question for finding the area that we can paint in 1 dL has not changed.	○ Help student pay attention to the difference between the problem used at the introduction and this problem.

	(2) Drawing the number line, and add the arrow that shows the relationship of numbers; understands that you need to do division to find the value in the □.  • $\square=\frac{2}{5}\div\frac{3}{4}$ $(\square\times\frac{3}{4}=\frac{2}{5})$ (3) If we think about the operation using word math sentence, (Area painted) ÷ (Amount of paint used (dL)) = (Area we can paint using 1 dL), we can write the math sentence as division.	• Ask students to draw a number line. Also help them to recall that they used □ to represent the value that they don't know and how they drew an allow pointing toward □. • If the arrow starts form □, it represents a multiplication math sentence. If this idea comes out from the students, ask the student to share his/her idea. Then, connect the idea of multiplication with the division math sentence. * In the textbook, the arrow start from 1. However, at this school, we instruct students to draw the arrow point toward the □. (All Course) O If the idea of (3) does not come from the students, provide three word cards so that students can construct the word math sentence; or create an arithmetic restoration problem using words.
Presentation And Discussion	O Students present their ideas • Identify the students who have the ideas (1), (2), and (3), and ask them to present the ideas in the class. O Students discuss their ideas • Discuss and check the idea of (1), (2), and (3).	• Provide opportunities for the students to explaining ideas by incorporating a pair learning. Also ask student to explain other students' idea. [Students explain why the math sentence is fraction ÷ fraction using diagrams.] (speaking/presentation, notebook)
Summary and Refraction	O Students summarize the lesson • Even if the amount of paint used was expressed in a fraction, to find the area we can paint in 1 dL, we use division similar to what we did with whole numbers and decimals. (Basic and Regular Course) O Students write refraction about the lesson. (Advanced Course) O Students demonstrate foresight for calculation of fraction ÷ fraction. O Ask students to think about how to calculate fraction ÷ fraction problems using the property of division.	• Help students understand that the divisor could be a fraction beside a whole number or a decimal. (Basic & Regular Course) • Help students to establish a task for next lesson that is "how they can calculate the division of fractions whose divisors are fractions" and make connections to the next lesson. (Advance Couse) • Ask students to think about fraction ÷ fraction problems. Inform them that the next lesson will start with some of the students presenting their ideas.

8. Instruction of the today's lesson
(the 2nd lesson of the 11 lessons)

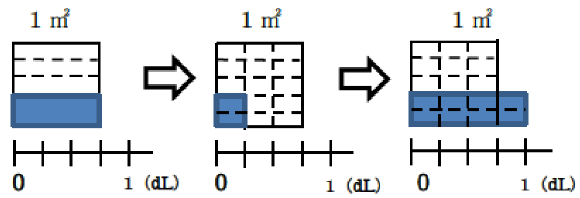
Advanced Course:
Place: Gymnasium
Teacher: Hiroyuki Shibata

(1) The Goals of the Previous Lesson

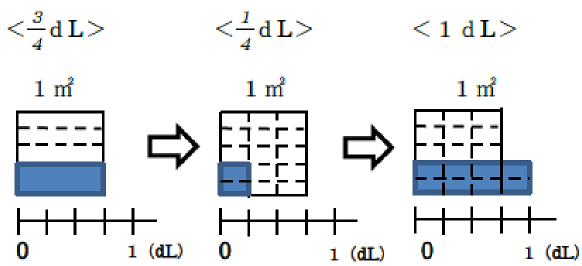
- Students explain and discuss the ideas of the calculations of fraction ÷ fraction and generalize the ideas to establish the generalizable formula.

(2) The Flow of the Previous Lesson

	○ Learning Activities	Instructional Points to Remember [Evaluation], ○ Support
Grasping Problem (5 min.)	**1. Reviewing learning from the last lesson and lean about the content of today's lesson.** T: What did we learn from the last lesson? C: We learned that the divisor of the division can be a fraction (as well as a whole number of a decimal). C: We thought about how to do the calculation when the divisor of the division is a fraction. T: Lets' confirm the task for today's lesson: Let's think about and explain how to do the calculation fraction ÷ fraction.	• Help students to recall that they established a math sentence $\frac{2}{5} \div \frac{3}{4}$ that represents the situation of the presented problem. • Talk with the students about how this was the first time they had a fraction as the divisor of a division problem.
Independent Problem Solving (5 min.)	**2. Confirming the process of calculation of $\frac{2}{3} \div \frac{3}{4}$ that was discussed about in the previous lesson.** C1: Make the divisor a whole number $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times 4) \div (\frac{3}{4} \times 4) = (\frac{2}{5} \times 4) \div 3$ $= \frac{2 \times 4}{5} \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C2: Make the divisor 1 by multiplying it by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$. $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3}) = (\frac{2}{5} \times \frac{4}{3}) \div 1$ $= \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$	• Select few students for the presentation. Pass out the mini white boards and ask them to write their thinking. • Ask students to put speech bubbles around the math sentences and the process of calculations as a way to make the explanation clear. * In class we write the process of calculation by aligning each part of the calculation at the equal (=) signs. However, in this lesson plan it is not written that way so that we can save space.

	C3: Make both dividend and divisor whole numbers by multiplying both number with the least common multiple 20. $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times 20)\div(\frac{3}{4}\times 20)$ $=(2\times 4)\div(3\times 5)$ $=\frac{2\times 4}{3\times 5}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C4: Use the ideas of fractions as division. $\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\div(3\div 4)=\frac{2}{5\times(3\div 4)}=\frac{2\times 4}{5\times(3\div 4)\times 4}$ $=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C5: Use invert and multiply the divisor. $\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\times\frac{4}{3}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C6: Use area model and expressions $<\frac{3}{4}\,dL>$ $<\frac{1}{4}\,dL>$ $<1\,dL>$  $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\div 3)\times 4=\frac{2}{5\times 3}\times 4=\frac{2\times 4}{5\times 3}=\frac{8}{15}$	• The method C5 will not be shared in the class, because those using it are most likely using it as a memorized calculation procedure.
Presentation And Discussion (30 min.)	**3. Present and Discuss ideas.** T: Please bring the portable whiteboards that you wrote your solution to the front of the class and paste them on the board. C1: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times 4)\div(\frac{3}{4}\times 4)=(\frac{2}{5}\times 4)\div 3$ $=\frac{2\times 4}{5}\div 3=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ T: Please write C1's idea in your notebook. T: When you finish writing, please talk with your partner. T: What kind of idea C1 is using?	• Select only 3 or 4 student ideas. • Read all students' notes from the previous lesson and select the students for presentation. • Show students the mini whiteboard with writing of math sentences. First students will think about the problem on their own and write down there thinking in the notebooks. Then, each will work with partner to discuss the process of calculations. ○ If I see student pairs having difficulty, I will ask them to come to the board, and think about the problem together. • If students want to add some explanation to the math sentences, ask them to write

	C: Making the divisor a whole number. C: Multiplying the dividend and the divisor by 4. C: The method uses the property of division. T: What is the property of division? C: Even if you multiply the both the dividend and the divisor with the same number, the quotient stay the same. T: Do you have anything you want to add, C1? C1: I think the explanation was great. T: The solution was using the property of division and making the divisor a whole number, wasn't it? T: Next, we see the person used different method. Do you understand the continuation of this math sentence? Please write it in your notebook.	explanation in a speech bubble. • Record student presentations and discussion on the board. • Select a student's work on the board and ask other students to describe his/her thinking. Also ask if students need to add something got the explanation.
	C2: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times\frac{4}{3})\div(\frac{3}{4}\times\frac{4}{3})=$________ C: $(\frac{2}{5}\times\frac{4}{3})\div 1=\frac{2}{5}\times\frac{4}{3}=\frac{2\times4}{5\times3}=\frac{8}{15}$ T: What idea did he/she use? C: he/she used the same idea as the last one. C: To make the divisor 1, inverse of the divisor was multiplied to both the dividend and the divisor. C: The last method, both the dividend and the divisor ware multiplied by 4 but this time, both of them were multiplied by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$, in order to make the divisor a whole number. T: This method change the divisor $\frac{3}{4}$ into 1. T: How about this one?	• Use projection equipment to show student writing in the notebook. Show only a part of the expression and ask other students to think about the continuation of the calculation. ○ Ask students to compare with the method C1 and ask how the ideas different. • Just like before, if students want to add some explanation to the math sentences, ask them to write it and use a speech bubble. • When students finish explaining the calculation process, the mini whiteboard will be posted on the board. Use the whiteboard examples to conduct the class discussion.
	C3: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times20)\div(\frac{3}{4}\times20)$ $=(2\times4)\div(3\times5)$ $=\frac{2\times4}{3\times5}=\frac{8}{15}$	• Introduce new ideas by asking student to bring their work as written the mini white board. • Select students to explain the solution.

	C: This time it is multiplied by 20. Why it is multiply by 20? C: I see, 20 is the least common multiple of 5 and 4. C: When both the dividend and the divisor is multiplied by 20, both of them become whole numbers.	• The number 20 was not randomly selected to use for the calculation. It is important for the students to look into and determine why the number 20 was used.
	T: I see, 20, the least common multiple of 5 and 4, was used to multiply both the dividend and the divisor.	
	T: These three solution methods used math sentences to explain the process of calculation. Here is another method. This time she/he used something other than math sentences.	• Make sure to prepare large area models for use on the board.
	C: I think the area model. C6: I used an area model and think the process of the calculation.  $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\div 3)\times 4=\frac{2}{5\times 3}\times 4=\frac{2\times 4}{5\times 3}=\frac{8}{15}$	• Provide the students the handout that include the area model and number line. Ask students to study the models, identify what part of the math sentence corresponds with what part of the area diagram. Ask students to use colored pen to highlight the important ides. • Show the area model to the students, and ask them to take turns to describing the presented idea. • Use chalk as baton, students will take turns to explain the calculation idea and develop a cohesive explanation.
	T: Let's look at it and figure out who she/he thought. C: First, the paint $\frac{3}{4}$ dL can paint $\frac{2}{5}$ m² of the boards, so it is like the diagram to the left. T: Okay, I will ask another person to describe next. C: The middle diagram shows the part that $\frac{2}{5}$ m² of paint can paint. T: Can we show that using a math sentence? C: We are splitting $\frac{2}{5}$ m² into three equal parts so it should be $\frac{2}{3}\div 3$. T: Let's pass the baton to the other person. So another person could continue to explain. C: We just find out the amount of boards we could paint with $\frac{1}{4}$ dL of paint. So we could paint 4 of that space for 1 dL of paint. (diagram on the right).	

	T: So how can we show the part we can paint with 1dL with an expression. C: We need to multiply $\frac{2}{5} \div 3$ by 4. So it should be $\left(\frac{2}{5} \div 3\right) \times 4$. T: Does the area you painted in the diagram matches with the answer you got? C: Wow, the diagram shows 8 fifteenth. So, it is $\frac{8}{15}$. **4. Compare the ideas and find the commonality.** T: This time we came up 4 different solution methods. They appear to be different but can you also find the commonality among the solutions? C: The method presented by C1, C2, and C3 all use the similar idea. They all made the divisor into a whole number. C: The method C3 does not have $\frac{2\times4}{5\times3}$ but that is included in the expression of the methods C1, C2 and C3. C: Wow, the expression right before calculating the answer is the same. T: Which one are you talking about? Let's ask 3 people to draw lines where it shows what she/he said. The method C3 does not have that so it is left out. C: Well, it has it also. You can see $\frac{2\times4}{5\times3}$ in the process of the calculation. If you switch the order of multiplication of the denominator from 3x5 to 5x3. C: Wow, we found $\frac{2\times4}{5\times3}$ in the method C also! So $\frac{2\times4}{5\times3}$ is common to all the methods. C: That means $\frac{2\times4}{5\times3} = \frac{2}{5} \times \frac{4}{3}$. T: Is there anything you notice when you compare the original math sentence and this sentence? C: When you compare $\frac{2}{3} \div \frac{3}{4}$ and $\frac{2}{5} \times \frac{4}{3}$, you can see the second fraction is changed into $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$. C: Also, the operation sign ÷ (dividion) is changed into x (multiplication).	[Students are able to describe the processes of calculating fraction ÷ fraction problems using diagrams, expressions, and words.] (presentation/speaking, notebook, observation) • Although, the ideas and processes of calculation represented with math sentences are different, ask student to look for the commonality among those different solutions. Using the discussion, generalize the calculation methods in to a math sentence. • Ask students to underline the commonality using a colored maker. • It is not easy to identify $\frac{2\times4}{5\times3}$ is in the process of the method C3. Therefore, fist discuss the commonality among the methods C1, C2, and C4. Then establish the commonality with C3 by asking if students could see the similarity. • Ask students to recall the original math sentence, $\frac{2}{3} \div \frac{3}{4}$. Then ask them to look for the math sentence that are similar in the process of calculation.

Summary and Refraction	**5. Summarizing** T: Let's summarize the lesson. Please summarize how to calculate fraction ÷ fraction problem in your notebook. C: When we divide by a fraction, we need to change the divisor into a reciprocal. C: The operation sign ÷ (division) is changed into x (multiplication). D: Can we wright the calculation process using math sentences with letters, just like we did with multiplication? C: Yes. we can. T: Please complete the math sentence, $\frac{b}{a} \div \frac{d}{c}$. When you finish writing, please write your reflection of today's lesson. T: Let's have someone share what they wrote. C: I understand how to do the calculation of fraction ÷ fraction. C: I know that fraction ÷ fraction can be calculated by multiplying inverse of the divisor but I did not know why. Now I understand why … so I am happy! C: Using the area model, I understand how to calculate fraction ÷ fraction. C: There were several different methods for the calculation, but we found the commonality among them.	• Using students' voices and construct the summary. • Using words to describe the process of calculation. • Try to establish the rule of division of fraction with the students using letters and symbols to generalize. • Establish a calculation rule $\frac{2\times4}{5\times3} = \frac{2}{5} \times \frac{4}{3}$. [Students able to generalize and establish the idea of calculation of fraction ÷ fraction problems from several different ideas and create a calculation formula.] (Speaking/notebook/observation)

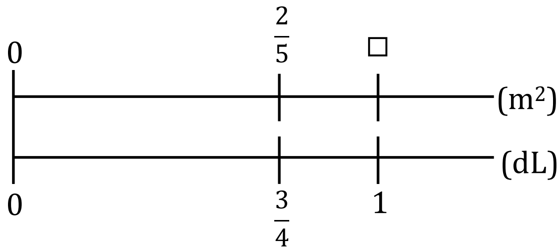
Regular Course:
Place: Grade 6, Class C Teacher: Yukiko Kobayashi

8. Instruction of today's lesson (the 2nd of 11 lessons)

(1) The Goals of the Lesson

- Students think and explain how to calculate fraction ÷ fraction.

(2) The Flow of the Lesson

	○ **Learning Activities**	**Instructional Points to Remember [Evaluation], ○ Support**
Grasping the Problem (5 min.)	**1. Reviewing learning from the last lesson and lean about the content of today's lesson.** T: What did we learn from the last lesson? C: We learned that the divisor in division could be a fraction (we knew from our previous learning that the divisor can be a whole number or a decimal). T: What math sentence did we establish in the previous lesson? C: It is $\frac{2}{3} \div \frac{3}{4}$. C: We have not learned how to calculate when the divisor is a fraction. T: I see, let's confirm the goal of today's lesson:	• Talk with the students about how it was the first time they had a fraction as a divisor in a division problem. • Confirm the problem situation and the math sentence from the previous lesson.
	Let's think about and explain how to do fraction ÷ fraction calculations.	
Independent Problem Solving (5 min.)	**2. Estimate the quotient $\frac{2}{5} \div \frac{3}{4}$ that we thought about in the previous lesson.** T: What do you think the value of the quotient could be?  C1: If you look at the number line, the value should not be less than $\frac{2}{5}$. T: How can we calculate $\frac{2}{5} \div \frac{3}{4}$? C: I could calculate this if the divisor was a whole number. T: If that is the case, if we could change the divisor to a whole number using what we have learned before, you could calculate it, couldn't you?	• Show the number line that the class used in the previous lesson. • Ask students to estimate the quotient. ○ Ask students to share what they are thinking about and having difficulty with this calculation. Confirm with students that the calculation is difficult because the divisor is a fraction.

3. Think about how to calculate $\frac{2}{5}\div\frac{3}{4}$.

C1: Changing the divisor to a whole number by multiplying the divisor by 4.

$$\frac{2}{5}\div\frac{3}{4}=\left(\frac{2}{5}\times4\right)\div\left(\frac{3}{4}\times4\right)=\left(\frac{2}{5}\times4\right)\div3$$

$$=\frac{2\times4}{5}\div3=\frac{2\times4}{5\times3}=\frac{8}{15}$$

C2: Changing the divisor to 1.

$$\frac{2}{5}\div\frac{3}{4}=\left(\frac{2}{5}\times\frac{4}{3}\right)\div\left(\frac{3}{4}\times\frac{4}{3}\right)=\left(\frac{2}{5}\times\frac{4}{3}\right)\div1$$

$$=\frac{2}{5}\times\frac{4}{3}=\frac{2\times4}{5\times3}=\frac{8}{15}$$

C3: Changing both dividend and divisor to a whole number by multiplying both by 20, which is the least common multiple of the divisors, 5 and 4.

$$\frac{2}{5}\div\frac{3}{4}=\left(\frac{2}{5}\times20\right)\div\left(\frac{3}{4}\times20\right)$$

$$=(2\times4)\div(3\times5)$$

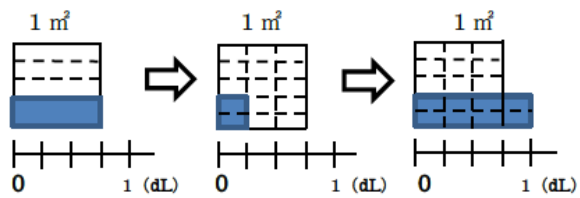
$$=\frac{2\times4}{3\times5}=\frac{2\times4}{5\times3}=\frac{8}{15}$$

C4: Find the inverse of the divisor and multiply the dividend by it.

$$\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\times\frac{4}{3}=\frac{2\times4}{5\times3}=\frac{8}{15}$$

C5: Using an area model and expression

$<\frac{3}{4}\,dL>$ $<\frac{1}{4}\,dL>$ $<1\,dL>$



$$\frac{2}{5}\div\frac{3}{4}=\left(\frac{2}{5}\div3\right)\times4=\frac{2}{5\times3}\times4=\frac{2\times4}{5\times3}=\frac{8}{15}$$

C6: I don't know what to do.

• For students who already figured out a solution method: Ask them to think about how to present their ideas to others clearly using a diagram, a math sentence, and words.

• When students come up with and show a solution method, encourage them to think about other solution methods.

• Do not ask students who used the "inverse and multiply" procedure to present their ideas to the class. Instead, ask them to think about why the quotient can be found by applying this procedure.

• If there is a student who is using the area model to describe the calculation process, provide a worksheet that has blank squares and number lines.

○ Prepare a hint card that says "even if the dividend and the divisor are multiplied by the same number the quotient remains the same." Give the card to students who are having difficulty thinking of a solution/idea.

Presentation And Discussion (30 min.)	**4. Discuss own ideas in small groups.** T: Let's talk about your solution ideas with your friends in small groups. **5. Present and discuss ideas.** T: Please bring your poster papers to the front and past them on the board. T: Let's start the presentation with the group No. 1. C1: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times 4)\div(\frac{3}{4}\times 4)=(\frac{2}{5}\times 4)\div 3$ $=\frac{2\times 4}{5}\div 3=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C1: We wanted to make a whole number divisor so we decided to multiply both the dividend and the divisor by 4. T: Is there anything that you did not understand from this explanation? C: Why did you multiply both the dividend and the divisor by 4? C: If you multiply only the divisor by 4, the answer will be wrong. C: We must use the property of division. T: What is the property of division? C: When you multiply both the dividend and the divisor by the same number, the quotient remains the same. T: I see you used the property to change the divisor to a whole number. T: Okay, how about Group No. 2? C2: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times\frac{4}{3})\div(\frac{3}{4}\times\frac{4}{3})$ $=(\frac{2}{5}\times\frac{4}{3})\div 1=\frac{2}{5}\times\frac{4}{3}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C2: Our group decided to multiply both the dividend and the divisor by $\frac{4}{3}$. When we did that the divisor became 1 and we could find the answer.	• Walk around the classroom to observe and monitor student work. Select different methods among the groups and ask the students to write their methods on poster paper. • Provide an enlarged copy of the worksheet that has squares and number lines to the group that used the area model to think about the calculation process. • Even if some of the students have not come up with an idea and aren't finished, ask these students to stop their work, so the presentation can be started when the specified work ending time is reached. Use the presentation and discussion to complete the full description and understanding of each presented idea. • Choose only 2 or 3 ideas for the whole group presentation. • Do not show the written poster papers all at once. These are presented one-at-a-time as each presentation is made. • If there are additional explanations, ask students to share these explanations with the class. • Record important points on the board. • Show the hint card to the students and review the property simply.

T: Do you have anything that you did not understand from their explanation?

C: Why did you decide to multiply by $\frac{4}{3}$?

C: Because they wanted to make the divisor into the whole number 1.

C: A fraction that changes a given fraction to 1 by multiplying it (by the given fraction) is called the "reciprocal" of the given fraction.

T: What more can we say about the idea this method used?

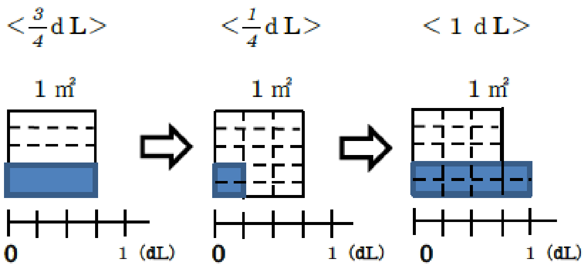
C: I think this method used the same idea as the previous one.

C: The number the group multiplied by was different from the last group's, but both methods made the divisor a whole number.

T. I see. So you are saying there is a commonality between the two methods.

T: How about Group No. 3.

C5: We used the area diagram and the math sentence.



$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \div 3\right) \times 4 = \frac{2}{5 \times 3} \times 4 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

T: Let's listen to how they were thinking carefully.

C: First, you could paint $\frac{2}{5}$ m^2 of the board with $\frac{3}{4}$ dL of the paint, so the diagram is like shown on the left.

C: Next, the middle diagram shows the area of the board that $\frac{1}{4}$ dL of the paint can cover in paint.

T: How could we represent that part with a math sentence?

C: We are dividing $\frac{2}{5}$ m^2 by three (3) equal amounts. So the math sentence would be $\frac{2}{5} \div 3$.

• If there is anything the students might want to add, ask these students to present.

• Record the points of discussion that come from the discussion among students.

• Prepare an enlarged worksheet for the board.

• Paste the area diagram on the board and ask the students to explain and build their understanding of the calculation method by taking turns to describe the method.

• Provide a worksheet that contains area diagrams and number lines to all students, so those who are trying to follow the explanations will have a chance to draw the diagram, also.

• The explanation of the method should not end in explaining the methods with the diagrams only. It is important to connect the diagram with the math sentence and clarify what parts of the diagram are represented in the math sentence.

	T: Then what happens next? C: We want to find out the area of the board that we can paint with 1 dL of the paint. So if we shade this 4 of the area that $\frac{1}{4}$ dL of the paint could paint. (See the diagram on the right in the above illustration.)	
	T: How could we express the last part with a math sentence? C: Because we need to multiply [$\frac{2}{5} \div 3$] by 4, so it will become $\left(\frac{2}{5} \div 3\right) \times 4$. T: Could you check if the area shaded on the diagram matches the calculation? C: Wow, it is 8 fifteenths so it is $\frac{8}{15}$!	• Make sure to use different colored markers, so the $\frac{8}{15}$ can be seen clearly in the area model. [Students explain the process of calculating fraction ÷ fraction clearly using diagrams, math sentences, and words.] (Speaking/presentation, notebook, observation)
	6. Compare ideas and find the commonality. T: This time we found 3 different solutions methods. They are different, but is there any commonality among the three methods? C: All of the methods C1, C2, and C3 are changing the divisor to a whole number. C: All of the math sentences include $\frac{2 \times 4}{5 \times 3}$ in the process of the calculation. C: Wow, that is true. The last math sentence before finding the answer is the same. T: What part of the math sentence are you talking about? I want three of you to come up and underline where you see $\frac{2 \times 4}{5 \times 3}$. C. It is true, that is common to all the math sentences.	• The methods or the ways that the math sentences are written is different among the presented ideas; however, help students to identify the commonality among these ideas. Drawing from this commonality, generalize the different math sentences into a single common math sentence.
	T: If you take this part of this math sentence, $\frac{2 \times 4}{5 \times 3}$, how can we express it? C: $\frac{2 \times 4}{5 \times 3} = \frac{2}{5} \times \frac{4}{3}$	• By underlining given parts, the similarity is emphasized visually.
	T: When you compare $\frac{2 \times 4}{5 \times 3}$ with the first math sentence, $\frac{2}{5} \div \frac{3}{4}$, do you notice anything? C: The denominator and the numerator of the divisor in $\frac{2}{5} \div \frac{3}{4}$ is switched. C: The operation sign ÷ is changed to x.	• Ask students to compare the initial math sentence and the new math sentence. Ask them to identify the part(s) of the math sentence that are different from the original one.

Summary and Reflection (5 min.)	**7. Summarizing** T: Let's summarize the lesson. Please summarize how to do the calculation for fraction ÷ fraction problems in your notebook. C: When we divide by a fraction, we need to change the divisor into its reciprocal. C: The operation sign ÷ is changed into x … division changed to multiplication. T: Let's write the reflection of today's lesson. T: I would like some of you to share what you wrote. C: I understand how to do the calculation of fraction ÷ fraction. C: The area model helped me to understand what we are doing with the math sentence. C: The methods presented were different, but there is a commonality.	• Using students' voices to construct the summary. • Using words to describe the process of calculation, fraction ÷ fraction. [Students understand how to calculate fraction ÷ fraction.] (Speaking/notebook)

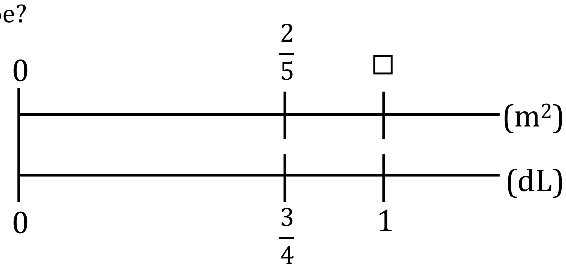
8. Instruction of the today's lesson
(the 2nd of 11 lessons)

Basic Course:
Place: No. 1 Study Classroom (3F)
Teacher: Koichi Hashimoto

(1)The Goal of the Lesson

- Students think about and explain how to calculate fraction ÷ fraction problems.

(2)The Flow of the Lesson

	○ **Learning Activities**	**Instructional Points to Remember [Evaluation], ○ Support**
Grasping the Problem (5 min.)	**1. Reviewing learning from the last lesson and learn about the content of today's lesson.** T: What did we learn from the last lesson? C: We learned that the divisor can be a fraction (besides a whole number of a decimal). T: What math sentence did we establish in the previous lesson? C: It is $\frac{2}{3} \div \frac{3}{4}$. C: We have not learned how to calculate when the divisor is a fraction. T: I see, let's confirm the goal of today's lesson:	• Discuss with students that it was the first time they had grappled with division where the divisors are fractions. • Confirm the problem situation and the math sentence in the previous lesson
	Let's think about and explain how to do the calculation fraction ÷ fraction.	
Independent Problem Solving (7 min.)	**2. Estimate the quotient $\frac{2}{5} \div \frac{3}{4}$ that thought about in the previous lesson.** T: What do you think the value of the quotient could be?  C1: If you look at the number line, the value should not be less than $\frac{2}{5}$. T: How can we calculate $\frac{2}{5} \div \frac{3}{4}$? C: I could calculate if the divisor was a whole number. T: If that is the case, if we could change the divisor to a whole number using what we have learned before, you could calculate it, couldn't you?	• Show the number line that the class used in the previous lesson. • Ask students to estimate the quotient. ○ Ask students to share what difficulty they may have had in thinking about how to do this calculation. Confirm with students that the calculation is difficult, because the divisor is a fraction.

	T: Okay, if the divisor is a whole number, do you remember how to do the calculation? For example, how do you calculate $\frac{2}{5} \div 3$? C: When we are dividing with a whole number, we can multiply 3 to the numerator. T: Okay, let's use what we learned before to make the divisor into a whole number. **3. Think about how to calculate $\frac{2}{5} \div \frac{3}{4}$.** C1: Changing the divisor to a whole number by multiplying it by 4. $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times 4) \div (\frac{3}{4} \times 4) = (\frac{2}{5} \times 4) \div 3$ $= \frac{2 \times 4}{5} \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C2: Changing the divisor to 1. $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3}) = (\frac{2}{5} \times \frac{4}{3}) \div 1$ $= \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C3: Finding the inverse of the divisor and multiplying with it. $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C4: Dividing the denominator by the other denominator, and the numerator by the other numerator. $\frac{2}{5} \div \frac{3}{4} = \frac{2 \div 3}{5 \div 4} =$ (Students don't know how to do this calculation.) C5: I don't know what to do.	• Make sure to review how to calculate when the divisor is a whole number. Use a concrete calculation problem example. • For students who already came up with a solution method, ask them to think about how to present their ideas to others clearly using a diagram, a math sentence, and words. ○ Walk around the classroom and grasp what students are doing. Provide a hint card to students who are having difficulty (like in the response C5). • C3: Do not ask the students who used the "inverse and multiply" procedure to present their idea to the class. Instead, ask those students to think about why the quotient could be found by using this procedure. • C4: Students who came up with this idea should be recognized because they are thinking about applying ideas learned from the previous unit, multiplication of fractions. However, the calculation 2÷3 cannot be done, so recommend to the students that they think about another method.

Presentation And Discussion (28 min.)	**4. Discuss ideas with a partner.** T: Let's talk about your solution ideas with your partner. **5. Present and discuss ideas.** T: Please share your ideas with your friends. C: I multiplied the divisor by 4. I have done the solution up to that point, but I could not finish it. T: What is the math sentence? C: $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \div \left(\frac{3}{4} \times 4\right)$ C: If you multiply only the divisor the answer will be wrong. T: That is right. Do you remember the property of division? $200 \div 2.5 = 80$ $\downarrow \times 10 \quad \downarrow \times 10$ $2000 \div 25 = 80$ The answer is the same! C: Well, I don't remember the property. T: Is there somebody who can explain the property? C: When you multiply both the dividend and the divisor by the same number, the quotient remains the same. T: When you use this property, is it okay to multiply only the divisor by 4? C: I think we need to multiply the dividend also. T: Okay, then what should we do to the math sentence? The sentence will be: $\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{4} \times 4\right)$ T: So what is the divisor going to be? C: It will be 3. C: Wow, the divisor is a whole number now. I think I can do the calculation.	• Walk around the classroom to see what things/ideas students are grappling and struggling with. • Be sure to encourage students to write down their thinking, even if they are still in the process of finding a solution. • Choose two ideas for the presentation and discussion. ○ Even if the students have not completed their work, start the presentation. Through the discussion of methods, try to construct a way that helps students understand each method and completes the task. • Show the property from the textbook using projector. • Make sure to record the important points on the board. • Make sure to carry out the discussion so that as many students as possible participate and complete the calculation.

	C: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times 4)\div(\frac{3}{4}\times 4)=(\frac{2}{5}\times 4)\div 3$ $=\frac{2\times 4}{5}\div 3=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ T: Well, we could find the answer. What do you think about this idea. C2: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times\frac{4}{3})\div(\frac{3}{4}\times\frac{4}{3})$ T: When you look at this, can you tell what idea she/he is using? C: The last time both the dividend and the divisor was multiplied by 4. But this time both the dividend and the divisor is multiplied by $\frac{4}{3}$. C: Why did you decided to multiply by $\frac{4}{3}$? T: Let's calculate the divisor part of the math sentence. C: Wow, the divisor is 1! T: Wow, is that by chance? Do you remember the name of the fraction that changes a given fraction to 1 when multiplied by the given fraction? C: A fraction that changes a given fraction to 1 by multiplying [to the given fraction] is called the "reciprocal" of the given fraction. T: What can we say about the idea this method used? C: I think the method uses the same idea as the previous method. C: The number the group multiplied was different from the last group's number, but both methods made the divisor a whole number. T: Let's continue and calculate it. $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times\frac{4}{3})\div(\frac{3}{4}\times\frac{4}{3})=(\frac{2}{5}\times\frac{4}{3})\div 1$ $=\frac{2}{5}\times\frac{4}{3}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$	• If there is a student who came up with this idea, ask the student to present it. • Make sure to record the important points on the board.

	C: We do not need to calculate ÷1. C: The answer is the same as the other one. **6. Compare ideas and find the commonality.** T: This time we found two different solutions methods. They are different, but is there any commonality between the two methods? C: Both methods changed the divisor to a whole number. C: I can see the same math sentence, $\frac{2\times4}{5\times3}$ in the process of the calculation. C: Wow, that is true. The math sentence right before finding the answer is the same. T: What part of the math sentence are you talking about? I want two of you to come up and underline where you could see $\frac{2\times4}{5\times3}$. C. It is true, that is common to the two math sentences. T: If you take a part of this math sentence, $\frac{2\times4}{5\times3}$, how can we express it? C: $\frac{2\times4}{5\times3}=\frac{2}{5}\times\frac{4}{3}$ T: When you compare $\frac{2\times4}{5\times3}$ with the first math sentence, $\frac{2}{5}\div\frac{3}{4}$, do you notice anything? C: The denominator and the numerator of the divisor in $\frac{2}{5}\div\frac{3}{4}$ is switched. C: The operation sign ÷ (division) is changed to x (multiplication).	[Students able to explain how to calculate fraction ÷ fraction clearly using math sentences and words.] (speaking/presenting, notebook, observation) • The methods or the ways that the math sentences were written are different between the presented ideas; however, help students identify the commonality between the ideas. Using the commonality, generalize the idea into a math sentence. • By underlining important parts, the similarity is emphasized visually. • Ask students to compare the initial math sentence and the new math sentence. Ask them to identify the part of the math sentence that is different from the original one.
Summary and Refraction	**7. Summarizing** T: Let's summarize the lesson. Please summarize how to do the calculation fraction ÷ fraction in your notebook. C: When we divide by a fraction, we need to change the divisor into a reciprocal. C: The operation sign ÷ is changed into x. T: Let's write the reflection of today's lesson. T: I would like some of you to share what you wrote.	• Using students' voices and construct the summary. • Using words to describe the process of calculation, fraction ÷ fraction.

	C: I understand how to do the calculation of fraction ÷ fraction. C: The methods presented were different but there is a commonality.	[Students understand how to calculate fraction ÷ fraction.] (Speaking/notebook)

6. The Unit Plan and Evaluation Plan (11 lessons)

Basic Course

	Basic Course		
Lesson	Goals	Learning Activities	Evaluation Method
	(1) Division of Fractions		
1	○ Students understand the meaning of dividing by fraction and are able to explain why the math sentence is a fraction ÷ fraction sentence.	• Students solve a problem related to the quantity of paint and area of board that the paint will be used to cover (in the order of whole number to fraction) • Student solve a problem whose math sentence becomes a fraction ÷ fraction math sentence. • Students explain why the divisor is a fraction. (Display number line and math sentence with words using projector)	A --- (1) [speaking/presentation, notebook] B --- (1) [Speaking/presentation, notebook]
2	○ Students think about how to calculate a fraction ÷ fraction math sentence.	• Students notice that they don't know how to calculate division when the divisor is a fraction. They will show foresight about how to change the divisor into a whole number, so they can use their prior knowledge (i.e., knowledge about solving fraction ÷ whole number). • Students think about how to calculate fraction ÷ fraction by using the property of division that the teacher underscores and/or shows. • Students present their ideas. • Students identify the commonality of the methods. • Students generalize the calculation method for fraction ÷ fraction problems using the commonality of different methods and verbally summarizes the process of calculation using words.	B --- (2) [Speaking/presentation, notebook]
3	○ Students understand that calculation can be done easily if fractions are simplified (reduced) in the process of calculation. ○ Students understand how to calculate whole number ÷ fraction and division involving mixed numbers. Students are able to	• Students review the calculation process of fraction ÷ fraction by solving a calculation problem. • Students solve $\frac{9}{14} \div \frac{3}{4}$. • Students learn that they could simplify (reduce) fractions in the process of calculation. Students think about common factors of 14 and 4, and 9 and 3 with the teacher. • Students notice that if they simplify (reduce) fractions in the process of calculating, the calculation becomes easier.	A --- (2) [Speaking/presentation, notebook. C --- (2) [Notebook] C --- (3) [Notebook]

	complete the calculation.	• Students solve $4 \div \frac{9}{2}$. • Students solve $\frac{2}{3} \div 3\frac{1}{5}$. • Students solve three application problems.	D --- (2) [Speaking/presentation, notebook]
4	○ Students understand that when the divisor in division problems is a proper fraction, the quotient becomes greater than the dividend. ○ Students understand how to calculate three fractions using a combination of multiplication and division. Students are able to successfully complete the calculation.	• Students pay attention to the quotients when a divisor is divided by either a proper fraction or a mixed number. • Students grasp the relationship of quantities visually by studying the number line that teacher shows. • Students understand that when the divisor is a proper fraction, the quotient becomes greater than the dividend. • Students think about how to calculate $\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5}$. • Students solve three application problems.	B --- (2) [Speaking/presentation, notebook] C --- (4) [Speaking/presentation, notebook]
5	○ Students understand that the multiplication/division calculations that include a combination of fractions, decimals, and whole numbers can be easily calculated by converting all the numbers to fractions. Students are able to successfully complete the calculation.	• Students think about how to calculate $0.3 \div \frac{3}{5}$. • Students present ideas and learn that the decimal can be converted to fractions. • Students notice that some fractions cannot be converted to decimals, therefore, these calculations are easier if decimals and whole numbers are converted to fractions. • Students solve two application problems.	B --- (3) [Speaking/presentation, notebook] C --- (5) [notebook]
6	○ Students develop deeper understanding of the process for making decisions about choosing appropriate operations using a number line.	• Students think about a math sentence that matches/represents the problem which asks students to find the weight of 1m, using the number line the teacher provides. • Students think of a math sentence that matches/represents the problem about finding the length for 1kg, using the number line the teacher provides. • Students think about a continuation to the word problem and write a math sentence.	B --- (4) [Speaking/presentation, notebook]

	(2) "Times as Much" with Fractions and Multiplication/Division of Fractions		
1	○ Students understand that the value that represents "times as much" can be found by division, even if both the comparing quantity and the base quantity are fractions.	• Students think about how many times $\frac{5}{4}$ m is as much as $\frac{1}{2}$ m. • Students identify the divisor and the dividend by thinking about how the same situation would be with whole numbers. • Students think about how many times $\frac{3}{8}$ m is as much as $\frac{1}{2}$ m. • Students summarize the concept of "times as much" by expressing the quantities on a number line.	C --- (6) [Speaking/presentation, notebook]
2	○ Students understand that the comparing quantity can be found using: (base quantity) x (times as much) = (comparing quantity).	• Students think about how to find the cost of something that is $\frac{6}{5}$ times as much as ¥600 and $\frac{3}{5}$ times as much as ¥600. • Students show the relationship on a number line and students estimate the cost using it. • Students understand that the cost can be found by using multiplication. • Work on the problem in the textbook, fill in the word in the ☐, and think about the meaning of the math sentence.	C --- (7) [Speaking/presentation, notebook]
3	○ Students understand that the base quantity can be found using: (comparing quantity) ÷ (times as much) = (base quantity).	• Students write quantities on a number line that represents ¥900 is $\frac{5}{3}$ times as much as the original price. • Students assign x to the cost and express the math sentence that represents the original price. • Students solve 1 appropriate application problem.	C --- (8) [Speaking/presentations, notebook]
	(3) Summary		
1	○ Students solve problems by applying what they learned in the unit.	• Students solve problems in Power Builder.	C --- (1) to (8) [Notebook]
2	○ Students check their understanding of the content of the unit and solidify their learning.	• Students solve problems in Mastery Problems.	D --- (4) [Notebook]

Regular Course

	Regular Course		
Lesson	Goals	Learning Activities	Evaluation Method
	(1) Division of Fractions		
1	○ Students understand the meaning of dividing by fractions and are able to explain why the math sentence is fraction ÷ fraction.	• Students solve a problem related to the quantity of paint and area of board that the paint can cover. (in the order of whole number to fraction) • Student solve a problem whose math sentence becomes fraction ÷ fraction. • Students explain why the divisor is a fraction. (Display number line and math sentence in words)	A --- (1) [speaking/presentation, notebook] B --- (1) [Speaking/presentation, notebook]
2	○ Students think about and explain how to calculate a fraction ÷ fraction problem.	• Students notice that they don't know how to calculate division when the divisor is a fraction. They show foresight about changing the divisor into a whole number so they can use their prior knowledge (fraction ÷ whole number). • Students think about how to calculate fraction ÷ fraction by using the property of division that the teacher shows. • Students present their ideas. • Students identify the commonality of the methods. • Students generalize the calculation method of fraction ÷ fraction using the commonality of different methods and verbally summarize the process of calculation using words.	B --- (2) [Speaking/presentation, notebook]
3	○ Students understand that calculations can be easier if fractions are simplified (reduced) in the process of calculation. ○ Students understand how to calculate whole number ÷ fraction and division that involves mixed numbers. Students able to successfully complete the calculation.	• Students review the calculation process of fraction ÷ fraction by solving a calculation problem. • Students solve $\frac{9}{14} \div \frac{3}{4}$. • Students present two solution methods: (1) simplify (reducing) fractions in the process of calculation and (2) no reduction of fractions until the answer is found. • Students compare the two methods mentioned above, and notice that it is easier to calculate if they simplify fractions in the process of calculating. • Students solve $4 \div \frac{9}{2}$. • Students solve $\frac{2}{3} \div 3\frac{1}{5}$. • Students solve six application problems.	A --- (2) [Speaking/presentation, notebook. C --- (2) [Notebook] C --- (3) [Notebook] D --- (2) [Speaking/presentation, notebook]

4	○ Students understand that when the divisor in division is a proper fraction, the quotient will be greater than the dividend. ○ Students understand how to calculate three fractions with a combination of multiplication and division. Students are able to do the calculation.	• Students pay attention to the quotients when a divisor is divided by either a proper fraction or a mixed number. • Students grasp the relationship of quantities visually by studying the number line the teacher shows. • Students understand that when the divisor is a proper fraction, the quotient becomes greater than the dividend. • Students think about how to calculate $\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5}$. • Students solve three application problems.	B --- (2) [Speaking/presentation, notebook] C --- (4) [Speaking/presentation, notebook]
5	○ Students understand that the multiplication/division calculations that include a mix of fractions, decimals, and whole numbers can be easily calculated by converting all the numbers to fractions. Students are able to do this calculation.	• Students think about how to calculate $0.3 \div \frac{3}{5}$. • Students present ideas and learn that there are two ways to do the calculations. • Students solve the problem $0.3 \div \frac{3}{5} \times 2$. • Students notice that some fractions cannot be converted to decimals; therefore, it is easier to calculate if both decimals and whole numbers are converted to fractions. • Students solve three application problems.	B --- (3) [Speaking/presentation, notebook] C --- (5) [notebook]
6	○ Students develop deeper understanding of the process for making decisions to choose appropriate operations using a number line.	• Students think about a math sentence that matches/represents the problem about finding the weight for 1m; they use a number line the teacher provides. • Students think about a math sentence that matches/represents the problem that is about finding the length for 1kg; their thinking is aided by drawing a number line without teacher or classmate help. • Students think about a continuation of the word problem and write a math sentence.	B --- (4) [Speaking/presentation, notebook]
	(2) "Times as Much" with Fractions and Multiplication/Division of Fractions		
1	○ Students understand that a value that shows "times as much" can be found by division, even when the comparing quantity and the base quantity are fractions.	• Students think about how many times $\frac{5}{4}$ m is as much as $\frac{1}{2}$ m and how many times $\frac{3}{8}$ m is as much as $\frac{1}{2}$ m. • Students identify the divisor and the dividend by thinking about the same situation with whole numbers. • Students summarize the concept of "times as much" by expressing the quantities on a number line.	C --- (6) [Speaking/presentation, notebook]
2	○ Students understand that the comparing quantity can be found using: (base quantity) x (times as much) = comparing quantity.	• Students think about how to find the cost of something that is $\frac{6}{5}$ times as much as ¥600 and $\frac{3}{5}$ times as much as ¥600. • Students and teacher together construct and use a number line to estimate the	C --- (7) [Speaking/presentations, notebook]

		cost. • Students understand that the cost can be found by using multiplication; they find the cost by making calculations. • Work on the problem in the textbook, fill the word in the □, and think about the meaning of the math sentence.	
3	○ Students understand that the base quantity can be found using: (comparing quantity) ÷ (times as much) = base quantity.	• Students and teacher together construct a number line that represents ¥900 is $\frac{5}{3}$ times as much as the original price. • Students assign x to the cost and express the math sentence that represents the original price. • Students solve 1 appropriate application problem.	C --- (8) [Speaking/presentation, notebook]
	(3) Summary		
1	○ Students solve problems by applying what they learned in the unit.	• Students solve problems in Power Builder.	C --- (1) to (8) [Notebook]
2	○ Students check their understanding of the content of the unit and solidify their learning.	• Students solve problems in Mastery Problems.	D --- (4) [Notebook]

Advanced Course

	Advanced Course		
Lesson	Goals	Learning Activities	Evaluation Method
	(1) Division of Fractions		
1	○ Students understand the meaning of dividing by a fraction and are able to explain why the math sentence is a fraction ÷ fraction sentence.	• Students solve a story problem that requires them to establish that the math lesson and problem is about fraction division, fraction ÷ fraction. • Student think about and explain why the divisor becomes a fraction (use number line, word math sentence) • Students think about how to calculate fraction ÷ fraction problems.	A --- (1) [Speaking/presentation, notebook] B --- (1) [Speaking/presentation, notebook]
2	○ Students think about and explain how to calculate fraction ÷ fraction. Moreover, they understand the presented ideas, find the commonality among them, and generalize the idea to construct the generalized formula for the calculation.	• Students present their ideas about how to calculate fraction ÷ fraction. (process the math sentence, area model, etc.) • Students deepen their understanding of other students' ideas. • Students discuss and compare the presented ideas. Then, they find the differences and commonality of the ideas. • Students generalize the calculation method of fraction ÷ fraction using the commonality of the ideas; they summarize the process of calculation using words and a math sentence with letters and symbols.	B --- (2) [Speaking/presentation, notebook]
3	○ Students understand that calculation can be made easier if fractions are simplified in the process of calculation. ○ Students understand how to calculate whole number ÷ fraction and division involving mixed numbers. Students able to successfully complete the calculation.	• Students review the calculation process of fraction ÷ fraction by solving a calculation problem. • Students solve $\frac{9}{14} \div \frac{3}{4}$. • Students present two solution methods: (1) simplify (reducing) fractions in the process of calculation and (2) no reduction of fractions until the answer is found. • Students compare the two methods mentioned above, and notice that it is easier to calculate if they simplify fractions in the process of calculation. • Students solve $4 \div \frac{9}{2}$. • Students solve $\frac{2}{3} \div 3\frac{1}{5}$. • Students solve eleven application problems.	A --- (2) [Speaking/presentation, notebook. C --- (2) [Notebook] C --- (3) [Notebook] D --- (2) [Speaking/presentation, notebook]

4	○ Students understand that when the divisor of a division problem is a proper fraction, the quotient becomes greater than the dividend. ○ Students understand how to calculate three fractions using a combination of multiplication and division. Students are able to do the calculation.	• Students solve two problems and compare the math sentences and answers. • Students draw number lines and grasp the relationships of quantities visually. • Students understand that when the divisor is a proper fraction, the quotient becomes greater than the dividend. • Students think about how to calculate $\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5}$. • Students solve seven application problems.	B --- (2) [Speaking/presentation, notebook] C --- (4) [Speaking/presentation, notebook]
5	○ Students understand that the multiplication/division calculations that are a mixture with fractions, decimals, and whole numbers can be easily done by converting all the numbers to fractions. Students are able to do the calculation.	• Students think about how to calculate $0.3 \div \frac{3}{5}$. • Students present ideas and learn that the decimal can be converted to a fraction. • Students notice that some fractions cannot be converted to decimals; but decimals and whole numbers can be converted to fractions to carry out the calculations. • Students solve two application problems.	B --- (3) [Speaking/presentation, notebook] C --- (5) [notebook]
6	○ Students develop a deeper understanding for making decisions about choosing appropriate operations.	• Students think about a math sentence that matches/represents the problem that asks about finding the weight for 1m, and the problem that asks about finding the length of 1kg (by drawing a number line). • Students think about a continuation of the word problem and write a math sentence. • Students create story problems and present their problem(s) to other students to solve.	B --- (4) [Speaking/presentation, notebook]
	(2) "Times as Much" with Fractions and Multiplication/Division of Fractions		
1	○ Students understand that the value that shows "times as much" can be found by division, even if the comparing quantity and the base quantity are fractions.	• Students think about how many times $\frac{5}{4}$ m is as much as $\frac{1}{2}$ m and how many times $\frac{3}{8}$ m is as much as $\frac{1}{2}$ m. • Students identify the relationship among the quantities visually by drawing a number line. • Students understand how to find "times as much." • Students solve two application problems.	C --- (6) [Speaking/presentation, notebook]
2	○ Students understand that the comparing quantity can be found using: (base quantity) x (times as much) = comparing quantity.	• Students think about how to find the cost of something that is $\frac{6}{5}$ times as much as ¥600 and $\frac{3}{5}$ times as much as ¥600. • Students estimate the cost using a number line.	C --- (7) [Speaking/presentation, notebook]

		• Students understand that that the cost can be found by multiplication. Students find the cost by calculation. • Students confirm the math sentence that helps them find the price of the pencil sharpener, and describe the math sentence using words. Students verbally describe how to find the prices of a colored pencil set and a notebook using words.	
3	○ Students understand that the base quantity can be found using: (comparing quantity) ÷ (times as much) = base quantity.	• Students draw a number line to represent the relationship of quantities in a problem: ¥900 is $\frac{5}{4}$ times as much as the original price. • Students assign x to the cost and express the math sentence that shows the original price. • Students find the original prices (finding the base quantity). • Students solve 1 appropriate application problem.	C --- (8) [Speaking/presentation, notebook]
	(3) Summary		
1	○ Students solve problems by applying what they learned in the unit.	• Students solve problems in Power Builder. They also solve additional problems.	C --- (1) to (8) [Notebook]
2	○ Students check their understanding of the content of the unit and solidify their learning.	• Students solve problems in Mastery Problems.	D --- (4) [Notebook]

Lesson Report

Report created by: Belle Cottingham, Sara Liebert, Camilla Pratt, Ruth Trundley
Name of Lesson: 6th Grade Fraction division
Date of Lesson: 28/06/2017

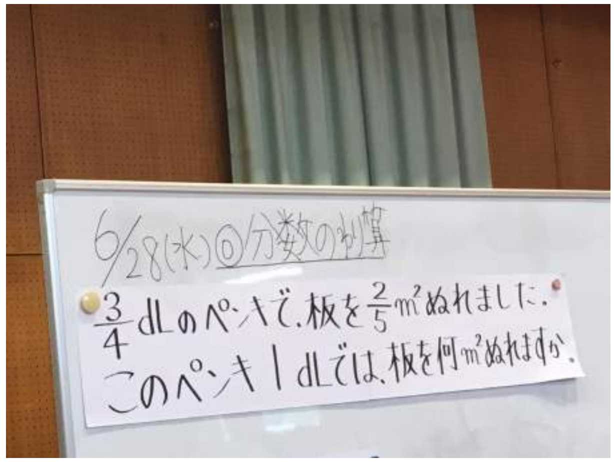
What are the primary lesson goals? Students explain and discuss the ideas of the calculations of fraction divided by fraction and generalize the ideas to establish the generalizable formula.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)? 2nd lesson of 11 lessons. The previous lesson explored the meaning of a fraction divided by a fraction. The goals of the unit are for students to understand the meaning and calculation process of division of fractions when the divisors are fractions, and develop an ability to apply their knowledge.

Summary of Lesson

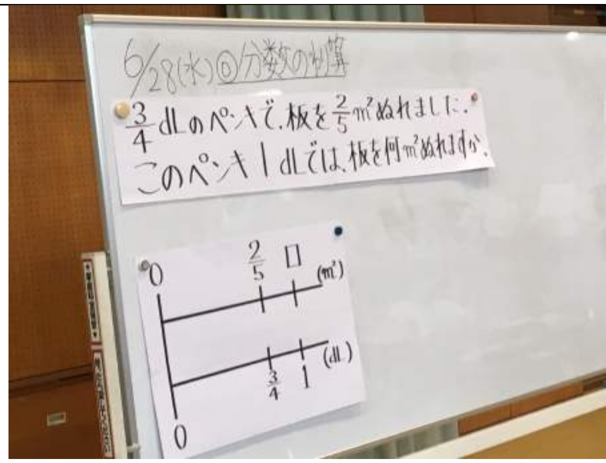
Start & End Time	**Lesson Phase**	**Notes**
13:35 - 13:43	**Introduction Posing Task**	**Strategies to build interest and to connect to prior knowledge** The lesson began by the teacher asking the students what they had done the previous day, "So we started a new unit yesterday, who can explain? Only 1?" Initially only one child put their hand up to respond. S13 said, " we did division of fractions" and S11 said, "we did how to do division of fractions". Other children then joined in and said, "we established the Maths problem". The teacher did not move on until the last statement was made. This shows the very specific response that the teacher was looking for - the response that made the distinction between whether they had learnt to calculate fraction divided by fraction or whether they had established the calculation needed to solve the problem. They had learnt the latter and it was interesting to see that the children recognised this. This is the basis of understanding the *structure* of the Maths and its relevance to a "real life" context versus understanding how to "do" the Maths. I put "real life" in inverted commas because, as pointed out in the post-lesson discussion, problems involving a fraction divided by a fraction are actually very rare in real life and in reality this problem would most likely be solved using decimals. At this point, the teacher then asked a child to read the Maths problem from yesterday which was stuck onto the board: "with ¾ Dl of paint,

we can paint 2/5m squared of a board. How much paint is needed for 1m squared?" (See the photo below).

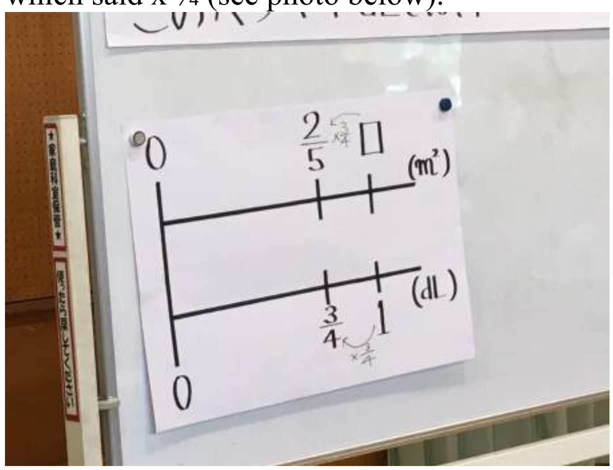


The teacher then asked the children how they did it to which a child replied, "double number line". This can be seen in the photo below.

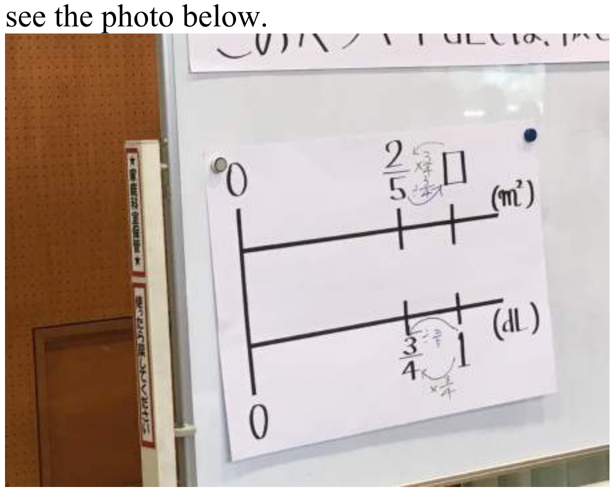
The teacher then stuck the double number line to the board (see photo below):



A child comes up to add to it to show how they had come to the conclusion yesterday that they needed to divide a fraction by a fraction. S24 added arrows pointing to the left to both number-lines which said x ¾ (see photo below).

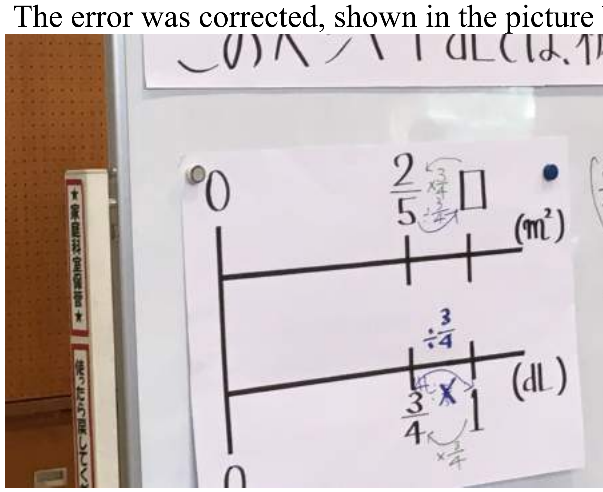


S30 then came up and added number lines in the opposite direction - see the photo below.



It can be seen in the photo that the additional arrows added by S30 are incorrect. Many students recognised this and there was a lot of mumbling from students with some saying, “something’s wrong”. This shows that not only were many students engaged in the lesson at this point but also that they had enough knowledge and understanding from yesterday to recognise the error. The teacher simply said, ‘we have some problems’. S30 looked through his notebook from yesterday and said, “I am so nervous, I keep making mistakes. I thought I knew this”. He then recognised and corrected his mistake, putting □ on the bottom instead of ¾ - this is quite a common error to make.

The error was corrected, shown in the picture below:



In the post-lesson discussion the teacher was disappointed that this mistake had been made and thought it something to criticize. Some may think that actually it was a good thing this mistake was made and

		proved a useful learning tool. It also kept the other children engaged as they were very keen to point out and discuss the mistake. For the observers, it showed that students use previous learning that is written down in their notebooks and that they look back at it to help them make sense of new learning (or to help them recap on old learning). After this, the teacher then asked who had finished solving the problem and who needed more time to solve the problem. As many students needed more time, the lesson then moved into the next phase (independent problem solving). Although it wasn't mentioned, the children had a strong understanding of the properties of division from work covered in 4th grade (e.g. if you multiply the dividend and the divisor by the same number, the quotient doesn't change; if you divide any number by itself the quotient is 1). The understanding is evident from some of the strategies children used during the independent problem solving phase of the lesson (see the below section for details of these). It is because the students had a thorough understanding of the *properties* of division that they were able to apply their knowledge to the unfamiliar situation of a fraction divided by a fraction. They could use this knowledge to attempt to *create* their own procedure (some more successfully than others).
13:43 - 13:49	**Independent Problem Solving**	The independent problem solving began by the teacher telling the students they had three minutes to consider how to divide a fraction by a fraction, and complete and improve the work in their notebook from the previous day's lesson. The teacher gave students a clue and said, yesterday we changed the fractions into whole numbers. Today you have to calculate and explain your solution. The teacher started a timer and students began to solve the problem and improve their notebook independently. The teacher immediately began to circle the room observing students working. Four students who used different strategies to solve the problem completed quickly and the teacher asked those students to present their work on a small whiteboard that could be magnetically posted to the larger classroom whiteboard. The students that the teacher chose to show their work used strategies from the lesson plan that were planned to be shared during the class discussion. These students worked behind the classroom whiteboard so that other students could not see what they were doing. The teacher moved between students working at their desks and students working behind the board. He observed students work and interacted verbally with students. The teacher used phrases such as, • "maybe you can put speech bubbles to explain what you did"

- "write in your book what the idea is"
- " write a description and an explanation about it"
- "talk to your partner about what you did"

This is a photo of a student working behind the large class white board. On the other side of the board are the rest of the students seated in their desks working on the same problem.

Individual, pairs, group, or combination of strategies

Photo 1 - Student was unsure and didn't do much in the time. Lots of rubbing out.

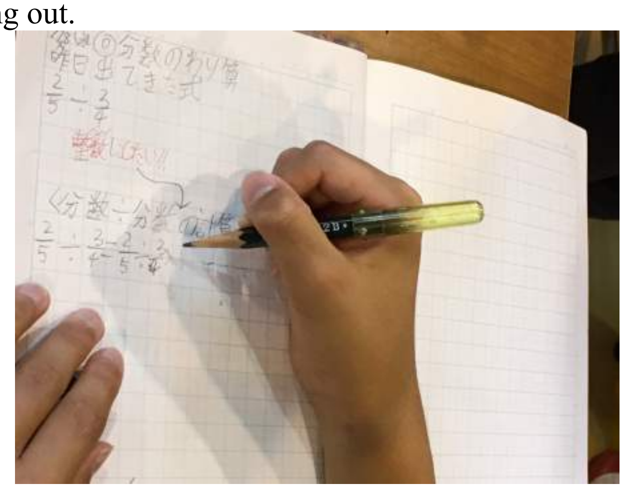
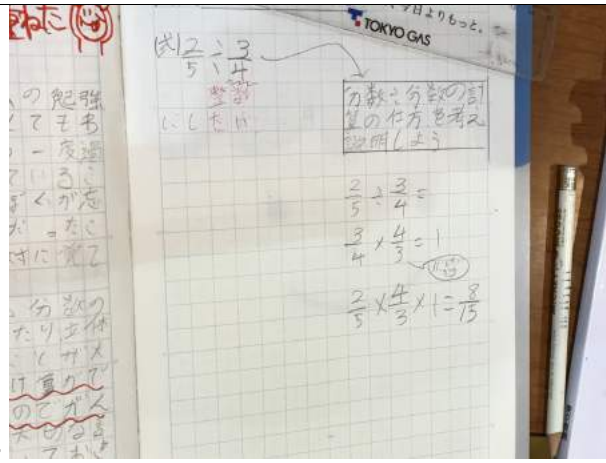
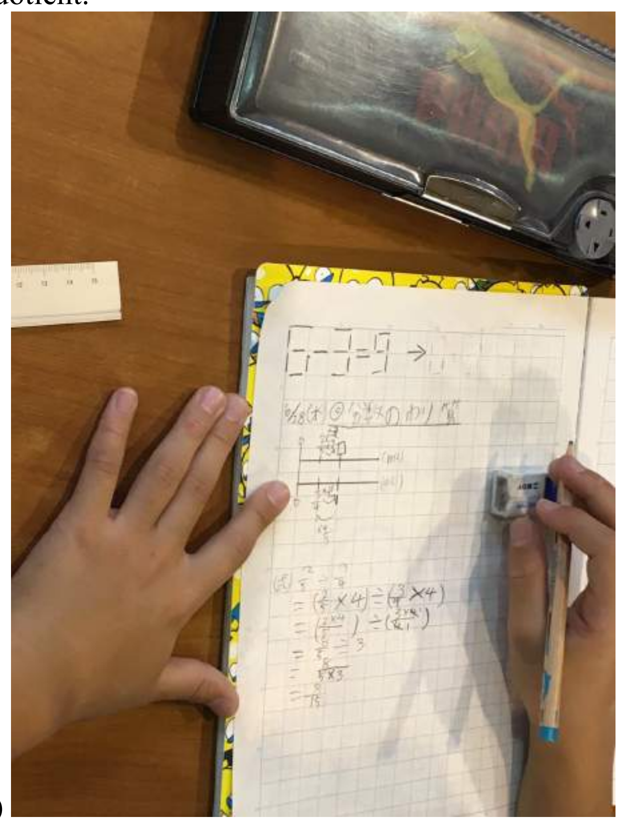
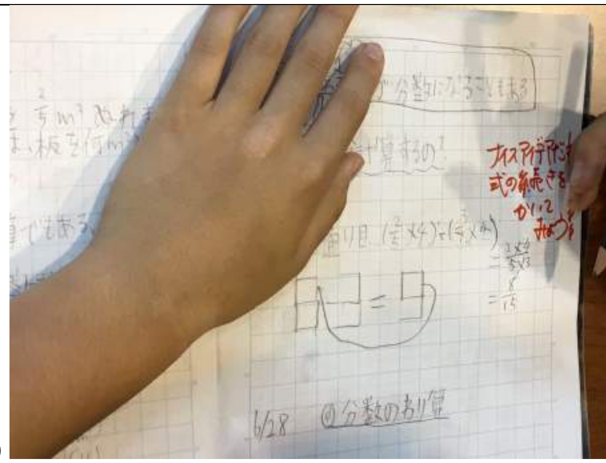
1) 

Photo 2 - This student worked out the correct answer. Student used the following properties of division: a) Multiplying by the reciprocal gives one. b) multiplying the dividend and the divisor by the same number doesn't change the quotient.

		2)  **Photos 3, 4 and 5** - All used the same strategy. Students made the divisor a whole number and then used the property: multiplying the dividend and the divisor by the same number doesn't change the quotient. 3) 

4) 

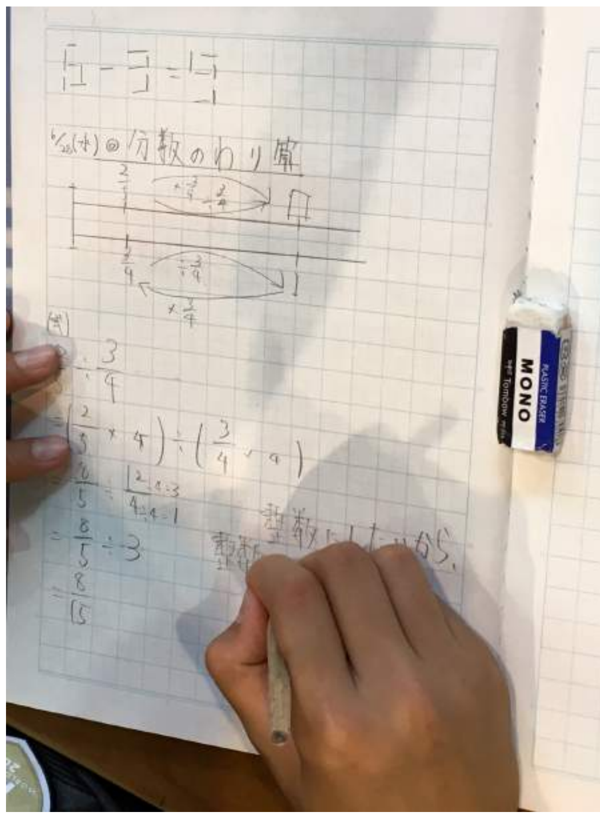
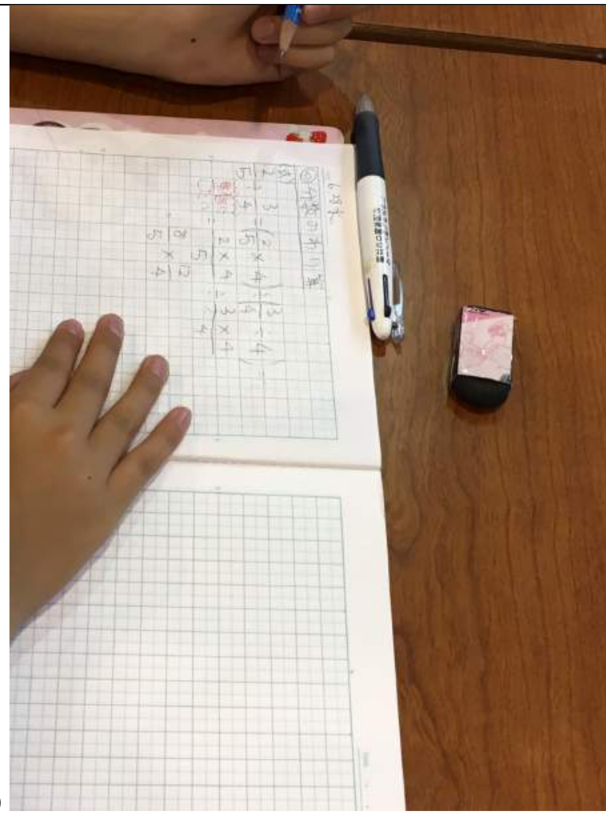
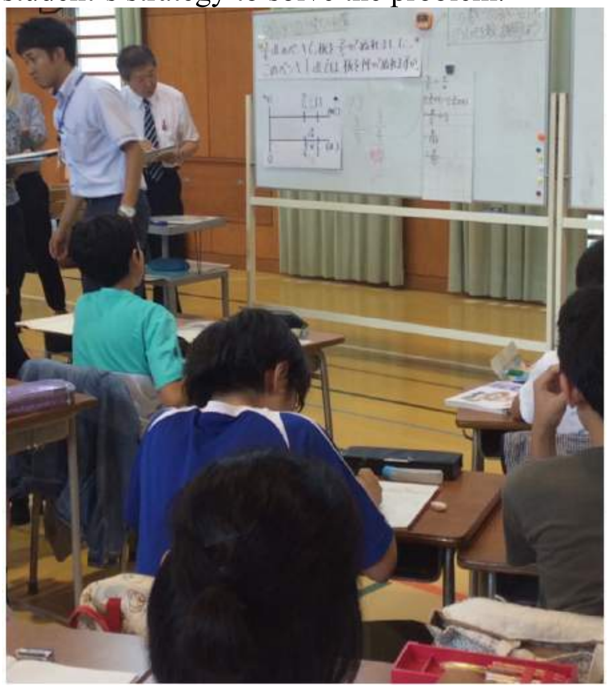
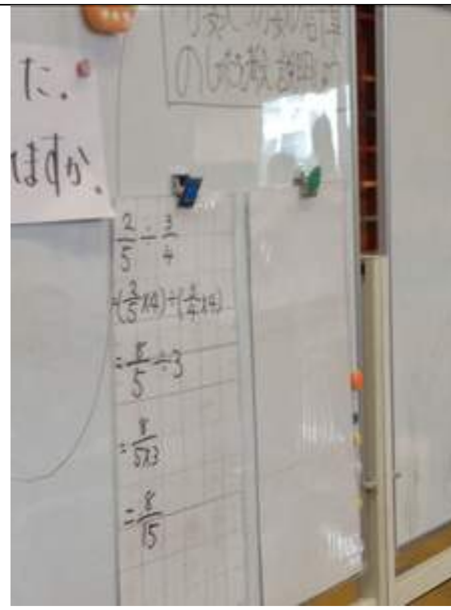
5) 

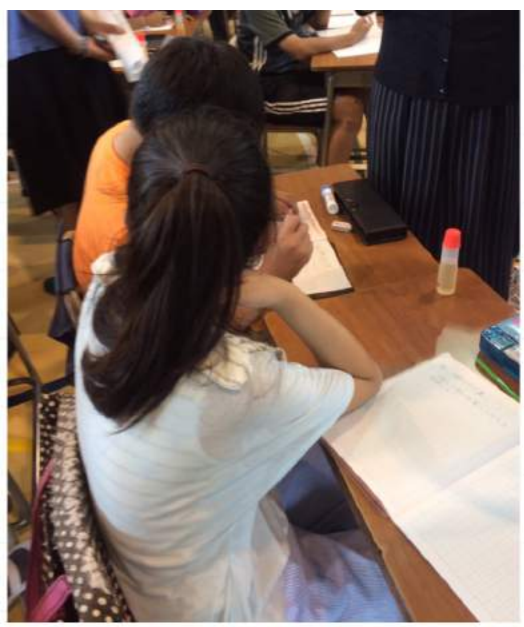
Photo 6 shows a students with an incorrect answer.

		6)  **Photo 7** shows a students getting stuck using the diagram to try to solve the problem. (7)  When most students were finished working, the teacher asked the students that had been working behind the large white board to take a seat. The teacher began to put the student work onto the whiteboard

		one at a time. As each of the students solutions were placed on the whiteboard, the teacher then lead a whole class discussion about the student's strategy to solve the problem. 
13:49 - 14:07	**Presentation of Students' Thinking, Class Discussion**	**Student Thinking/ Visuals/ Peer Responses/ Teacher Responses** The students were asked to put their pencils down and look up. The teacher said "We are going to think about how each person did this" and he revealed the first of the sheets that had been prepared by the four selected students during the previous section. **Student 1:** The first method selected matches the first method set out in the lesson plan. The teacher identified that about eight students had used this method and asks: "Who can explain? Write a description and then talk to your partner." The teacher wanted all the students to understand this method, whether or not it is the method they used.



The talking between pairs at this point is varied. S01 and S02 talk animatedly but the four pairs of students sitting behind them spend very little time speaking to each other. In the middle column of pairs, two pairs of students talk intently, one student does not have a partner and does not turn to speak to anyone else and the remaining two pairs look at their own books without talking.



Some of the students seem keen to talk but their partners do not engage, such as the pair in this picture (photo above).

After a minute the teacher asks "Who can explain? What did this person do?" pointing at the board. Four students put up their hands.

S09 is invited to explain: “In order to change the divisor into a whole number, he has multiplied by four.”

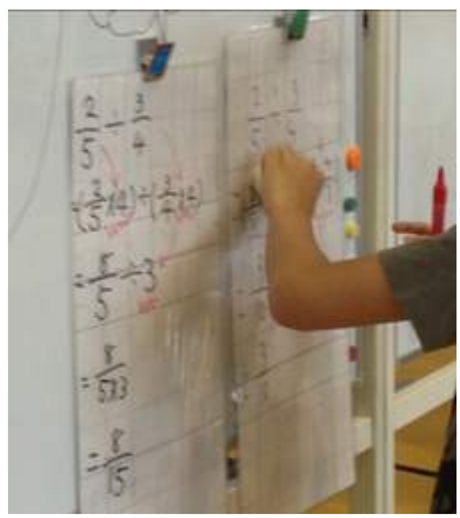
S15 adds: “So the three quarters I wanted to make it into a whole number so multiplied by four; we used the property of division and multiplied both numbers by four.” This is an explicit reference to one of the properties mentioned above, the identity property.

Understanding of the identity property, in relation to division with whole numbers, is explored in an earlier grade but the teacher does not assume that everyone will immediately be able to recognise to what S15 is referring and asks: “Do you know what he means by ‘property of division’? How do we know to multiply by four to make a whole number?”

The second question draws attention to the fact that the choice of four is not random, it is based on understanding. About two thirds of the students put up their hands in response to the second question. Students offer explanations and the teacher writes on the board to accompany this, annotating the method accordingly (see photo below). This is the solution used by students in the previous sections - see photos 3, 4 and 5.

Student 2

The teacher reveals the second of the sheets. This shows the second method set out in the plan. The method contains a mistake and the teacher asks “What did he try to do?” inviting the students to make sense of the method and so notice the mistake and how to correct it.

		 S21 says: “To make the divisor ¾ into one, multiply by 4/3.” The teacher asks “Who understands this idea?” and some students, including S19, respond that they don’t understand it. S25 explains that you have to multiply the dividend as well as the divisor by 4/3, using understanding of the identity property again. The teacher then says: “Let’s look to the students to explain” referring to the students that were responsible for laying out the method on the sheet. S24 says: “It must be multiply by 4/3” and is invited to come to the board to ‘fix it’.  S12 then comes to the board and the teacher says “Let’s continue to fix it.” He tells the student not to explain, just to write to fix it.

The teacher summarises this method by saying “Use the reciprocal to change to a whole number” and presents this idea in a speech bubble above the sheet. Several students look unsure at this point. This is the solution used by a student in the previous section - see photo 2.

Student 3
The teacher reveals the third of the sheets, saying “Next one.”. This shows the third method set out in the plan. He asks “How many of you understand it?” and eight students indicate that they do. The teacher asks them to talk to their partner and to go to the board if they can’t see the method. He identifies that S06 needs to go to the board and takes them over.

Only some students are talking to their partners. About six students gather at the board without their partners who have are left to sit on their own. Some pairs talk whilst others look in their books and wait.

After a minute and a half the teacher asks “Have you had a chance to talk? What did you multiply by, is it random?”

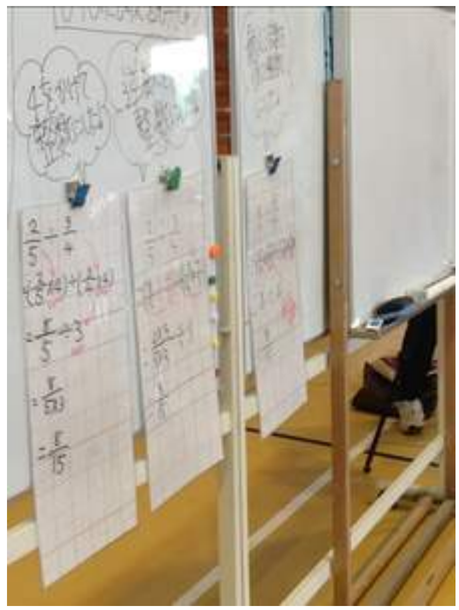
S07 says: “Both □ and ¾, if we multiply them by twenty both will become whole numbers.”

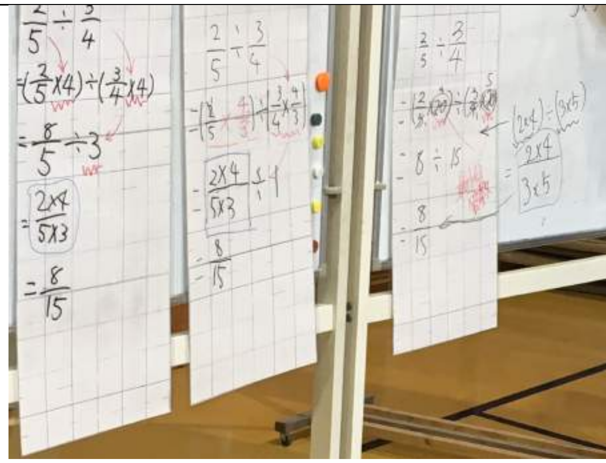
<table>
<tr><td></td><td></td><td>The teacher probes for further explanation: “How did you find this number?”

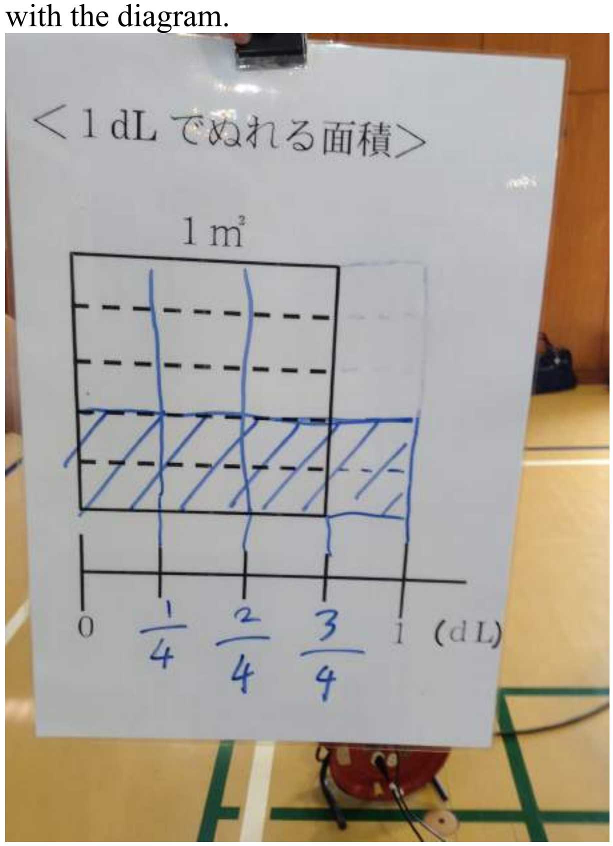
S09 says: “If you look at the denominators four and five, and find the lowest common multiple.”

S15: “You have got eight divided by fifteen.”

The teacher asks: “What shall we do to summarise?” He then suggests “Using lowest common multiple to make both whole numbers” and writes this in a speech bubble above the sheet.


The teacher decides not to show the fourth sheet which has the fourth method set out in the plan, using a diagram. This is to allow time for comparing the three methods and identifying commonalities which the teacher identifies as critical to the purpose of the lesson.</td></tr>
<tr><td>14:07 - 14:25</td><td>Summary/Consolidation of Knowledge</td><td>Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals
The teacher focused on three different ideas and asked children to compare them. There was a lack of group or paired work and the class discussion was dominated by only a few children.</td></tr>
</table>

		 S11 says : “All of them involve changing to whole numbers but different expressions.” S04 says : “All of them are making the fractions into whole numbers!” S05 says: “It is actually making the divisor whole.” The teachers asks: “Three different ideas all making the divisor into a whole number! Anything else?” S01 is the only one to put her hand up. The teacher says: “No one else?” and starts to tell them to talk to their partner. There is not much discussion, hence the teacher asks S01. S01 says: “All doing division, dividing by one or dividing by 15.” The teacher says: “So the eight on the left hand side, how did we come up with 8?!” S19 says: “No eight in □ divided by ¾...not sure!” S05 says “We did □ multiplied by 4 so two is the numerator multiplied by four is eight.” Teacher points out that this step was skipped, hence he added this information in (2 x 4 divided by 5 x 3 as in the photo above). Teacher then explains 2 x 4 divided by 3 x 5 on a different solution Some children have noticed the commonality of expression. S04 claps hands. S01 says: “Oh!” S15 says: “The expression is the same!” Teacher asks: “Do you see the same maths expression?” S11, S16 and S17 come to the board Teacher asks: “What do you think is it the same?” One of the children says that the order is different. Teacher asks children to clarify. S11 says: The order 3 x 5 - the order is different.” Children at the back were not engaging at this point and it was not clear whether they were clear about the points that were being made. The teacher than circled the commonality in all three (see above) and says 2 x 4 divided by 3 x 5 is included in all three calculations Very specific discussion about 3x5 in the third solution being different to 5x3 in the first two. This again linked back to the properties of

		division. Teacher kept asking questions and encouraging students to discuss their findings. Teacher says: “Do you always want to change to a whole number? Do you have something you can explain?” S17 says: “We can multiply the reciprocal of ¾ so 4/3.” Teacher says:”When we do division with fractions we use the reciprocal of what?” S17 says: “Of the divisor.” The teacher asks children to make a reflection of this in their books. The summary leads children to come up with a rule for dividing a fraction by a fraction. Do all children understand this as not all participated in the discussion? Children at the back write the reflections independently. It would be interesting to see what their reflections were, which children copy exactly from the board and which show a more in depth understanding. During the last couple of minutes the teacher shows the fourth method with the diagram.  There was not enough time to discuss this in the lesson and the teacher says: - “When you look at this you might not be able to figure it out. Let’s maybe talk about this tomorrow.”

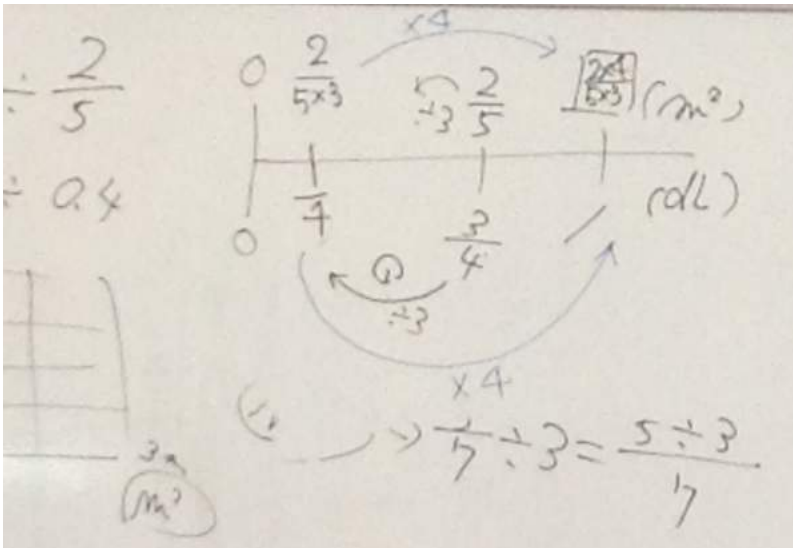
		

What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?

- The importance of understanding, as a teacher, why you are teaching a topic, what understanding it is built on, from previous grades, and how it fits with future learning. This then leads to a greater understanding of the mathematics in the topic and allows you to consider contexts that will be useful and appropriate and numbers that will give rise to the need for the new understanding. Sometimes contexts can be problematic; for the mathematics in this lesson it is difficult to find a convincing ‘real life’ context; most often calculations with measures will involve decimals rather than fractions. The fractions used in the lesson had to be ones where converting them to decimals to calculate was not obviously easier and so prompted the need to be able to divide a fraction by a fraction.
- Having clarity about the understanding that students bring into a lesson allows a teacher to draw attention to this understanding, making explicit connections between what is already known and understood and the new learning. With regard to this lesson, the properties of division had been taught and understood in the context of whole number calculations prior to grade 6 and in this lesson the students were then applying this understanding to calculations with fractions. In the plan the properties were identified: the identity property (which was used throughout the session), the commutative property (which was important in the discussion when looking for commonalities during the summary), the associative property and the distributive property.
- By focussing on using understanding of division and its properties, the methods shared demonstrated why the mathematics works, rather than simply giving the students a formula for how to divide with fractions (teaching them a ‘trick’). This is the difference between instrumental understanding and relational understanding as set out by Skemp (1976). By focussing on relational understanding the students

will not only be able to generalise about how they can solve divisions involving fractions (using the reciprocal) but they will also understand how and when to use this method, rather than using it indiscriminately (for example to solve 6/7 divided by 3), and will be able to use it as the foundation for future learning and understanding.

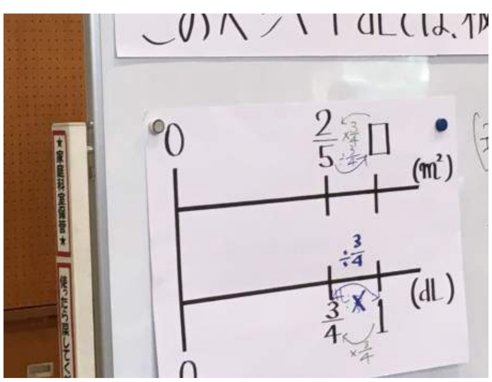
- The importance of teachers grappling themselves with the mathematics planned for the lesson before teaching it or observing it being taught, otherwise it is not possible to anticipate the children's responses and it can lead to assumptions. The diagram method which was not shared in the main part of the lesson but was in the plan, appeared problematic when looking at it prior to the lesson and sparked a lot of debate in our group. In the post-lesson discussions it was clear that many observers were not convinced that it should have been included, partly because it focussed on solving the problem rather than focussing on how to divide a fraction by a fraction which was the purpose of the lesson.
- Anticipating responses and sequencing them are vital to the planning of a lesson and allow the teacher to prepare a pathway through the lesson, leading to the summary of learning which will still come from the students. The teacher must manage their time in the lesson, often multi-tasking; in this lesson the teacher identified children who had used methods he wanted to share and set them up to reproduce these on sheets whilst the other children were working independently so that the methods were ready to share in the next part of the lesson.
- The distinction between teaching WHY a certain calculation is needed to solve a problem in a given context and HOW to solve such calculations, leading to generalisation. The lesson prior to this one focussed on solving a problem set in a context; this problem could be solved using a diagram (see above). Today's lesson focussed on how to solve a calculation which involved a fraction being divided by a fraction; the teaching sequence then goes on to build on this and use the understanding to solve other such calculations. It is important, as was pointed out by the final commentator, that students understand that one example does not allow you to generalise but it does allow you to hypothesise and then any hypothesis can be tested out in future lessons.
- The importance of focussing children on simplifying and on the process they are using so that they attend to and model all the intermediate steps. This then allows everyone to find connections and commonalities and leads to hypothesising.
- Any image used (manipulatives, diagrams etc.) needs to support the understanding that is the focus of the lesson. In this lesson we were able to consider a diagram that supports students to reach a generalisation and a diagram that supports students to solve one problem. The latter was not useful in this lesson. The former was the 'double number line'; this really exposed why the calculation involved dividing by three and multiplying by four.



- Listening carefully is key; choosing students who have something important for everyone to hear to share with the class. The lesson plan is a guide but student thinking should be the focus for the teacher and lead them to make decisions in the lesson which might not fit with the plan.

What new insights did you gain about how administrators can support teachers to do lesson study?

- In the post-lesson discussion it was mentioned that this topic was chosen because it is one that is traditionally hard to teach. I think this would give teachers the confidence to realise that it's ok to admit some topics are hard to teach and will help them realise that others also find this hard.
- The above allowed us to understand that lesson study should not be a "show-case" lesson but should be a real lesson where things will go wrong (like at the beginning where a child made an error when writing on the board work from the previous lesson - see photo below).



- It was useful to hear a lot of critical comments and questions about the lesson and to see that the teacher (and to some extent the planning team) were taking these on board and not becoming defensive or offended. This will allow teachers new to lesson study to understand that it is ok to receive critical comments and that this will help them to move forwards in the future.

- The most useful lessons are the ones that tackle something difficult as they prompt everyone to think. This lesson not only prompted students to think but truly engaged the adults in the post lesson discussion around several topics. (1) the usefulness of the diagram (2) teacher leading the students too much in the summary and (3) the importance of students showing their work, rather than holding calculations in their head.
- How to engage participants, even if it is group discussion before the whole group discuss. This lesson, the facilitator of the debrief first had group discuss guiding questions. Each group had a facilitator that shared out the group's discussion points and key observations to the lead teacher- who could then response. We thought this was an effective way to engage such a large group of observers.

How does this lesson contribute to our understanding of high impact practices?
This lesson contributes to our understanding of a high-impact instructional practice in several ways.

Firstly, it emphasises the importance of detailed planning and more importantly the anticipated students' responses. This process enables the teacher to sequence the solution strategies that come from students and use their voice and thinking to build understanding rather than simply teach students to calculate, or teach them 'a trick'.

The teacher is very skillful in managing the time so that children could compare the three methods that he had anticipated them to achieve. The process also highlights the importance of teacher's own subject knowledge and shows that even complicated lessons such as dividing fractions can be solved in more than one way.

The lesson demonstrates the importance of using prior knowledge as a stepping stone in solving problems in new situations or contexts. In this case the new context was dividing a fraction by fraction and students had to understand prior concepts so that they could independently construct new ideas.

Listening to the children's answers including their mistakes and using their voices and thinking to build understanding and develop their mathematical thinking is a very important feature of high impact practices.

Finally, the lesson highlighted the fact that to be able to generalise, students need to be allowed time and given the opportunity to work through a few examples of how to solve a problem, rather than just a single one.

Section 7

Other Resources

Elementary School Teaching Guide

for the Japanese Course of Study: Arithmetic (Grades 1-6)

LESSON STUDY RESOURCE SERIES

GER
Global Education Resources
www.globaledresources.com

LESSON STUDY RESOURCE SERIES

Lower Secondary School Teaching Guide

for the Japanese Course of Study: Mathematics (Grades 7-9)

Global Education Resources
www.globaledresources.com

Section 8

Evaluation Tools

IMPULS Pre-Program Survey Items

1) **How long have you been involved in lesson study?**
( ) No involvement yet ( ) Less than 1 year ( ) 1-3 years ( ) 3 years or more.
2) **In what content area(s) have you experienced lesson study?**
(mathematics, music, language arts, etc.?)
3) **Please describe how your current organization uses lesson study.**
4) **Please indicate the number of times you have participated in any of the following lesson study roles.** Research lesson observer (not on planning team): ___ Facilitator: ___ Planning team participant: ____ Research lesson teacher: ____Final commentator: ____ Member of a steering committee: ___ Other role :___ Please describe.
5) **How did you learn to perform these roles?**
6) **Please describe what you currently see as the essential features of lesson study.**
7) **What do you think are the potential strengths/ benefits of lesson study in your current organization?**
8) **What do you think are the challenges to using lesson study in your current organization?**
9) **To what extent do you expect to learn about each of the following during the immersion trip to Japan?** Indicate for each ( ) Not at all ( ) A little ( ) Some ( ) Quite a bit ( ) A lot.
   (a) Important features of lesson study
   (b) How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)
   (c) Supporting participants to have powerful and effective lesson study experiences
   (d) Writing a good lesson plan
   (e) Evaluating the quality of a lesson plan
   (f) How to observe research lessons
   (g) Organizing a successful post-lesson discussion
   (h) The role of the knowledgeable other
   (i) How to support student problem-solving
   (j) How to build students' mathematical habits of mind and practices
   (k) How to organize the board
   (l) Teacher questioning techniques
   (m) How to summarize a lesson
   (n) Anticipating student responses
   (o) How teachers support whole class discussion (neriage)
   (p) Strategies for making students' thinking visible
   (q) Student note-taking
   (r) Knowledge about the Japanese educational system in general
   (s) Mathematics content
   (t) Knowledge about Japanese curriculum materials
   (u) How to develop a mathematics unit/curriculum
10) **Are there additional learning experiences (not listed above) you hope to have during the immersion trip?**
11) **Please select and rank in order of importance five items from question 9) that you believe will be most professionally useful for you within the next year.** (Drag and drop your top five from the left-hand list to the right-hand column.)
12) **Is there anything else you'd like to add?**

IMPULS Post-Program Survey Items

1) **In looking over all the research lessons during the immersion program, name one that was especially meaningful to you, and why?**
2) **In looking over all the post-lesson discussions during the immersion program, name one that was especially meaningful to you, and why?**
3) **In looking over all the lectures during the immersion program, name one that was especially meaningful to you, and why?**
4) **Was there a conversation among participants during the immersion program that stands out to you?** (This might have been an informal conversation, outside the official program.) **Please describe, and provide reasons that this stood out for you?**
5) **How did your views about teaching and learning mathematics change as a result of this trip, if at all?**
6) **How did your views about the essential features of lesson study change as a result of this trip, if at all?**
7) **Please describe the biggest or most crucial changes you are now considering in your lesson study work as a result of this trip, if at all**.
8) **How much did you learn about each of the following during the immersion trip to Japan?**
   Indicate for each ( ) Not at all ( ) A little ( ) Some ( ) Quite a bit ( ) A lot.
   (a) Important features of lesson study
   (b) How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)
   (c) Supporting participants to have powerful and effective lesson study experiences
   (d) Writing a good lesson plan
   (e) Evaluating the quality of a lesson plan
   (f) How to observe research lessons
   (g) Organizing a successful post-lesson discussion
   (h) The role of the knowledgeable other
   (i) How to support student problem-solving
   (j) How to build students' mathematical habits of mind and practices
   (k) How to organize the board
   (l) Teacher questioning techniques
   (m) How to summarize a lesson
   (n) Anticipating student responses
   (o) How teachers support whole class discussion (neriage)
   (p) Strategies for making students' thinking visible
   (q) Student note-taking
   (r) Knowledge about the Japanese educational system in general
   (s) Mathematics content
   (t) Knowledge about Japanese curriculum materials
   (u) How to develop a mathematics unit/curriculum
9) **If you had other learning experiences (not listed above) during the immersion trip, please describe them below** (you can add up to 3) **and rate how much you learned about each of them** ( ) Not at all ( ) A little ( ) Some ( ) Quite a bit ( ) A lot
10) **Please select and rank in order of importance the five items from the previous two questions that you believe will be most professionally useful for you within the next year.** (Drag and drop your top five from the left-hand list to the right-hand column.)
11) **Please comment on the schedule/timetable of the program. Was there enough time for preparation and review? Other issues that would help in planning future programs?**
12) **Are there remaining questions you still have**?

Daily Reflection Questions

Reflection Questions Days 1-6

1. What are one or two "Aha's" or insights you had about lesson study?
2. What are one or two "Aha's" or insights you had about mathematics for teaching
3. What are one or two "Aha's" or insights you had about pedagogy or classroom instruction?
4. Please describe any ideas that you would like to share with your colleagues when you return?

Additional Reflection Questions Days 7-8

5. Please describe any ideas that you would like to share with your colleagues when you return?
6. Please share any remaining questions or wonderings you have?

Final Day's Reflection Questions Day 9

1. What opportunities for teacher learning does Lesson Study afford?
2. How did your views of lesson study change from your participation in the IMPULS program?
3. How do you view teaching and learning now?
4. How will you take this back to your own context?

Lesson Report

(Annotate with pictures, quotes, student work examples, board work etc.)

Report created by:
Name of Lesson:
Date of Lesson:

What are the primary lesson goals?

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

Summary of Lesson

Start & End Time	**Lesson Phase**	**Notes**
	Introduction, Posing Task	Strategies to build interest and to connect to prior knowledge
	Independent Problem Solving	Individual, pairs, group, or combination of strategies • experience of diverse learners • teacher's activities
	Presentation of Students' Thinking, Class Discussion	Student Thinking/ Visuals/ Peer Responses/ Teacher Responses Student 1: Student 2: Student 3: Student 4: Student 5:
	Summary/Consolidation of Knowledge	Strategies to support consolidation, e.g. blackboard writing, class discussion, math journals

What new insights did you gain about mathematics or pedagogy from the post lesson discussion and IMPULS participant discussion?

What new insights did you gain about how administrators can support teachers to do lesson study?

How does this lesson contribute to our understanding of high impact practices?

Protocol for Creating an Action Plan

In your assigned small groups (see below), write out recommendations you would make for:

- Your own classrooms (instruction, pedagogy)
- Lesson Study Teams at your site (or in your context)
- Your School (for your admin, ILT, or other decision-makers regarding instruction, pedagogy, or PD)
- Central Office if applicable (may include math department lesson study supports through central office, etc.)
- Networking across sites, districts, external partners

Groups may choose present their plans through a visual, a timeline of actionable next steps, a chart, or any other format conducive to generating a concrete plan

Groups

1. Acorn Woodland Elementary	7. Prieto Math & Science Academy
2. Argonne Elementary	8. San Francisco Community School
3. Hillcrest Elementary	9. University based (Ed, Millie)
4. Lawton Elementary	10. Math Consultants (Belle, Karen, Ruth)
5. Middle Schools (David/Stephanie)	11. Building lesson study at a new school (Meghan, Nakachi, Rory, Shelby)
6. Muir Elementary	

Section 9
Evaluation Data

IMPULS Pre-Program Survey (May 2017)

1. How long have you been involved in lesson study?

Value	Percent	Responses
No involvement yet	10.7%	3
Less than 1 year	21.4%	6
1-3 years	42.9%	12
3 years or more	25.0%	7
		Totals: 28

2. In what content area(s) have you experienced lesson study (mathematics, music, language arts, etc.?)

Count	Response
20	Mathematics
1	ELA
3	Mathematics and ELA
1	Mathematics, ELA, and Science

3. Please describe how your current organization uses lesson study.

Response
As a collaborative tool to enhance our math instruction individually, grade-level, and school wide.
As part of action research projects with groups of schools/teachers
At our site we have two lesson study teams. We use lesson study to create research lessons that we hope will address a specific learning gap at our school.
I have been running a lesson study group at my school for 6 years. 4-5 teachers.
I supervise the Master Teacher Program in SFUSD, which requires all master teachers to recruit and lead a lesson study team in the content area of their choice. We have roughly 30 lesson study teams supported through our program, with the hopes of expanding next year.
I work in a K-8 school. This year the 4 Kindergarten and first grade teachers took part in lesson study. I attended as the school's math coach.
In SY 2016-2017, I was on a team of 3 other teachers at my site working with TTP. In SY 2017-18, I plan to apply to be a Master Teacher and lead a team at Lawton.
My current school has two planning teams (k-1 and 2-3) and each team plans 2 research lessons per year.
My math department conducts research lessons once per semester.
My school engages in mathematics lesson study school-wide. Each grade level does one cycle per year, where one teacher on that grade level teaches the research lesson.
Myself and 2 teachers from my school have been part of an IoE led programme (Connecting Knowledge) to improve children's writing.
Our school engages in Lesson Study as a form of school-wide professional development. 4-6 people serve on the planning team, and all members of staff are invited to the demonstration lesson and debrief.
Our school holds 3-4 lesson study cycles/year. We have cross-grade level teams that plan the lesson for one teacher to implement. School-wide we have a yearly theme/focus that we align to. One teacher teaches the lesson and the entire school participates in an observation/feedback cycle.

Response

Response
The Maths Hub runs TRGs (Teacher Research Groups) and I have led one this year. These last half a morning and take place every half term. Typically there are ten teachers from five different schools. The usual format is that I deliver some CPD and then we all watch a lesson. After the lesson we discuss it in relation to mastery concepts.
The math department at our site works together on lesson study. We did two public lessons this year for our entire staff at Bret Hart.
We are working on academic discourse in mathematics and across subjects, dovetailing with our schoolwide goal of increasing student voice.
We currently use lesson study as our main form of professional development for teacher teams.
We do multiple lesson study cycles for math throughout the year with different cohorts of teachers. The whole school comes for the pre discussion, to observe the lesson, and for the post discussion to learn from the team.
We have lesson study teams based on grade level bands. Each team designs, plans, and implements two public lessons per year. Next year we will be expanding lesson study to the whole school and more of our lesson study time will be fine during grade level collaboration time as opposed to after school.
We have planned two public lessons over the past year. We worked with grade level colleagues and our math coach at the K and 1 levels. We are hoping to grow lesson study to other grade levels next year.
We have two lesson study teams that meet bi-weekly to plan public lessons. Our lesson study team uses Teaching Through Problem Solving for mathematics.
We use LS to improve the way we teach mathematics.
We use it to improve the quality of our content. We also film lesson study and supply it as training videos for our audience.
We use lesson study to dive deeper into math. We meet weekly to discuss and plan and then we have a public lesson day. We did two cycles last year.
professional development

4. Please indicate the number of times you have participated in any of the following lesson study roles:

Research lesson observer (not on planning team)

Count	Response
6	0
5	1
2	2
1	3
1	3-4
2	5
1	5-6
1	7
2	10
1	10+
1	12
1	20
1	25

Facilitator

Count	Response
16	0
2	1
1	3
3	4
1	20
1	35-40
1	Unspecified number over 6 years

Planning team participant

Count	Response
2	1
8	2
4	3
2	4
1	6
2	7 or 7+
2	10
1	12
1	35-40
1	Unspecified number over 1 year
1	Unspecified number over 6 years

Research Lesson Teacher

Count	Response
7	0
9	1
7	2
1	8
1	Unspecified number over 6 years

Final commentator

Count	Response
23	0
1	2
1	6

Member of a steering committee

Count	Response
24	0
1	1

Other role. Please describe:

I have been a panel member for the post-lesson discussion.
I have been on our school's lead team and my participation has impacted my leadership.
Panel Participant - leading small group discussions and then reporting out questions.
Supervisor of SFUSD's lesson study program
observed two public lessons at a neighboring school

5. How did you learn to perform these roles?

Response
As part of the Connecting Knowledge programme. Training sessions and working with an experienced lead teacher.
Coaching, reading articles/books/ and experience over time.
From our consultant DR. Yeap
I attend the Chicago Lesson Study PD in August of 2016, read various books and articles, talked to participants, and generally self-taught. While I had not done lesson study specifically, I had participated in many similar forms of PD.
I have attended multiple professional development cycles that were put on by Dr. T/OUSD. Our Math Lead team also provided additional onsite trainings around Lesson Study, its components, and it's implementation.
I learned by observing others in the same position.
I learned from following the protocol.
I learned to perform these roles through Impulse Summer Institutes and through lesson study meeting at our school.
I've sort of learned "on the job". Coaches have also given feedback to help deepen my effectiveness.
Information from the facilitator and my teaching experience.
Lesson study alliance conference, lesson study alliance leadership conference, professional development meetings with LSA
My coach facilitated most meetings and walked us through the process of lesson study.
Nora Houseman (SFUSD), collaboration, and through observation
Observation of others, conversations with our school math/science coordinator, participating in conferences with Lesson Study Alliance.

Response
Participating in sessions on lesson study and collaborative lesson research (including attending the seminar in London last September), develping an approach within our team and reading.
School District Coach and through Mills College
Some knowledge from training and meetings at the Maths Hub but I'm not sure what we are doing follows exactly the same structure as lesson study - it isn't as formal.
Through collaboration with other teachers, and the OUSD lesson study summer institutes. I learned what constitutes substantive observation data from listening to other more experienced teachers.
Through my coworkers and Dr. Takahashi's support. We also had Jan and Shelley support.
Through observation and participation as well as collegial collaboration.
Through the week long summer institute last summer and through working with more experienced master teachers on my team.
Was part of a lesson study team for a year. Then joined the master teacher program at SFUSD and received coaching around facilitation. I have led a team for 2 years.
We have had excellent guidance from Mills College professors ; -) and from Dr. Takahashi directly. Additionally, there is a huge amount of trial and error.
Working with my lesson study team and with guidance from our Master Teacher. Also, as an observer, I was offered the opportunity to observe a public lesson at Hillcrest Elem.
lesson study manual, lesson study facilitator/team

6. Please describe what you currently see as the essential features of lesson study.

Response
Problem Solving
- Learning happens over time, part of a culture of continual improvement. - Collaborative. - Happens in real classrooms with real students. - Addresses a shared problem / challenge. - Based on research.
A clear focus or purpose T ime to explore what is already understood about the area of focus and what might be planned for - done both together through discussion and individually through reading relevant articles/research Meetings to develop research proposal take place over time and detailed proposal agreed Live research session with observors who may or may not have been involved in the planning Discussion of live session Involvement of a 'knowledgable other' Sharing of results with a wider community
A proven way to inspire teachers to engage in ongoing, high-quality professional development.
Collaboration Diving deep into content Observation Reflection Revision
Collaboration amongst teachers to collectively plan a research lesson with a question/theme at the center. Depth over breadth approach to PD - that is directly applicable, relevant, contextualized - and that allows for all realms of the classroom experience to be integrated (social-emotional needs, behavior, oral language, etc.).
Collaboration with peers, feedback from lessons. Modifying my lesson after observing my colleague lesson during our public lesson day.
Collaborative planning, student understanding, and thoughtful reflection as three essential components of the lesson study process. Collaborative planning involves a team using multiple resources and multiple curricula to develop a deep understanding of the content. Gauging student understanding includes planning out possible misconceptions, giving students agency to own the lesson/their learning, and interpreting student understanding through a variety of assessments. Lastly, thoughtful reflection includes allowing observers to give feedback on the lesson, taking what you learned from that feedback, and using it to further strengthen that lesson and future lessons.

Response

Essential features of lesson study include: -a planning team that is both collaborative and flexible -Opportunities for onsite staff and outside observers to observe a lesson and provide feedback -Final Commentators to provide reflection and opportunities for next steps -A planning document to capture team thinking, the lesson-planning process, feedback/reflection, and the lesson itself

I see lesson study as an inquiry cycle around one lesson, where the lesson is a rich task that tests an aspect of teaching. Lesson study is a way to develop a shared vision within a math department.

I think the essential features of lesson study are learning to see through the eyes of children. I also think it helps teachers learn how to teach through problem solving and gain a critical eye for analyzing curriculum.

I think the very thoughtful and careful design of a lesson with peers is one of the most valuable aspects of lesson study. Mapping out every piece of the lesson from the standards to expected student responses leads to important learning for every teacher involved. The public lesson itself is important, but all of the reflection after the lesson is so essential for growth.

It enables teachers to address an academic gap through a public lesson. These lessons inform our instruction and enable us to improve our practice every year.

Lesson Study provides opportunity for collaboration amongst and across grade level teams, focused planning to address curricular needs as identified by team, observation opportunities of colleagues, debriefing time.

Lesson study allows for continual refinement of instructional practices and content in a collaborative setting. In teaching through problem solving, we determine new learning for students, plan the unit and specific lesson, think through possible misconceptions and responses, do a mock lesson to get feedback, and finally hold the public lesson. We co-created the lesson and two of us taught the lesson. We all observed students working and brought back these observations for our discussion, revision and reflection phase.

Planning the research question. Observing students and recording the effects of the research question. Discussing with the research group. Drawing conclusions and making suggestion to further research.

Teachers working together to plan and implement a specific lesson design in order to find out how students will respond to that teacher strategy.

The ability to plan and explore student thinking with grade level colleagues.

The collaboration process. The unpacking of a lesson. Talking about math with other math teachers. Observing student interactions.

Response
The essential fearures are the planning team, the research focus, the documentation, and the pre and post discussion.
The essential features of lesson study are thoughtful design, implementation, observation, reflection, and adjustment. Lesson study has to be continuous and collaborative to be truly effective. The lesson study team needs to plan a lesson with intention and a specific goal. After carefully planning the lesson with feedback, a teacher demonstrates the lesson publicly. Teachers and experts reflect on the lesson, give more feedback, and make adjustments that make the lesson more effective.
Time (hardest factor); teachers willing and excited about collaborating together; teachers open to being observed and reflecting with others about what was observed.
Time to discuss the plan. Time afterwards to discuss and evaluate the lesson.
Unit plan, public lesson plan, lesson plan feedback and revision, mock lesson, public lesson, lesson debrief/discussion, final commentary
collaboration
collaboration observation discussion reflection
collaboration, learning from students, other teachers, growing professionally, improving quality of instruction.
planning, anticipating student responses, facilitating the class discussion, summarizing and reflecting

7. What do you think are the potential strengths/ benefits of lesson study in your current organization?

Response
- Builds on existing good practice. - Shares good ideas / knowledge. - Supportive / collaborative / non judgmental. - Teachers have agency / ownership over changes / improvements to their teaching. - Learning organically spreads across school.
Almost half of our teaching staff is on the lesson study team. We have done public lessons with support from Dr. T and the Mills team. We are thoughtful in our public lessons and incorporate feedback.
Collaboration, reflection, and revision
Grassroots approach that is teacher driven; supports a teacher-leadership model of growing lesson study Ability to expand and grow amongst schools Led by a philosophically-aligned team (QTEA) with a critical, progressive approach to PD
I am in the middle of writing a Problem Solving series of books. I think the Lesson Study will help my writing (in terms of books for children and guidance for teachers).
I believe that lesson study supports and encourages student learning, discovery, and engagement by providing them with authentic problem-solving situations. It also provides opportunities for differentiation through its multiple access/entry points.
I have already learned so much in my year of doing lesson study not only about math content and pedagogy, but also on how to collaborate with peers and how to be extremely reflective about my teaching practice. I am so excited for lesson study to expand school wide at my site so that we can see even more benefits in terms of vertical alignment.
I think one strength is the vertical alignment of teams. It's beneficial for teachers to learn the standards above/below the grade level they teach.
I think teachers being able to spend an intensive amount of time looking closely at a focus and a lesson designed around this, and then reflecting afterward about how the lesson was received by the students is extremely powerful. It changes the way we look at ourselves as teachers, as well as changes how/what we teach. I would like to be able to do more of this type of collaboration across the school and different grade levels. Time is always the inhibiting factor.

Response

I work at a University as a tutor on a teacher training course. I would love to set up lesson study as a way to enable students to plan collaboratively, teach and evaluate. I think it would prevent students being limited by a lack of ideas - together they can generate better plans. It would also take the pressure off students when teaching as the lesson would have been planned collaboratively. In addition to this, they would be able to have a group discussion about the lesson and learn what works and what doesn't. I think it is an excellent way to help student teachers develop.

Lesson study asks teachers to carefully observe and discuss student learning and misconceptions, and to use this as the basis for deciding whether a teaching strategy is helpful or effective. THIS IS HUGELY POWERFUL. Further, in my school setting we are specifically using the process of lesson study to explore the Japanese problem solving lesson design. We have found problem solving lesson design to support our students in a) gaining better conceptual understanding of the mathematics b) evaluating and discussing one another's thinking and c) using diverse strategies to solve problems, all of which are key goals of the Common Core.

Lesson study has provided opportunities for teachers to collaboratively work together in identifying concerns in units/scope and sequence; has generated stronger and more consistent mathematical instruction from teacher to teacher; increased the amount of student discourse across content areas; provided a school-wide focus for professional development.

My current school has gone from one math curriculum to the next, many of which have been direct-instruction heavy. We could benefit from this strong teaching through problem solving framework for developing deep conceptual math understanding in our students despite the curriculum.

Same as above

Strengths: encourages teacher-teacher observations, allows for collaborative planning, strengthens content knowledge, creates a space that is focused on students understanding

Teachers desire to engage in LS, reflect on teaching practices, engage in thinking about and planning math instruction

The amount of time in planning a lesson. Going over all the small details to help improve student learning.

The strengths of my math department are that we have strong collaborative relationships with our grade level planning partners, and for the most part a shared vision across the department.

To sustain in-school action following training and to skill up schools to offer and manage higher quality professional development for all staff.

Response
Very high impact for schools who seek to improve their practice.
We are a united cohort and work well together. We are curious, and excited to explore this new area of learning.
We are going to be using the lesson study model school-wide next year. I expect that it will create authentic, teacher driven professional development. In addition, I hope this will allow us to be more consistent and vertically aligned.
We are implementing TTP within research lessons and adapting that with the SFUSD CCSS-M curriculum for our mathematic instruction.
We now have a better understanding of the importance of the planning stage and have improved this which has made the work we are involved in this year much higher quality PD. We act as the 'knowledgable other' so ensure this is built into what we do with schools.
a tool to learn from experts in the field and disseminate good practice to others.
collaboration clarification of standards networking
professional development increased student achievement/efficacy
whole school implications for more reflective math practice

8. What do you think are the challenges to using lesson study in your current organization?

Response
Challenges: following the same format in the same subject area can get redundant and feel inorganic, teachers are not always excited to open their classroom doors to what might be viewed as scrutiny
Engaging schools so that they take on the practice themselves. Funding for releasing teachers to be involved in the different phases, particularly the live sessions and discussion that follows, is a challenge - we have managed to get some of this from maths hubs over the past two years. In England one of the big challenges is shifting the way that teachers view lessons from one of judgement to one about the impact of decisions made when planning.
Finding time / cover so those involved can plan thoroughly.
Given the funding crisis, there are attempts to move lesson study into a department (math) and utilize the funding for a content-driven approach rather than a pedagogical/PD approach. These attempts also mean moving toward a top-down compliance model rather than a grassroots, teacher-driven model. I have concerns about all of this.
I wish it was school wide.
It can be very time consuming and not everyone is always on board with putting the necessary time in. Also I think it is hard for people to want to put so much time and effort into designing just one or two lessons per year when we are struggling to design all of the curriculum for our own classrooms because of the serious flaws in the curriculum that we have to work with. If we had better currículum i think people will feel more confident going in depth with a few lessons and making those better.
It is very laborious for us to be attempting to create curriculum from scratch. Additionally, serving on a planning team requires many many hours of work simply to choose an appropriate task, and even with strong facilitation it can be challenging to collaborate effectively, to distinguish between "productive struggle" and being "stuck" as a team.
It requires a large amount of time for a very focused component of a wide, evolving district-developed curriculum.

Response
Maximizing learning from research lessons that have an impact our practice. i.e. taking what we observe and learn from a research lesson and making changes to our daily practice that result some kind of positive outcome for students. We usually learn a lot about particular content or structures from a research lesson, but do not usually walk away immediately with a new idea to test out in our practice.
One challenge is that our current curriculum is not aligned to this style of teaching. Another challenge is that our school has inclusion classrooms, and with that comes a variety of skill/ability levels, and we are unsure how to utilize lesson study in a way that meets their unique needs.
People don't know much about it.
Scheduling time that works for everyone. Making sure the process is equitable for all the participants. Makong a decesion that everyone can agree on.
Schools and teachers rarely understand the process of lesson study and need to be better informed about the steps before and after the lesson.
Schools prioritising and investing financially in approach. Not allowing or planning for action over a sustained period of time. Strategic organisation being affected by day-to-day running of school in challenging times.
Teachers with less than 5 years experience, time to plan, see one another and reflect.
Team facilitation will be difficult. 3 of the teams will have new facilitators.
The biggest challenge is that my current organization uses a very scripted, non problem solving based curriculum. We are not allowed to stray from this other than for research lessons. This makes lesson study feel like a separate entity versus something that our school uses to inform our day to day teaching.
The challenges are getting all teachers to feel comfortable with the TTP practice.
Time and time-tabling implications. In addition to this, finding enough schools who would be willing to place students who are doing lesson study.
Time to meet
Time; not understanding it's power and benefits.
We are new to lesson study and need more experience. We are currently the only team, and we would like our colleagues to engage in this process.

Response
We need all our teachers to be on the lesson study team so we can plan and reflect more often. We need support in choosing thoughtful math goals and planning lessons that meet those goals to move students forward in their mathematical thinking.
With limited time for professional development, being creative in planning out lesson study cycles;
accuracy of interpretation
adaptation for inclusion/students with learning differences
lack of curriculum and cohesiveness because we are bouncing between sfusd (which we do not like) and trying to obtain Japan Math
time buy-in from other staff

9. To what extent do you expect to learn about each of the following during the immersion trip to Japan?

(a) Important features of lesson study

Value	Percent	Responses
Some	3.6%	1
Quite a bit	32.1%	9
A lot	64.3%	18
		Totals: 28

(b) How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)

Value	Percent	Responses
Some	21.4%	6
Quite a bit	39.3%	11
A lot	39.3%	11
		Totals: 28

(c) Supporting participants to have powerful and effective lesson study experiences

Value	Percent	Responses
Some	14.3%	4
Quite a bit	39.3%	11
A lot	46.4%	13
		Totals: 28

(d) Writing a good lesson plan

Value		Percent	Responses
Not at all		3.6%	1
A little		3.6%	1
Some		28.6%	8
Quite a bit		25.0%	7
A lot		39.3%	11

Totals: 28

(e) Evaluating the quality of a lesson plan

Value		Percent	Responses
A little		3.6%	1
Some		21.4%	6
Quite a bit		32.1%	9
A lot		42.9%	12

Totals: 28

(f) How to observe research lessons

Value	Percent	Responses
Some	14.3%	4
Quite a bit	32.1%	9
A lot	53.6%	15
		Totals: 28

(g) Organizing a successful post-lesson discussion

Value	Percent	Responses
A little	3.6%	1
Some	14.3%	4
Quite a bit	35.7%	10
A lot	46.4%	13
		Totals: 28

(h) The role of the knowledgeable other

Value	Percent	Responses
Some	35.7%	10
Quite a bit	25.0%	7
A lot	39.3%	11
		Totals: 28

(k) How to organize the board

Value		Percent	Responses
Not at all		3.6%	1
A little		3.6%	1
Some		7.1%	2
Quite a bit		28.6%	8
A lot		57.1%	16

Totals: 28

(l) Teacher questioning techniques

Value		Percent	Responses
Some		17.9%	5
Quite a bit		25.0%	7
A lot		57.1%	16

Totals: 28

(m) How to summarize a lesson

Value		Percent	Responses
Some		7.1%	2
Quite a bit		39.3%	11
A lot		53.6%	15
			Totals: 28

(n) Anticipating student responses

Value		Percent	Responses
Some		14.3%	4
Quite a bit		42.9%	12
A lot		42.9%	12
			Totals: 28

(o) How teachers support whole class discussion (neriage)

Value		Percent	Responses
Some		14.3%	4
Quite a bit		28.6%	8
A lot		57.1%	16
			Totals: 28

(p) Strategies for making students' thinking visible

Value	Percent	Responses
Some	7.1%	2
Quite a bit	50.0%	14
A lot	42.9%	12
		Totals: 28

(q) Student note-taking

Value	Percent	Responses
Some	25.0%	7
Quite a bit	35.7%	10
A lot	39.3%	11
		Totals: 28

(r) Knowledge about the Japanese educational system in general

Value	Percent	Responses
Some	39.3%	11
Quite a bit	28.6%	8
A lot	32.1%	9
		Totals: 28

(s) Mathematics content

Value	Percent	Responses
A little	3.6%	1
Some	42.9%	12
Quite a bit	32.1%	9
A lot	21.4%	6
		Totals: 28

(t) Knowledge about Japanese curriculum materials

Value	Percent	Responses
A little	7.1%	2
Some	35.7%	10
Quite a bit	25.0%	7
A lot	32.1%	9
		Totals: 28

(u) How to develop a mathematics unit/curriculum

Value	Percent	Responses
A little	10.7%	3
Some	46.4%	13
Quite a bit	17.9%	5
A lot	25.0%	7
		Totals: 28

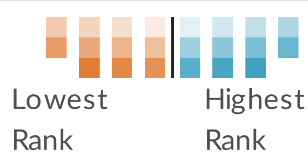
10. Are there additional learning experiences (not listed above) you hope to have during the immersion trip?

Response
Brainstorming how to make lesson study work for my school context.
I am interested in seeing what areas of student thinking that the educators will be discussing and examining. I am curious to see if there are similarities to students within our district/school site.
I hope to also observe how Japanese culture influences schools.
I hope to observe kindergarten and first grade lessons so I can see the format from beginning to end. I'm especially interested in the board design, strategies for showing student thinking, the neriage, and summary.
I would like to see the textbooks that children use.
Joining with and learning from fellow educators participating in the trip.
Learn about how various schools and districts support and organize lesson study.
None that I can think of! I am excited to see how they utilize this model in lower grade classrooms!
This might be covered in (b) - opportunities to hear from others participating in the program (not from Japan) about their experiences using lesson study; in the planning, I am curious the types of evaluation questions we will see and how/why the teams decided on those specific evaluation points.
Understand what supports the lesson planning team in effective collaboration
What learning progressions they use to inform their curriculum
general strategies of teaching mathematics and the thinking behind them
how inclusion works in japanese schools

11. Please select and rank in order of importance five items from the previous question that you believe will be most professionally useful for you within the next year. (Drag and drop your top five from the left-hand list to the right-hand column.)

Item	Overall Rank	Rank Distribution	No. of Rankings
(o) How teachers support whole class discussion (neriage)	1		19
(i) How to support student problem-solving	2		16
(p) Strategies for making students' thinking visible	3		14
(j) How to build students' mathematical habits of mind and practices	4		12
(c) Supporting participants to have powerful and effective lesson study experiences	5		11
(l) Teacher questioning techniques	6		11
(m) How to summarize a lesson	7		9
(g) Organizing a successful post-lesson discussion	8		9
(k) How to organize the board	9		7
(n) Anticipating student responses	10		6
(e) Evaluating the quality of a lesson plan	11		5

Item	Overall Rank	Rank Distribution	No. of Rankings
(b) How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)	12		5
(q) Student note-taking	13		5
(a) Important features of lesson study	14		5
(f) How to observe research lessons	15		5
(h) The role of the knowledgeable other	16		4
(d) Writing a good lesson plan	17		3
(u) How to develop a mathematics unit/curriculum	18		3
(t) Knowledge about Japanese curriculum materials	19		3
(r) Knowledge about the Japanese educational system in general	20		2
(s) Mathematics content	21		1
(v) Learning experiences you added in response to Question 10	22		1



12. Is there anything else you'd like to add?

Response

Having had other team members from our school attend, I am interested in ideas/suggestions for bringing my new learning back and creating a new learning experience for other staff members.

I'd be interested in hearing about challenges and the ways leaders of LS have addressed these. I'd be particularly interested in the Japanese equivalent of my role where I don't work in the schools and the schools must invest financially in the support.

I'm excited about this privileged opportunity and will give 100% to get the most learning I possibly can out of this experience.

I'm looking forward to this experience!

I'm very excited for this incredible opportunity and look forward to applying my learning in the classroom next year. Thank you.

What are one or two "Aha's" or insights you had about lesson study?	What are one or two "Aha's" or insights you had about mathematics for teaching	What are one or two "Aha's" or insights you had about pedagogy or classroom instruction?	Please describe any ideas that you would like to share with your colleagues when you return?
We should do lesson study as a district so that we can have district-wide research lessons to improve practice and student learning. Eventually it would be helpful to do lesson study between districts. Another important part of lesson study is publishing the information learned from post-lesson discussion.	It was helpful to learn about the progression of geometry. Starting off with 3D shapes makes more sense.	It is important to let the students struggle and learn how to access their own prior knowledge. We should not tell students what information to access prior to the lesson.	We need to set up a committee so that we can have more cross grade level alignment.
How systematically it is done in Japan.	How real life context is attached to so much in Japan.	The division of a fraction by a fraction lesson plan really showed me how to contextualise this and show the mathematical structure on a double number line.	The importance of contextualising all maths even concepts that are difficult to understand.
I was struck by how much more effective LS is as a whole school structure. There seem to be so many obstacles to get that to happen in our school but I want to think about it more and see what could be done.	It was interesting to see how geometry is broken down by grades, and how kids have much more "hands-on" experiences before they are given any formal names or definitions.	I continue to see the power of using problem solving to teach math. I want to continue to see how I can do this better.	Still reflecting.....I definitely want to talk with some of the younger grade teachers about what we saw with geometry.
There was validation today that what we are doing with lesson study has our site on a good path with much more work to do.	The big aha for me today, and it is now sinking in after hearing it multiple times, is not to start with a review of what children's prior knowledge should be going into the lesson but letting them access their prior knowledge without me. A very difficult task.	The insight above also speaks to pedagogy across curricula. Being more thoughtful about shaping discovery.	
The need to involve the whole school in order to maximise impact and the way this is achieved in Japan and could be achieved in England. Using lesson study for preparing for and implementing curriculum change.	The clear progression in geometry avoids a reliance on children remembering and 'barking' shape names with no understanding of the essential features of the shapes. It arises naturally from play; the English curriculum is not fit for purpose in this respect and the end of key stage 1 assessments encourages teaching based on memorization rather than understanding. The use of models and images to support understanding is well-established in England	Most teachers in Japan don't tell the children what to think about'; this is linked to the three levels of expertise. In England there is a cultural shift needed for primary (elementary) teachers to believe they can and should aspire to level three. We have too many teachers and school leaders who believe level 1 is good enough. The use of models and images to support understanding and the orchestrating of classroom discussion using the five stage from Mary Kay Stein etc al resonates with our approach in our work with schools; I am looking forward to seeing them in action.	How we can run CLR with whole schools next year. How teachers were supported to improve their research proposals. The sequence 'define, examine, construct' in geometrical reasoning.

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That lesson study is most effective when embedded across a whole school	It is possible to start with a problem only when you choose the right level of challenge.	Grouping ideas as a way of developing a shared understanding. Using definitions to help support learning	All of the above.
-value in cross grade level team planning; "develop a common vision of education at the school through teacher collaboration." -the roll at the end of the post lesson discussion in which someone summarizes and gives next steps leading to newsletter publications.	- geometry sequence; very interesting the lack of vocabulary emphasized in 1st grade and then how students create, investigate, design leading to definitions in subsequent grades. - in the Riko and Kota problem very surprised by the number of ways solutions were presented and ability to find commonalities between solutions.	-the value of prior learning in math... began to think how we need to change mindset in math to be like that of reading, where we are developing/establishing foundational skills in the primary grades. -importance at the end of the lesson to make clear connections.	So many! Current top two ideas are: p.15 of Takahashi & McDougal article "structure of school research program;" and we need to implement summary with next steps including newsletter.
The piece about sharing the LS work with colleagues within the school in a monthly newsletter.	Folding a paper any way twice makes a right angle and defining parallel lines through the definition of perpendicular.	In Japan the stats noted that less content is explicitly taught but more is retained and students have confidence to build and score higher in competence.	I plan to share the geometry sequences and my math "aha's".
I walked away with a deeper understanding of the structures that need to be put in place in order to successfully implement school-wide lesson study.	Anticipating student responses must always involve teachers trying the problem on their own first.	connecting student responses is important during the discussion portion	Anticipating & connecting student responses. Solving the task/problem as adults in order to anticipate student responses.
How wonderful it would be to have parents involved in this process with us! I want to make that happen at my school.	Makoto's demo lesson was an excellent entry into our own math thinking and I want to see us do that with our LS teams at the beginning of each cycle to help us formulate our research lessons.	Define ---> Examine --> Construct	See "ahas" above
That the approach is totally consistent across the whole country. The level of professionalism shown (and expected) by teachers and the way they work collaboratively to support each other.	That the approach of lesson study actively supports children's resilience and willingness to tackle unfamiliar problems. The ' Define Examine Construct' approach	Anticipating student responses and considering the order in which ideas could/should be presented to construct understanding. Not using shape names in initial teaching of geometry but instead, focusing purely upon exploration and developing meaningful language related to properties.	The need for co-construction of learning journeys deeply embedded in a high level of subject knowledge. Having teachers of all year groups working together to see the journey and understand more about how to provide a consistent approach to learning through problem solving which promotes student thinking and conceptual understanding.

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1) I learned that there are 3 types of lesson study, before I only understood the school-based one. 2) I learned that the United States' poor math performance is directly tied to the ways we teach - we focus on teacher-centered instruction while other countries who are successful provide opportunities for student to construct their own ideas.	1) It is important to strategically structure the lessons and problems to create a sequence that gradually builds student understanding. 2) It is critical for students to be involved in creating the definitions of mathematic vocabulary through a process of exploring and trial/error because it deepens their understanding of the mathematical concepts.	1) I loved the idea of designing a sequence of lessons that allows students to define, examine, and construct concepts about mathematical practices. 2) I also was really interested in how we can adapt existing Curriculum to make it emphasize independent practice/constructivist methodologies, rather than teacher telling/explaining	I am excited to explore how we structure our Research Steering Committee, and think about who on our faculty would be the most impactful in that role, especially in a site as small as ours. I am also intrigued about the idea of doing some sort of district-wide lesson study, where we can share our learnings and dive deeper into curriculum modifications.
The district-wide approach and the amount of interest from educators, administrators, and family members. I think the overall take away is just understanding the process and seeing how embedded it is in the climate of education in Japan.	When examining the geometry units throughout the grade levels, I was very interested in the curriculum's design and flow. I really had an aha moment with the "Define, Examine, Construct" approach as introduced by Dr. Takahashi.	Seeing the structure of Lesson Study was profound because of the commitment of all involved, especially the Steering Committee and the Grade-band teachers and the amount of time and dedication they have towards its success.	Being equipped and having access to the slide shows that we observed with Dr. Takahashi will best serve us as a platform to develop our own presentation when returning. We very much appreciate the access to those slideshows and the research in which they represent. Thank you!
Lesson study can be done on a district wide level. Would love to do this in Chicago public schools!	Teachers in Japan seem to place a great deal of emphasis on deep content knowledge.	Starting a lesson with yesterday's learning may hinder student learning.	I would love to help implement grade band teams at my school along with a research steering committee.
A model for whole school lesson study-ideas for how the Research Steering Committee could work at my school site.	n/a	I thought it was interesting how the Japanese introduce and teach geometry - particularly how students explore shapes before they know their name and learning how to create and draw shapes and lines.	The model that Dr. T shared for whole school lesson study, and the idea of creating a steering committee.
I'm very interested in how lesson study is used for alignment and analysis across schools and districts. I'm very interested as well in understanding the systems by which lesson study research is reported and shared across schools, districts and to the ministry of education.	continuing to think about teaching with the textbook versus teaching the textbook.	how to create independence so students are monitoring their own learning and studying to meet their own needs (homework here is often students having the freedom to study and prepare as needed to ensure understanding and previewing of upcoming lessons - this is a crazy concept for an American!)	how important is the quality of the textbook/curriculum? should this be a continued focus or should we let it go to focus on pedagogy?
There are three different types of lesson study and teachers are simultaneously participating in both school wide and district wide lesson study.	Students need to figure out what prior learning is needed to solve the problem. Also, shape names aren't introduced until grade 2. When names are given, they're given with definitions.	The shape scope and sequence (vertically) really gave insight in how to teach shapes and other topics. It solidifies the importance of meaning before vocabulary terms.	School wide lesson study to inform instruction.

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I learned about the different types of lesson study and the purpose of each one.	I enjoyed dissecting the lesson with Tad to build my background knowledge. I learned that good problems allow students to reaffirm the knowledge and extend their learning.		
I didn't realise how much work is involved to plan and conduct a research lesson designed to achieve its goals. Today's session has been an eye-opener!	The simplicity of answers in a mathematical task. When we were asked to draw a right angle triangle, some people looked for a protractor. Folding the paper twice does makes a right angle triangle. Or using two rulers...Today I have been thinking of how the easiest way to teach maths is by doing maths...	Something that Dr Akihiko Takahashi said "A good teacher does not teach the book but teaches with the book." I don't think I will ever forget that!	The whole day has made me reflect on the importance of good maths books and how to help teachers to make the best of them. I haven't got one clear idea on how to make this possible, but I know that it is an issue and I am thinking about it.
Insight into the purpose of impuls project Japan, and its wide scale use	None	Impact of whole school observation on pedagogy is interesting	Notion of lesson study for professional development
Just how interwooven Lesson Study is within Japenese teaching. I like that there were students teachers working with college professors in order to improve student learning outcomes. Just the idea that all teachers despite how long they have been teaching are working on lesson studies. I think it is extremly valuable for teachers of differernt levels of expierence to work with eachother in order to improve their craft.	The problem solving approach being defined so clearly. I think giving the a problem so that the student can learn something new is something I need to remind myself when I am teaching. I just want to step back and ask myself, "What's new that the students' are learning today?"	The idea about differentiating the entry point as opposed to differentiating the lesson.	I would like to share with my collegues that there is tremendous value in talking and planning time with collegues to improve your practice. I think we tend to close of ourselves from other teachers and that may hinder our developement as teachers
District-wide lesson study	The college professors being so enaged in the lessons and learning	The impact it has on the student learning when so many stakeholders are involved	District-wide lesson study and oberving teachers from other schools.
I had never heard of district wide and cross-district-wide lesson study! What a meaningful way to open up our practice and learn from one another for the true benefit of all students. It is here that new ideas can be developed and shared on a broader scale.	When the array of circles was presented, I learned that 45% of U.S. students used verbal addition to find out how many circles there are. 37% of students in Japan used a multiplication math expression for the exact same problem. This leads me to believe that it is how you APPROACH a problem that is important, not solely a correct answer. When we tell students what they should think, we rob them of the opportunity to construct their own learning and advance their mathematical skills. We need to shift the framework for how we teach math in the U.S.	Students in Japan are taught less content but score higher on the achievement test where as students in the U.S. are taught more content yet have lower scores on the achievement test. This is due to our pedagogy. "Covering" content does not equal students learning. Teaching through problem solving gives students opportunities to develop the skills necessary to tackle a new problem making good use of their prior knowledge.	

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The conversation touched on very specific details (such as the specific word used for "price" in the lesson) to more broad articulation of pedagogical philosophy. I think it's important to have both and anything in between involved in the conversation in order to make sure the team is aligning with the larger vision, but also honing in on instructional practice.	The question of "is division appropriate?" was very interesting, we often don't dive in as deep when conceptualizing division in this case is more difficult as opposed to thinking backwards in multiplication. I learned that saying that "multiplication is the opposite of division so division works" may be insufficient if we really want an understanding of division to deepen and stick.	Careful planning of boardwork can depict a clear mathematical narrative for students. There is one caveat though, as teachers must be willing to change course based on students' thinking.	This lesson encouraged me to think about boardwork and its role in a lesson. Is perfect boardwork is attainable or even desirable when math can be messy? Some of our research lessons could probably do a better job at being more detailed with boardwork planning.
Lesson Study is one of the primary ways that Japanese educators improve their practice.		It is worthwhile to analyze the ratio of time spent on three types of mathematical practice: Practicing routine procedures, inventing new procedures, and applying these new procedures to new situations. The US far exceeds other countries in our emphasis on practicing routine procedures which were introduced by the teacher - seemingly to the detriment of student performance.	Teachers' performance in TTP instruction can be thought of on a 1-3 scale in which a level 2 is a teacher who connects content with conceptual aids, but who develops strategies or "leads" students. Level 3 is when the students are able to develop the strategies more independently.

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Having kids formulate questions; board organization	Still thinking about the conversation about where to put the 1 on a double number line.	When does solving the problem help you prove understanding and when is it not as important to solve the problem?	I want to think more about how to get kids to formulate the questions and then share that with my colleagues.
- post discussion was much more lengthy than what we seem to have and I greatly appreciated the number of people involved and the diverse roles they had at the school; -also thought it was good to have the teacher speak again after the knowledgeable other, we don't always allow time for this to occur.	-emphasis on not solving problem but developing proof(s) of equation 1st was a flip to what I would traditionally see or think of doing; -pleasantly surprised by several students using term "proportional" and seemingly to understand the proportional relationship to the problem presented.	-amazing board work and student notebooks! Found interesting the discussion about the summary, how much is student driven verses teacher created. The time given for students to write some summary on their own we refer to as "reflection," which is then followed by the summary. This did not seem to be differentiated today. -would love to understand more about the class being split. How often does this occur? Are groupings flexible based on pre-assessment as shown with data in the lesson plan? What are teachers thoughts on how students reponded to questions 1 and 3 in the math "attitudes " questionnaire? Huge disparity in data for these two questions but not addressed in text explanation.	-with further info. (Hopefully tomorrow) the hint cards are a potentially good strategy to address differentiation in the classroom. -plan for more post discussion time; include more staff with specific roles during Lesson and post lesson
The knowledgeable other prepares for the final commentary before the live lesson takes place much in the way the teacher prepares for the lesson, by anticipating what will arise, the elements that will cause discussion, the 'tricky' bits. They then adjust this in response to what happens and what is discussed just as the teacher adjusts their teaching in response to the children. Doing some maths with the teachers as part of the final commentary to make them think in a way related to the theme of the lesson. Discussing the decisions made by the teachers in the planning especially where they deviated from the text book - this was exemplified by the discussion about the summary at the end of the lesson with the lesson being the first half of a pair of lessons focused on one idea.	The way the context was introduced in order to give the children a real purpose to engaging in the mathematics - not just having the ribbon but showing the 1m of ribbon that she needed that another teacher had purchased for her, the cost of 300 Yen being identified and then revealing that there was in fact 2.5m - this was absolutely delicious. The children thought they knew what the situation was and then it was revealed to be not quite as they thought and the reason for the problem had been established. This gave them an understanding of the context by understanding how it could come about and this supported thinking and understanding. The unrelenting focus on the mathematical question - not what is the answer but can you justify that the calculation is appropriate for the context.	Planning ahead where different things will be written or presented on the board in order to support the learning /understanding. Asking the children to write down the problem after they had made sense of the reason for the problem allowed them to consider the problem for themselves and identify their own questions which they were also asked to record. These were then used to shape the rest of the lesson. The decisions about sequencing would be interesting to investigate further.	Ideas about the role of the knowledgeable other and the structure of the final commentary. The simple but effective start to the lesson, the focus on thinking supported by having a key mathematical question for the lesson and thinking about what will be recorded on the board and how.

IMPULS 2017 Reflections Day 2 (Wednesday June 21st) (Responses)

What are one or two "Aha's" or insights you had about lesson study?	What are one or two "Aha's" or insights you had about mathematics for teaching	What are one or two "Aha's" or insights you had about pedagogy or classroom instruction?	Please describe any ideas that you would like to share with your colleagues when you return?
1) The way children didn't seem to worry about who was around them. It made me think that if children were timid or didn't participate, the lesson study would not work very well. 2) The reflection of the teacher after the lesson study was something I hadn't thought about.	Children managed to prove in different ways and also generalise. The journey the children went through from 'concrete' (ribbon) to writing a formula on how to calculate one unit, was very inspiring.	a) A good teacher talks less and listen more. That was evident in the lesson I saw. b) Involving the children (by showing the ribbon) from the start. The attention of the children was kept from the very beginning and held throughout the lesson. The children were more interested in finding the answer, but the teacher 'navigated' the lesson in such a way that the focus was on finding out why you divide, rather than simply divide.	a) Have more 'proof' questions. The answer is not important, the journey on how to solve a question is. b) Encourage children to work together and share ideas. c) Turn real life situations into mathematical tasks.
It's ok to not reach the lesson goals, balance positivity with constructive criticism.	how can you be sure that students understand division by a decimal?	Motivate a problem where students generate their own question. Be very intentional about board work.	A lesson could be centered around a conceptual question (like why do we use division?) without actually finding the answer to the problem.
I am happy to see some of what I saw is happening for us already in the states.	Loved the hint cards on the wall.	I still wanted to see more student to student interaction.	Doing lesson study, with all the trials and tribulations, is putting us on a VERY positive path.
1) color-coded boardwork!!!! It has the capacity to reveal so many connections for the students! 2) math lessons need not be solely focused on deriving the answer, they can also be process-based in order to deepen students' conceptual understanding.	1) I need to be more thoughtful about how I lay out my boardwork and think about intentional way to structure it to maximize learning. 2) I can carry one lesson over multiple days, with different objectives, in order to emphasize process as well as accuracy/efficiency.	1) I really struggle with the summary piece. I liked seeing the teacher give the students ownership in creating their own summaries and am interested in building a similar system in my own classroom, as right now I feel like they are mostly prescribed and teacher generated. 2). I want to create a resource area with prompts and tools for students to use when they are struggling to gain access to a concept.	I love the idea of giving our students a series of tools/resources that they can self-select from, based on need. I think it will support with our differentiaton, particularly with our struggling students.
Whole school lesson study should or can include all teachers and support staff (not just teachers of math). Using early release day for public lesson.	We don't teach proportional relationships in elementary school, and I'm wondering why not.	I need to practice my board work more.	I'd like to explore the idea of using early release days to do public lessons with all staff being observers and engaged in the pre and post lesson discussion.
I'm thinking about the pros and cons of including the "evidence share" within the protocol. I'm also fascinated by the formality of the approach and wonder how that is linked to the professionalism of teaching here in Japan.	I really liked having students discern the mathematical question for the day, then giving students space to write down their own questions or hypotheses before beginning independent work time.	I'm thinking about contrived vs authentic partnering and pair-sharing. How importance is oral participation and equity of voice? What does accountability and engagement look like when there is full buy-in? Must there be equal talk time and must all students speak? Are there other ways to consider authentic engagement?	see above...I want to bring back the concept of students formulating and deriving the question to explore, then brainstorming all initial thoughts about that question before attempting to solve it.
I really appreciated that Japanese schools include all teachers in this process (art, special Ed, etc.)	The concept/discussion of giving the answer and then having the lesson focus on the process was very intriguing to me.	Beautiful board work was reflected in student notebooks.	Process over content

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The systems and structures that schools have to conduct lesson study; students are dismissed except for the LS class.	The teacher spent 45 minutes with the class on one question and that was if division was the correct operation. The time spent on conceptual learning was commendable.	I wanted to know more about the hint cards and how that support is set up. The concept of hint cards felt "aha" like to me.	The beautiful board, hint cards, including all staff members...
1. The lesson plan was so thoughtfully planned out. It made the lesson easy to follow and allowed my attention to focus on the actions of the students. 2. During the lesson, it immediately became clear to me that the development of the mathematical question became the guiding course of the lesson. This was an "aha" moment for me because in my experience with TTP, my emphasis was placed on finding the answer in order to develop a summary. Today's lesson demonstrated that that method does not always have to be the case.	As mentioned before, today's exposure to developing and using the mathematical question to lead to meeting the goals of the lesson was key. As our first observation, this is something that I will definitely pay close attention to in the following lessons.	Konohara-sensei demonstrated spectacular board writing that was well thought out and prepared. Her level of dedication to the anticipated student responses as well as her own teaching goals made the lesson successful. Also, the hint corner was a teaching strategy that I would like to learn more about. I think it would be helpful for us to have an understanding and translation of the forms that she made available for the students to access.	I take away from the lesson a lasting impact of the joyful learners that I observed. I look forward to sharing the approach and shift away from finding the answer to the development of proving why the mathematical thinking is valid for answering the mathematical question. I believe that this will help my students see the value and connection to why such math is helpful in their academic careers.
How 'normal' it is to carry out lesson study and feel that this is how you learn and develop as a teacher. The honesty and whole-school involvement in the process and post lesson review. Teachers being able to critique each other.	The level of preparation that went in to what would be recorded on the board and how to support the transparency of the learning process. That only one calculation with no answer was the focus of an entire lesson. The focus instead being on making sense of the mathematics. I loved the use of 'worded equations' as part of this process.	The quality of language modelled and therefore used by the children. 'No hands up'; instead there was a respectful discussion throughout the lesson where the the teacher avoided responding facially to the children's contributions but instead listened and captured there thinking on the board.This included use of the board (written, symbolic and diagrammatic) and visualiser to share and organise the children's contributions.	Planning the sequence of anticipated student responses and how this will be shared as a journey visually (and referred back to as we saw children doing). Giving children far longer to work on and share their ideas whilst teachers observe, listen, capture and probe (when appropriate) whilst they are working independently. Also, 'independent work' occurring as a key part of the teaching sequence and not 'after the teacher input'.

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Today I realised the importance of the lesson plan. It is so detailed, in depth and thorough and every element of it is analysed and discussed. I was particularly interested in the thought that had gone into writing down the anticipated student responses along with which responses would be looked at in more depth and which wouldn't.	I had known for some time that in Japan more time is spent looking at just one problem but it was fascinating to see this in practice. I can really see the benefit of scrutinising one problem and really trying to understand the underlying structure of the maths behind it. This will enable the children to have a really good grasp of what division by a decimal means and how it relates to real life.	It was excellent to see the children trying to prove that the problem needed division to solve it. This allowed them to look so much deeper than the problem itself. I also thought that by allowing children to just call out their thoughts and suggestions, the pace of the lesson was much better than what you may traditionally see in England where children must be asked individually to comment. The children were fully engaged and keen to discuss the mathematics with one another. One girl was even looking back in her notebook to relate it to previous learning to help her construct new ideas. I watched as she was then very enthusiastic about showing and discussing this with a peer.	I definitely want to share the idea of looking at one problem in depth and I want to discuss with them how to do this in reality. I also want to discuss the idea of allowing children to freely call out their ideas and suggestions.
Whole school partcipation is a powerful way to engage in lesson study. This would help to create vertical alignment around a common theme.	Student work in notebooks can be used as a reference to support new learning. How can I support this in the Kindergarten or 1st grade classroom?	There is balance needed in focusing discussion tailored to the lesson plan but allowing for space for new insights and student wondering/ideas.	
All staff participate in lesson study.	The purpose of the lesson wasn't about finding the answer but instead about the process and reasoning behind what math was needed to solve the problem. Proof!	After posing the problem, teacher asked students to write down questions they had in their notebook. Then teacher wrote the questions on the board. From there students helped come up with a mathematical question.	All staff participating in public lessons.
The board plan and the organization of student ideas on the board.	Level 2 vs level 3 teaching and what characteristics from the lesson helped us identify what level the teaching was.	Teacher led versus student led discussions.	How to support teachers in moving from level 2 to level 3 teaching.
The in depth discussion with all the teachers, admin and expert commentator. The attention to detail and the structure of lesson study.	The board work was amazing, using the ruler, taking time to use different colors, collaborating on the math question with the students.	Students knew and understood format and were actively engaged. Teacher had excellent repore with students and they wanted to show what they knew about math.	The importance of collaboration and sharing leadership with the students. Highlighting student thinking vs. teacher thinking
There's always more to critique, even in a seemingly immaculate lesson. This orientation is deeply professional and growth oriented.	Double number lines support conception of multiplication and division in terms of proportional relationships and ratios, and allow for flexible use of place value in solving multiplication or division of decimal numbers.	Students can and should be asked to craft the language of the daily problem by being given the real-world objects and situation. The "quiet mutterings" that students say to or ask themselves at the start of problem solving are valuable to capture, praise and refine. Also, the daily math lesson focus does not have to be simply solving a given math problem.	All of the above.
Children are unaffected by presence of lots of adults in the room .	The use of double number lines to show proportional relationships.	It is possible to use of one question for 2 lessons and still challenge children.	All of the above.

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A teacher should never say something or write something that a student can say. Also, the difference between a level two teacher and a level three teacher involves the acute listening of student ideas and adapting the lesson and boardwork accordingly.	When teaching division, it's important to take into account the fact that each problem and context lends itself to a specific type of division. In fifth grade, (and perhaps also before,) students should begin thinking of division as proportional reasoning. The equal sharing/partitive division that is taught in 3rd and the 4th grades needs to be extended strategically. It might be even more beneficial in the long run to teach this type of division after teaching quotative division, which is the opposite of what many American teachers currently do. These understandings of division should be extended and then unified with division as proportional reasoning, fractions as division, and finding unit quantity with division.	Student thinking should always be represented on the board and embedded into the lesson in the moment. This requires the teacher to listen carefully to student ideas so that they can be integrated into the flow of the lesson. Sometimes, when a lesson plan is executed exactly as intended, student voice might not be accounted for.	Be sure to hear all student ideas, not just the ones that you want to hear. Be careful not to pick and choose certain parts of what students say in order to make the lesson fit exactly with the original plan.
Today, we talked about how the public lessons should always be new learning.	I learned about the levels of teaching. Often times we try to rush lessons without listening to students and become level one teachers. With less explaining and more guiding students to come up with their own ideas leads us to level 3 teaching. I also learned that we should introduce quotative division first to build an image of division.	Sometimes we weave in and out between the different levels of teaching. Our goal should always be level 3 teaching.	I would discuss the levels of teaching and push them to have level 3 teaching as their goal.
First hand experience of how it works	Slow pace and instruction was interesting	Emphasis on thinking time and determination was refreshing	The notion of waiting time extension to allow for deeper thinking
Just how candid the feedback section was in the post discussion. I think it was very difficult for the teacher today to take much of the feedback because I felt that she was dissappointed with her lesson. But I think she was couragous and did a good job receiving feedback.	The was the first time I have ever been in a discussion about teaching where there was so much thought about boardwriting and how serious board writing is in Japan. In America we just talk about having neat writing on the board and making sure students can see it. In Japan there is more then bieng neat going into the board work. The idea of how to use the board in order to convey a clear message to the students?	I really liked when the teacher asked the students,"What type of problem do you think this is ?" When students said it's not addition or subtraction was powerful. I just wish she would have asked why is this problem not addition, subtraction, or multiplication?" I guess I just wanted to hear the students justify their responses a bit more.	The importance of asking students what type of operation is this problem? I need to spend more time with students determing the operation involved because I am seeing really stuggling setting up problems due to their lack of a basic understanding of operations.

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One thing that I observed that we don't always do during the post lesson discussion is start out with the teacher's reflection. I think this is extremely important in helping frame the important points of discussion. Furthermore, the participation of the staff members was very authentic. At Acorn, it often felt forced into a certain structure. I personally prefer the inner and outer circle with specific discussion points where teachers are sharing out whole group. It was clearer and allows everyone to hear everyone else's input.	Today's task was "Is it okay to use the mathematical expression 300 / 2.5 to find the price of 1 meter of ribbon? The students were focused on how to solve it with whole numbers, and from there finding a range of prices that it must be in between. The summary concluded that it was okay to use that math expression, but since the students were focused on the range or prices using whole numbers, maybe that summary was too far ahead. Perhaps the summary could have been if we want to find 1m, division is okay, since the students discovered that. However we as teachers need to always assess where students are really at because tying up the summary with a pretty bow isn't for the students, its for the teacher to feel good about sticking to the lesson plan. This is much easier said than done because it requires teachers to simultaneously balance the end goal, their lesson plan, and what the students in front of them are bringing to the table.	This teacher engaged the students in a true TTP problem that the students had a genuine interest in solving. The problem was well thought out and her questions put the cognitive load on the students and sparked their interest. For example "How much do you think I should pay?," to which a student responded "We should ask her." The teacher also said, "I only want 1 meter. How can I find out how much it will cost?," to which a student responded, "we should measure the ribbon." The students were coming up with how to approach the problem in a real life situation, not the teacher. I will strive to make the mathematics this engaging and student centered in my future lessons. In order to do so, I will need to be thoughtful about the numbers I choose, the problem situation, and the way I present the problem and elicit ideas from the students for how to solve.	When we use a big ruler to make our lines straight on our board work, we are simultaneously making the board work clear and easy to read while also demonstrating to the children that mathematicians use tools to draw straight lines accurately. This also send the message that student notebooks should look organized and sparks the desire for students to take the initiative to organize their notebooks independently.
The post-lesson discussion was super hard to understand because of the translation so I didn't get a lot out of that part.		I like how the teacher had the students develop the questions and problem on their own instead of posing it herself.	I like the idea of hint cards and would like to develop some with my colleagues for different units.

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I'm wondering about the role of humor to keep the process during the debrief engaging and exciting.	The importance of connecting visuals to mathematical expressions toward deep understanding of each variable and component of the expression.	I am so impressed with the content knowledge of the teaching and relatively unimpressed by the pedagogy and lack of genuine conversation, collaboration and engagement (S to S; S to T) amongst students. Wondering about how to ensure both are in place.	levels of teaching and how we can utilize this within the lesson study process to analyze/critique the teaching.
There seem to be different approaches to lesson- I think I was expecting more uniformity.	I want to explore more this idea of developing an image of division, as we talked about today, based on 1 and scaling.	I really liked seeing the emphasis on tying expressions and diagrams together. We do this in the younger grades with simpler expressions, but I haven't seen it done in older grades.	Continued ties between algorithms, expressions, and visuals/diagrams.
During the discussion the teachers feel very comfortable in speaking honestly with each other and in questioning decisions made by the teacher who taught the lesson. Similarly, the teacher who taught the lesson was also open and honest and he acknowledged things that he thought didn't go well or could be done differently next time. This is a contrast to England where teachers would most likely become defence or think they were being criticised. A mind-shift is needed to create that open and honest professional discussion.	Again, the whole lesson was spend looking at just one problem. I thought it was an excellent idea to get the children to express the tenth term in as many different ways as they could. Consequently this will lead them to deep understanding of pattern and its links with algebra. It will also allow them to see where the simplified expression (4n) comes from and why it works as opposed to telling children to just look at the difference and use this to create the nth term. Like was mentioned in the post-lesson discussion, I also would have liked to have seen the teacher link all the expressions the children came up with to 4n. This would have enabled the, to see that no matter how you express the pattern, it can always be simplified to the same ultimate expression. During the post-lesson discussion I found it interesting to see the progression in this across different year groups and how patterns of dots can also be used to explain and prove more complex upper secondary mathematics.	I found it interesting how all the children knew exactly what they needed to do in order to begin solve the problem. Every child started to write something. If this was in the UK you would have many children raising their hands and asking for clarification of the task, wanting reassurance or wanting support. It is clear that independent working is natural to these children and has been learnt from a young age. They are resilient and not afraid to try something. The children are also fully engaged and appear to be enjoying the mathematics.	I am going to provide the a pattern and ask them to discuss how many different ways they can express it and look at the links to these expressions and the simplified expressions. I am also going to share the ice of spending a whole lesson on one question.

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The idea about assigning a level to a teacher for an observed lesson took me by surprise. I think the three levels are extremely useful for having a dialogue about teaching but labelling teachers is something we are trying to get away from for many reasons and my question in the post discussion discussion this morning still stands as I did not receive a direct response - there was a bit of a reference in the summing up but I want to know what they think they are aiming for in Japanese lessons in terms of the levels. Is it not the case that there can be a time and place for use of each ; for example, would you be expected to teach how to use a ruler in a level 3 way? I loved the clarity about where was the students' thinking in yesterday's lesson, was the teacher teaching for the students or for herself (or for the visitors?)	I have a lot of questions from the lesson regarding the maths - before the lesson started I wondered how the children would describe the way that the shape 'grew' in the sequence in order to link it to what could be seen very obviously from the numbers for the different shapes. This was exactly where the challenge lay but there was a piece of the puzzle I wanted to see and I am now questioning why I did and whether for the focus of the lesson this would have been necessary. That is the students talking about how they could see the same change happening from one shape to the next and using this to describe what the tenth shape would look like , not just how many stones it would have but what it would look like. For me the barrier for the children was the fact that they couldn't describe the tenth shape or weren't required to - the teacher had prepared the tenth one. If they had described it I wonder if it would have been the same shape resulting from all of the students. Fascinating to hear that in Japanese text books only one structure for division is identified - we spend a lot of time getting teachers to understand the importance of both grouping and sharing, with grouping being the most useful way to think about many divisions because asking 'how many___are in...' connects to known multipications.	I want to unpick further the notion of 'most important point from the lesson' - what does this mean and what do different people commenting on it think it means? Most important in terms of the learning and understanding of the students? Most important in terms of the focus of the lesson? These are not the same thing. Again planning for what the board might look like - this is not something we explicitly attend to England.	Knowledgeable other stays silent until the final commentary. We could do this by working in pairs in our team. The Japanese are consistent with us in terms of writing multipications. How easy it is for children to abandon their thinking when presented with an alternative they believe the teacher favours. More examples of the knowledgeable other doing some maths as part of the final commentary. Three levels. Children's thinking reflected on the board.
No lesson is every perfect, it is a continued process to improve teaching and to fine tune our craft based on what students are showing us that they know.	Hearing critical feedback from other math teachers and expert commentators is crucial to growing and evolving as a teacher of math. We can improve far faster and greater with the insight of our peers and experts in the field. They help us see things we can't see and guide to make changes when necessary.	Schoolwide lesson study benefits the staff, administrator and the students far better than isolated groups of teachers working on their own. The impact is greater when school wide.	Research lessons give us an opportunity to examine our teaching practices and help us improve our craft. Listening to what students are telling us is far more important than what we are telling them. We are there to guide students and provide opportunites for them to learn new information based on what they already know rather than tell them what they need to know.

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I thought the advice Dr Nishimura gave and the examples he presented were very inspiring! It showed the power and the importance of the Post Lesson Discussion.	The use of all the different answers that the children gave on the first part of the question including use of diagrams, tables, algebra and number patterns.	On the first part of the task, the children started the question without much hesitation. On the second part, I felt that the teacher's instructions and guidance wasn't very clear and this affected the children's work.	The lesson today made me think about the ways that the mathematical thinking can be developed from an early age. 'Making connections to interpret ideas' is something that I will explore more in my future work.
I am having a bit of a problem staying engaged in the discussion for the full hour and a half, especially following translation at the same time.	One of my VERY long term goals is to understand curriculum alignment up and down the scope and sequence of the standards, but the algebra of this lesson put me to task big time!		
1) it is important to be strategic about the strategies you choose to highlight/isolate in order to best build towards the learning goal.	1) teachers need to teach to the students and meet them where they are, even if that means abandoning or redirecting a lesson, otherwise the lesson is for them, not for the students.	1) it is critical for students to have opportunities to go back and make connections between their initial strategy and their new learnings, otherwise they may not actually be able to apply what they learned. In addition to reflecting on their learning, students need time to do applied practice using the new strategy.	I think it is important to highlight the need to keep the work student-focused. It is so difficult to abandon or redirect a lesson when you have spent so much time and energy on it during the lesson study cycle, but if you find that isn't actually what the kids need, then it is important to change course to address that gap.
I thought it was really interesting to see an older grade's lesson on a topic that I have seen in the US. I also thought there plan was more detailed than other plans I have seen.	The need to have students revise their work or go back and reference earlier work to connect to a new topic.	Carefully selecting and connecting student work for the board work.	How to get students to reference prior knowledge and connect their learning to prior learning.
The post-lesson discussion was very interesting to me for this research lesson. There seemed to be more clarifying questions and a deeper research approach to the discussion rather than having the members of the panel share out what they noticed with regards to student thinking. I could really gain a sense of understanding about the continued research that has been occurring with this team.	I really appreciated the lead-in that the teacher did. With having 40+ adults in the room, he was able to provide an ice breaker activity that the students, although somewhat shy and timid, really appreciated and enjoyed. The transition into the lesson was smooth and had a lot of carry over from the launch.	It is always difficult for me when doing TTP to gauge how much student work I should be discussing on the board with the class. This lesson seemed to draw out every expression the students could share which of course had the lesson run long and a bit rushed at the end. Overall, it was wonderful to see a research lesson like this so that I can also witness similarities with difficulties that I also face with pacing.	One of the notes that I had written was that even with such experience, this lesson study team was still learning new things about the lesson. I wrote down that lesson study lessons should not be viewed as trying to teach a perfect lesson, rather they are lessons to be taught for us to continue to learn from base on the goal we have set out to meet and what we did/still need to do to achieve it.
There are many different models in Japan, but the goals seem the same.	still thinking about this one...	I'm thinking about the balance between listening to students and bringing the mathematics out during the lesson.	I'm interested in shifting the culture at my school during the post lesson debriefs so that we are more focused on the lesson, the math and the student learning without feeling defensive around the discussion, questions, and feedback of observers.

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That even with planning that is far more detailed than I have seen in my previous experience of LS things will not be perfect but it is possible to see more clearly/precisely where improvements could be made.	Children can learn / understand algebra earlier than we in the uk think when it is taught meaningfully.	It was exciting to see children tackle a problem with no input.	I will be keen to discuss how lessons and in class discussion could be structured differently to make children more independent in their learning and more confident in their maths.
Even when your lesson plan is 20+ pages, the instructional decisions you make in the moment of the lesson can affect the your students' understanding more than you intended. Also, the way that you pose the task can greatly affect the way students try to solve the problem, as in the case with 4n vs n x 4 that we saw in this lesson.	The process of learning math, seeing patterns, and finding various expressions is just as important as the mathematics content itself.	I want to do more work with dot patterns to expand my students' understanding of math as a process and not as a means to an end of finding the answer to a problem.	There is value in encouraging students to think of a variety of strategies with which to solve a problem. The math learning and discussion should continue long after the problem has been solved. Further, even if a child has learned a strategy from a peer that is not considered a higher level than the one they first tried, they should learn multiple methods in order to open their minds to new perspectives on the same problem or diagram.
I used lesson note successfully for the first time. It helped me to better track the lesson and conctrate on student work and the flow of the lesson.	The knowledgeable other talks was fascinating. I appreciated the connections between the various grades and how formulas appear in geometry in different grades that tied into expressions with unknowns.	I appreciated all of the student participation and the balance of boy and girl participation.	Lesson note and ideas about the crossover between math domains,
Trying to follow the lesson plan too closely can lead a teacher to stop being responsive to students' needs.	Integrating the three levels of math teaching to analyze a lesson is a valuable tool.	The post lesson discussion is a powerful tool to push our practice and reflect on classroom instruction.	I would like to introduce the three levels of math teaching to my colleagues.
It's important to connect lesson study cycles to each other to preserve and build on the learning.	When students can manipulate expressions they learn to see the what these expressions represent in different ways.	If the goal is to deepen student understanding, returning to a simpler representation (from student prior learning) and building from there is ok.	Let's not allow our work to get lost in the shuffle of the work but carry insights forward from cycle to cycle.
Students are well-versed in the structures and routines of lesson study.	Planning a well-thought out and construced lesson does not always mean that students will understand the math in the way that it was intended by the teacher. Teachers need to listen to student voices more in order to understand what their needs are and not what the goals of the teacher might be.	It seems that there is an understanding in Japan that research lesson means that most feedback will be critical, and teachers accept and encourage this criticism without defensiveness. Every research lesson is therefore, a chance to deepen one's understanding and practice regardless of the feedback given.	Embracing critique without defensiveness

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-the written plan was very detailed in its rationale and sequencing of the lesson which helped me in understanding the context of where students were with equations.	-4x10 vs 10x4 this was an important reminder to previous we have had at Prieto regarding consistency with use; -I was impressed by the various equations that students came up with and how the teacher compared the equations. It was amazing to watch one of the girls in the back row complete her calculations. She was very detailed and did a hand motion when solving in her head; -the learning progression of equations and introduction to variables during the final comments provided additional clarity;	- the need to make clear connections back to the diagrams during student discussion, and in this case before the 2nd question was posed.	- multiplication order (4x10 vs10x4) and explicit connections to diagrams - previously discussed at Prieto but consistentency in implementation is not there yet.
Lesson study helps me learn and reflect about my own practice. Research lessons should be open to others.	Students seem to be stuck in the beginning to understand phase. Students don't have a chance to allow their learning and revise their work and thinking.	Students never collaborate and talk to one another.	Lesson note!
The teacher did not have time for a summary. I am wondering if this is a normal occurance	We need to be intentional in the examples we choose to put on the board. This lesson had way too many examples and lost student engagement.	In this lesson Dr. T explained that there was a huge gap between summary, students' idea, and their friend's idea.	Make sure our lessons are intentional and the their is not such a huge gap between students' idea if learning and teacher's goal.
The impact of whole school discussion on improving lessons	Fascinating lesson outcomes when time is spent discussing the thinking behind different student approaches	Allowing students to discuss in depth each other's ideas	Spending time discussing others ideas
The feedback sessions offer very specific insights and advice about how things could have been improved e.g. using smaller numbers would have allowed the students to access the concepts more effectively and the benefits of acting out the scenario. This seemed to be a very effective way we could move forward in the UK and US to help support the anxiety teachers feel about feedback.	The use of language, both spoken and written, is highly valued, modelled and expected. There is a consistency and accuracy in the teachers' use of terminology that we have seen throughout and students seem to use mathematical vocabulary with far greater ease than our students due to this.	Although the pictorial element is clearly being developed, the concrete stage is generally absent, particularly with regards to the students handling resources and sharing ideas. As the lesson was focused upon 'difference' and the total was relevant but of less importance, modelling would have revealed where the students were in their thinking and how the teaching needed to be adapted to move the students' thinking.	As always, many things! I will, however, focus specifically upon the language aspect. Developing this focus would not only improve student thinking and communication but also give teachers a very pertinent reason to improve their subject knowledge.

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I really thought about the knowledegable other in this particular lesson. He talked about why this problem was important and took the sequence of the problem all the way to calculas. The power of content knowledge and how it can be used to create problems that go deeper. Very rarly do I think about what my students are learning in math multiple years down the line and when I do I think about the problem when I learned it and that is a different approach to how it was taught. I think about my content knowledge is very limited to 7th and 8th grade and a tiny bit of 6th grade and how this limited content knowledge limits my teaching ability.	The teacher today had a really great idea about working backwards with expressions in order to have a deeper understanding. Currently the way most expression are taught is that students are always told to simplify expressions and they get very good at doing it. But learning how to work backwards with expression is a very usful technique especially when dealing with summation notations and limits.	I think about this lessons and two area of difficulty mant students had with this particular problem was that they were unable to find exactly where the fours stones were being added to each figure. I wonder how relevent to the problem knowing where to put the new stones for each figure was? Also I think about the other tools that students were using tables in this case can be used to aid in this problem as well? I also like the defining the act of adding more stones to each figure as a phenomenon. This ties back to the one of the major pillers in Japanese instruction of how students see math in their life.	Sometimes diagrams are not as usefull to a student as we think. Just because students have a daigram it may not help them understand the problem. Diagrams can also stiffle a students thinking and add confusion.
As we were in the midst of our own IMPULS post lesson discussion, an interesting discussion about level1, 2, and 3 teachers came up. We were debating whether or not it was okay to make a level 2 or level 1 decision within an overall level 3 lesson because that may be what the students need in that moment. Although we did not come to a conclusion, I found it very profound when we did decide that that's the point of lesson study. That is why we discuss the same exact lesson from all of our unique perspectives to establish our consensus eventually and come to a more comprehensive understanding.	Partitive division, or fair share, is very strong among students in Japan. However, quantitive is very weak. As a second grade teacher, I do not feel that I have the content knowledge to know how to strengthen either. However, I do feel it should be a shared responsibility among all teachers in the school or district. With that said, lesson study is so important in deepening teachers content knowledge below and above their grade level in order to best serve all students. This makes lesson study all the more important if we want to have a strong vertical alignment.	A question that is permanently stained into my brain from today is, "Are you teaching for your students or for yourself?" I think we would all immediately say without hesitation that we are teaching for our students. But are we thoughtful enough in our planning to really live by that? I cannot say with complete confidence that I always am. Sticking closely to the lesson plan may be with the intention of doing what we think is best for students. But in reality, if we stick closely to our lesson plan and turn a blind eye to what is happening in real time, we are not listening to the students. When we are not listening to students, we are not making choices that are best for their learning.	We need to be willing to open up our classrooms, be vulnerable, and all come together with our own unique perspectives in lesson study in order to push everyone to grow with the students' at the heart of our work. If it is the students that we truly care about, we will let go of our own fear, worry, and pride about allowing others to see us in action and instead approach lesson study with an understanding that none of us are perfect. It is not about judgement, it is about the math and the students.
Sometimes the lesson does not fit in the specified time frame, but the teacher still gave students the work time they needed (in the first individual student work time). It was good that the teacher got to the extension question though because students' started going down unexpected paths which made room for the further investigation and critical feedback. So although it's important to give students the time they need, there are key points in the lesson to get to that generate the most salient student data.	There are different kinds of pile patterns. 4n could mean "n groups of 4" or "4 groups of n" depending on what the pattern looks like. "4 groups of n" where the number of stones in each group increases is arguably a richer task.	In the post-lesson discussion, someone raised a question of whether digging deeper into how the table relates to the diagram would have better helped students understand what to do with the extension question. I think this connection is easy to overlook because we are so familiar with tables, but finding a causal link to student confusion is very powerful.	I think most of the pile patterns that we work with in 8th grade math are "n groups 4" (quotative) kinds of patterns. It would be useful to have "4 groups of n" (partitive) patterns to make students think more creatively and flexibly. In general, I need to get better at identifying when a task implicates quotative or partitive division.

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You can teach a public lesson in the gymnasium or auditorium without trouble: if students are prepared ahead of time for the environment, then they can be just as focused, and a larger group of observers can thus be included.	Dot talks have wonderful application for algebra instruction up through middle school - the visual terms can provide the concrete basis on which an algorithm is based - but some visual terms more clearly reveal that algorithmic relationship than others.	More is not always better when it comes to student strategies. Also, it is very important to ensure that students are asked to follow or explain one anothers' reasoning	The 5 practices for facilitating effective inquiry-oriented classrooms (from NCTM): Anticipating what students will do--what strategies they will use--in solving a problem Monitoring their work as they approach the problem in class Selecting students whose strategies are worth discussing in class Sequencing those students' presentations to maximize their potential to increase students' learning Connecting the strategies and ideas in a way that helps students understand the mathematics learned

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It seems to keep coming up that teachers feel rushed to get to finish and sometimes stop hearing kids.	How to best get kids o understand how to use diagrams.....other methods for visuals	Teachers over talking- selective questions. Not leading kids in the lesson set-up.	Still thinking....
The school today were excellent hosts and very proud to show us their school and their children. I felt welcomed. It is clear that every person involved in the lesson study also has a sense of belonging and understands their responsibility within the lesson study. I was also interested to hear a lot of criticism of the lesson and much of this was also things I'd thought myself (see below). It is good that this discussion can be had in a professional environment and that participants feel comfortable with this.	There is always more than one way to do something and all ways should be explored. To help the children to construct meaning from the problem I think manipulative a could have been used. An example of this would have been to split 60 sheets of paper into two piles of 30 and then to move one across and ask "how many in each pile now"? "What is the difference?" This would have drawn the children's attention to the idea of difference. They could also have added difference to the table shown at the start of the lesson (as discussed). If this had been done first then I think more children could have accessed the problem and looked at it in different ways (such as trial and improvement). Different children could then have shared solutions with the class and hopefully one would have had a diagram solution which the teacher could have focussed on.	I felt today that The children would have benefitted from having more time to discuss with each other and share their solutions. This was brought up in the discussion and was justified by saying that because the children were young the discussion needed more facilitation from the teacher. I feel that the children would have been ok to have conversations amongst themselves. I also felt that the lesson was too structured and teacher led. I wanted the teacher to allow the children to explore the problem independently sooner and to not to have provided them with a structured diagram so quickly. I think this confused some.	For my final year students (third year undergraduates) I am going to present them with a problem at the start of each of their lectures (such as the problem given to the 4th graders today). I am going to ask them to solve it and then put the multiple ways up into the board to generate discussion. This will help to consolidate and extend their ability to teach mathematics through problem solving.
The discussion was quite different because the teachers had been assigned children to observe and probably because they have not been focussing on maths. It felt like a more muted discussion and didn't go as deep as the others had, in terms of the contributions from the participating teachers. Was really pleased to hear the final final commentator say that there is a value in messiness and lessons can be messy. I am still puzzling about what it is they are hoping to see in the lessons but this was reassuring as it is not about a performance.	I found the final commentators made some really useful comments on the maths; this included the suggested use of the question 'why have you subtracted 12? Can you show why? And the context for understanding the effect of subtracting from one number and adding to another ; I gave thought about this with football teams before but did not make the link until it was suggested.	There have been two things in particular that have struck me about the approach we have now seen in three lessons. There has been little evidence of children being expected to engage in talk with each other. This could be to collaborate for example when lots of children did not have an answer today, using this as an opportunity for the children to talk to each other about what they have understood and either see if they can come to agreement on a solution or identify what it is that they don't understand. It could be in response to another child, saying whether they agree or disagree and why or asking a question of a child who has spoken. The talk seems to always go through the teacher which is surprising to me. The second is the absence of manipulatives to support and demonstrate understanding. I also wonder what the balance is between these lessons and other types of lessons focused on fluency.	Different roles during a lesson study session.
The more buy in, the better. Having state support and presence at a public lesson is how it should be.	Keep it simple and messy.	Network with all and whomever you can.	Lunch in the room and quality food.

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Using guiding questions related to the research theme to facilitate the lesson debrief. Providing time for observers to ask the presenting teacher questions about the lesson, decisions s/he made during the lesson, and the mathematics.	The importance of understanding the scope and sequence of how the math progresses throughout the grades- what students have learned and where they are going in mathematics.	"Are we laying the tracks for students thinking or facilitating students thinking."	I want to bring back the idea of creating guiding questions based off our research theme- to specify for observers what they should be looking for and also to guide the lesson debrief.
The first aha moment is how much the teacher of the lesson puts him/herself out there. It is a challenge for a teacher to recieve feedback and not take it personal.	I really am impressed by the challenging topics teachers are taking on. They are are very challenging areas of mathematics. There has not been any real focus on the answer in any of the lessons I have observed. The goals of the of the lessons have been based more around absract ideas like: WHy is this a division problem? Can you use a diagram to show your thinking? Can you manipulate a diagram in order to reconize a phenomenom?	One thing I have noticed is that in the lessons that I have observed is that there has not been much cross talk between students. I am curious the reasons for this. I noticed that that students share their mathematical thinking in a whole class discussion but not in smaller settings.	That goals of the lesson are more then finding an answer. Sometimes the goals of the lesson can be about which mathematical tools can you use to solve the problem and how did you make your decesion on which mathematical tool to use.
During today's debrief, I found it helpful to have the Observation and Discussion points. It helped me focus on what each team was presenting and sharing. This was very helpful for me because during my observation, I knew what my role was and which topics I was specifically looking for.	The teacher's questioning of the students helped him with his board work, however, I kept in mind the three different levels of teaching and wondered and reflected about his attempts at achieving level 2 & 3.	Today's final commentators reinforced the idea that if a teacher's goal is to have the students make use of diagrams, then they have to be given specific time and opportunities to create authentic diagrams.	Today's lesson once again demonstrated that with Lesson Study, it's okay to plan and try new methods with an expectation of feedback from both the students and the lesson planning team to help move lesson thinking/planning forward. Additionally, I will be sharing this teacher's experience regarding "teacher talk" and questioning and how much is too much vs. allowing the students to try things on their own.
All the teachers across districts are deeply engaged in this process and provide valuable feedback. Teachers collaborate and work hard to make the best lesson they can, but expect the criticism.	Undersatnding the progression of mathematical instruction and learning is crucial to developing rigourous lessons that meet students needs and levels of understanding.	Student leaders starting lessons and ending lessons is effective and creates shared leadership.	More shared leadership with students
All of my "Aha's" are reiterations of what I have already said. It's the way to go. It is hard work. It is good work.	When we teach math well, it is especially helpful when the "dots" are connected.	Make it student focussed! Always! WAIT--Why Am I Talking.	Listening to the public comments on minimal sleep is REALLY challenging.
- appreciated the amount of participation during the post conference. With grade levels receiving organizers to gather specific data it held everyone accountable and therefore encouraged more conversation. -post discussion set-up was a very defined square with others in fishbowl - provided additional structure to conversation and greater accountability. -reflecting on previous learning when students do not generate ideas/strategies you had planned, in this lesson diagrams.	-diagrams were a large part of the lesson and post discussion. Value of diagrams much more clear when idea is generated from students; -during third set of final comments - the idea of connecting back to the table to show the difference provided great clarity in the table and its potential purpose/ value.	- what manipulatives could have been provided to students? Paper tape or counters? - students needed to have drawn diagrams and shared out verses teachers ideas being created on the board	-organizer for observations and assigning specific observational roles - analysis of diagram usage in our school wide scope and sequence

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How professionally discussions are conducted and the level of analysis, openness and honesty. Most teachers in the UK would really struggle to not take comments personally and I'm hugely admiring of the attitude and focus these discussions have. There seems to be a huge emphasis, in all of the lessons we've observed, on children speaking to the teacher in front of the class rather than each other. The goals in their planning specifically refer to 'learning from each other and with friends' and yet the discussions are all primarily led and addressed to the teacher. I'd like to know more about when pupils have opportunities to work in pairs and teams where they can discuss, explain, convince and argue with one another before presenting their ideas to the class or another group.	How important diagrams are and that they MUST make complete sense to the children. This lesson demonstrated what happens when teachers lead discussions too much and impose their own ideas on children. It was interesting to hear the range of experiences teachers within our group have of this way of working and the journey that must be undertaken to fully understand the essential role of manipulative and diagrams.	Making sure that the focus of the session (both in term of mathematical content and learning behaviours) is where the children's attention is drawn. The concept of difference was paramount in this session but was not the focus of the teaching and so was not the focus of the children's thinking.	Listening to children. The teacher must be able to listen and respond to children very effectively and genuinely hear what they are saying and how their existing knowledge an misconceptions are being revealed. Teaching needs to be focused upon 'drawing out' pupil understanding and helping them construct their new thinking and connections.
1) students need to generate tools in order for them to be useful. If a teacher gives the students a tool to use, and they weren't a part of the creation process, it won't hold any meaning and result in more confusion than support.	1) teachers can strategically select portions of the problem to give students and withhold certain portions of the problem in order to generate deeper thinking about the process and operation. 2) A teacher can really set students up for success by doing an introduction piece in a lesson, in order to activate prior knowledge (though this can also hinder student understanding if it tells rather than develops too much).	I really want to think about how I am structuring/introducing the problems that I am using. I could be making slight tweaks that could change the trajectory of student learning, in both positive and negative ways, and I need to be more conscious about how I am rolling them put to my students.	I think it will be really important to communicate to teachers that they can have some freedom and flexibility in their problem introductions. You can strategicaly include/withhold information in order to best activate prior knowledge and develop schema around a learning target.
Having a research goal of a specific type of representation can be tricky: what if that representation is not as meaningful or helpful to students as another representation?	Actual strips of paper are a more concrete model of quantity, both part part whole and comparison situations, that can then lead to effective use of Tape Diagrams.	Giving only the first half of the prompt at first engages learners and supports generative thinking. It helps ensure a shared visualization of the problem context.	
You cannot follow the lesson plan so closely that you ignore students' needs and knowledge they are presenting during the lesson.	If you want students to use a strategy or tool, you need to know what they have previously learned and what tools they currently use.	Students need opportunities to overcome challenges. Students need time to think and grapple with the problem and reason about the problem.	Planning is only a part of the big picture. It is important that we listen and are responsive to students.
Lessons don't need to be so neat. They should be messy to allow students to grapple, persevere, and be excited about finding answers.	Too often teachers want to conclude lessons in a good way and they lull kids in that direction. Kids do not get their themselves.	Communication between students is done through the board work and classs discussion, rather than student to student collaboration.	Teacher should have a wide pathway for students to learn. Teachers must deviate from the original lesson. Otherwise teachers are not listening to students.
In our school, we focus on students when debriefing. When they are doing the post lesson discussion, they focus on the teacher. They also discuss a lot of the content which is helpful.	Today, we focused on the diagram. The diagram is not a strategy. The diagram is to show your ideas, understand others' ideas, and prove your solutions.	When creating lessons, we need to keep in mind the accessibility of the numbers we choose. The numbers we use have to be intentional. Also, we have to teach students to understand the reasonableness of their answers.	We need to have a clear purpose for our lesson study team. With that, we need to discuss what we want it to look like and make sure we hold eachother accountable to our goals in lesson study.

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I'm thinking about the pros/cons to doing just one lesson, doing one and redoing it, and doing two on a similar research theme or content strand. There seems to be many pros/cons to each and perhaps this year I will ask each LS team to decide for themselves which route to go.	The importance of building to a mathematical climax in the lesson, where students realize a critical misunderstanding or AHA and want to deeply engage. I felt there was a key point in the boat lesson when students recognized that 12 could not be added and subtracted from both, and were hooked. This would have been the perfect point for groupwork or more independent thinking, but instead the teacher directed the lesson from here.	When the teacher stops listening and being responsive to student thinking/student learning, the student thinking and learning stops. Many of the teachers began directing the lesson mid-way through, focusing on the lesson plan rather than the possibilities in front of them. This inhibited the lesson flow and learning potential for the math block.	I'm considering all sorts of tools and flow charts I'd like to develop for our master teacher program and LS teams...
I appreciate how formal the discussion was and the knowledgeable other commentaries.	It is messy.	I liked the idea that if the students were to use diagrams then they should have made their own.	I loved how beautiful the school was and I like how kids ran everything. I am thinking about how that could apply in our k-8.
The post Lesson Discussion was very interesting. The respect across the table and the fact that teachers are very receptive of the ideas and suggestions is something I really enjoyed.	I don't think the children really understand the "12 more" idea. The use of manipulative at the beginning would have helped more to grasp what what happening.	The teacher spent 20 minutes explaining the lesson and even after 20 minutes children were not sure what was going on. Children spent 7 minutes solving the task. I wonder whether this affected the whole lesson and whether if the children had spent more time talking and working together would have brought different results.	Use of different methods including use of diagrams to express addition or subtraction.
It is okay if a lesson doesn't go exactly according to plan and if the "math gets messy". When a teacher tries to stick too closely to the lesson plan, e.g. Using one particular diagram, this may actually limit students thinking because it doesn't allow students to use strategies that organically come from them. Furthe, when a teacher sticks too closely to the research lesson plan, he/she likely is not able to listen to students as efffectively.	Each diagram, table, and chart being used in a lesson should have a clear use that is directly tied to the objective of the lesson. In this lesson, the teacher used a table to show various sets of numbers that added to make sixty, but because this table didn't contain rows for difference and total, it may not have be mathematically useful.	There may have been too many teacher questions in this lesson. A teacher should be selective about the types of questions he/she asks. When a teacher speaks so much, students cannot think. If you want to develop students' reasoning skills, you need to let them speak!	We need to spend more time anticipating student responses so that we can get as wide a variety of possible strategies as we can. We should be very careful not to unintentionally limit students' problem-solving strategies by introducing a particular diagram/chart/table too early in the flow of the lesson.
How consistent the process seems to be	Subject knowledge of teachers is far greater than that of many western teachers	Seeing that planners of the lesson observed take responsibility for it alongside the teacher who taught it	An increased emphasis on subject knowledge
The opening introduction of the post lesson discussion, when the administrator said that he has been in the position of that teacher before, and that honest (potentially harsh) feedback is how you grow.	What is the role of a diagram? In this lesson, it was a different way of seeing the problem, but you need to consider how students will be motivated to see the diagram in that way.	Teachers have been doing a good job of opening up the problem or question, but student engagement and learning becomes more varied towards the end of the lesson. How can you assess students where they're at, at points where you anticipate that students may not be clear?	Think about intentionally the sequencing of when we teach different representations. Do we start with a diagram? Or with calculations? What will students feel comfortable with first before moving onto the next representation?

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Doing this problem as a group with Makoto was incredibly valuable. I was able to access the content more readily. I was able to come into the research lesson with an understanding of the different solutions and misconceptions. I realize the value of doing the math as a research team and it's essential place in the process of Kyouzai-Kenku. Knowing there are deficits many of us have in content knowledge in the US, I believe doing the math together cannot be left to chance or done in isolation. I gained so much from the discussion we had about our different solution strategies. I learned, not only how to do the math, but the different conceptual understandings it required and, in turn, developed. These are gains it very well could have taken hours, even weeks or more, to discover if I were working through the problem alone.	When the objective of a lesson is that students notice commonalities through the use of diagrams, it is important that commonalities and differences be brought out in the investigation of their prior learning. In this lesson, a 3rd row in the table of possible distributions of the paper between Riko and Kota that showed the differences between each possibility would have allowed students to see the effects of manipulating the amounts of paper each had and they would see (rather than depending on abstract mental reasoning abilities they have not yet developed) why Riko having 6 additional sheets would give a difference of 12. Observing this reinforced for me the value of visual representations for teaching math as well as how important it is to be very intentional in how I guide students to access the elements of their prior learning that will be key for the lesson.	I know that visual representations are crucial for deep mathematical understanding, but if all the diagrams, tables, and visual representations of the problem come from me the chance that they will make sense to the students is low. When I allow students to develop these representations themselves, this is the "messy" place in lesson where rich learning can happen. I need to trust my students more and make sure I give them the space to navigate their own path into the math together.	Assigning students to grade level teams to observe. Not sure if this was done for this research lesson, but I would also consider giving grade level teams guiding questions ahead of the research lesson to consider as they observe their assigned students and to address in their post-lesson discussion comments.
It was very interesting to see how Lesson Study could be done at a deep and meaningful level in a school context (rather than at a wider level). It was also interesting to see the value of having such a wide range of adults from the school involved in the debate although how to organise this in my school would be a challenge.	It was a useful example of a class discussion where different methods were shared and discussed.	Reinforced the importance of giving children time to tackle problems before discussing them.	The importance of providing students with malleables for children who are strugling to visualise.

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A comment made at the beginning of the post lesson discussion was "Don't hold back your ideas, that's how teachers sharpen their skills." I really appreciate how honest yet respectful the team is during the whole lesson study process. There was never a feeling of competition or defensiveness because everyone is in it to learn and for the students. The teachers are always open to feedback because of the culture of teachers as life-long learners that lesson study inherently creates.	If we use tools during a lesson, they should be meaningful to students and useful throughout the lesson. During the discussion, many people brought up how the table used at the beginning of the lesson was not an effective tool. Furthermore, the idea to even make a diagram in the first place should have come from the students if they are meant to see the value in using it. I think this can be generalized for many mathematics lessons. The tools should be student generated in order for students to construct meaning and use them throughout the lesson to get to the end goal.	During the post lesson discussion someone brought up the idea that if the goal of the lesson was to help students recognize the merits of using diagrams that help them visualize the relationships between quantities clearly and easily in the context of word problems, then students should have had more opportunities to create meaningful diagrams. If another goal was to develop students way of communicating, then students need more opportunities to talk. This sounds like very obvious feedback for a very experienced teacher. But in my experience with lesson study, it is often the findings that seem so simple and obvious that are the most important and profound. We as teachers develop habits and have blind spots, so it is so valuable to hear the feedback of people who are observers. This feedback can sometimes sound very obvious, but may not yet be a part of our practice until we make a conscious effort to implement it. As a person who tends to feel a bit jumbled in the mind with so many different train of thought, it feels helpful to remember these simple golden nuggets of information.	Our school has had a big focus on minimizing teacher talk so that we are giving students more opportunities to talk. In the lesson we observed, a lot of the discussion went from student to teacher to student to teacher to student to teacher. All the information went through the teacher who was at the center. We need to put students at the center. Instead, the conversation should be between students, and the teacher should be acting as a facilitator. If not, the students learn that they do not have to justify their thinking or explain their thinking clearly because the teacher will do that for them. If we want to develop independent problem solvers and excited mathematicians who can justify their thinking, then we must step back and provide more opportunities for them to build off of each other and explain their thinking to one another.

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That the post lesson discussions structures vary greatly in different schools. Depending upon the administrator's role and how the teams have been set up regarding observation, the feedback focus changes quite dramatically. I have gained most when discussions are centred around a specific goal and illustrated as well as talked about. Would like to know more about any research that has been conducted around the effectiveness of different types of post-lesson discussions.	The deep understanding teachers have about the journeys pupils need to take in order to begin to understand a concept fully throughout primary into secondary.. The way the series of lessons are planned and understood (by subsequent and prior year groups) so that pupils have consistent experiences which build upon one another is very evident. The focus upon 'interpreting the remainder' lesson where children were respected for giving answers that matched their existing knowledge and then challenged to modify these ideas in the context of the problem was very important.	This was the first time we'd seen manipulative being used. There were other lessons so far where manipulatives would have been enormously helpful in generating student reasoning and it was very interesting to see how the counters developed student explanation and engagement. The image that remained on the board throughout the lesson allowed all children to return to and use the reasoning being modelled and being constructed by a child and not the teacher, made this even more accessible. The use of terminology is very strong for me; 'expression' and 'equation' are used throughout and with full understanding by the teachers and pupils.	Use of children's interpretation and evolving understanding of the problem using manipulatives. Leaving manipulative models visible for the class to refer to and physically move is hugely important in justifying whether a mathematica expression supports the actual mathematics taking place. Secondly, the use of mathematical terminology as described above.
It was interesting to see how they did a district-wide PD. I'm wondering about what the "take-away" was for the other teachers in the district, and how those lessons might be discussed in their various schools.	I was intrigued by the division lesson, and how the kids had to think about why the remainder is never larger than the dividend. I want to try this back home.	I'm still wondering about their processing ideas as partners/groups. I did see the one "turn and talk" yesterday, but I'm wondering if that is enough for them to get to the goal of valuing the idea of "learning with friends." Partner/small group work hasn't been seen much in our lessons so far.	See above
I thought that the cross-district lesson study was very different to the others that we have seen. There were some obvious things such as lots more observers, it being on a Saturday and it being in a hall....but what I was most surprised by was the difference in the post-lesson discussion. It seemed that it was a lot less interactive with it mostly being the class teacher and expert comment at the end. There were some questions and comments from others but not as much as we have previously seen. I was also wondering why the other people who planned the lesson don't have more comments to make or don't refer back to the plan. (For example they may make reference to how somehing went - or did not go - as expected). I also think it's beneficial how the lesson plans and discussions often link the learning to how it's relevant later on in education. For example in the division lesson plan it discussed a grade 9 national assessment question about algebraic expression which directly linked to the third grade lesson.	This division lesson was the first time that I have seen physical pictures and manipulatives and diagrams used in a similar way to how they would be used in the U.K. I think the use of magnets to represent the people in the boats made it very clear. I found the whole lesson question in the statistics lesson extremely interesting and I though it was relevant to the children. For me, it really allowed the children to explore and talk about the data with a depth that is not usually seen in the UK. In the UK children are taught procedures for finding the mean, mode etc but would never be asked to apply this to relevant situations and even move towards deciding which is most appropriate and why.	As I've said previously, both lessons were quite teacher led. In the division lesson it would have been nice if pairs of groups of children actually had manipulatives (such as counters and laminated boats). This would have enabled them to understand the problem with more depth and would have also helped those children who struggled. In the statistics lesson I would have liked to observe the students having more discussions with one another and possibly presenting their recommendation and tryin to purseude the rest of the class it's the correct one. They said in the post-lesson discussion that they are working on student talk and collaboration but I don't feel like we see much of it.	I am definitely going to share the content from both these lessons but especially the statistics lesson because I feel this is an area of weakness for many teachers in the U.K.

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The final commentator can add significantly to the thinking and learning of participants and the best examples we have experienced have been people who have understood how they could offer something to make people think more deeply and go beyond simply commenting on the lesson. Being an effective final commentator requires a deep understanding of the teaching of the mathematics planned and the ability to make connections during the live sessions which is quite demanding.Today's were not the best examples but without them it would be more difficult to identify what are the key features of an effective final commentary. The post-lesson discussion is variable and I am interested in whether this is dependent on the level of the lesson study (school, district, open); today's wasn't as penetrating as previous discussions. I was surprised that the discussion did not focus more on the school's theme for lesson study 'students who continue to learn together with their friends through lessons in which students feel the value of learning.' I love this theme and would have liked to have heard what the two teachers felt they did differently in their lessons because they were attending to this theme.	It was really good to see an image being used to support understanding but I wondered why the teacher chose not to allow all of the students the opportunity to manipulate images themselves or draw something to make sense of the mathematics and demonstrate their understanding. Research suggests that children need to manipulate images themselves in order to make sense of the mathematics that the image is demonstrating. I wanted to know why the teacher went away from her plan and why she did not plan to get the children to generate their own examples instead choosing to do nine examples of her own all in a connected sequence. This seemed like a missed opportunity and more about her than the children but maybe she was aware of something that I wasn't.	The wording of 'I got it' rather than 'I did it' is really helpful; for me it fits with John Mason's wording of 'working on' rather than 'working through'. I think it is accessible for teachers and something they can use to reflect on the experiences they are providing. We have seen a lot of teacher talk in the lessons and very little student to student talk ; everything seems to have to pass through the teacher. We have schools who have worked very hard on getting their students to believe they have a responsibility to listen to everyone expecting to be able to challenge, support or question as appropriate. There has been a tendency for the teachers to want to control the journey of the lesson and I wonder how much of this is about the lessons being research lessons. I also still want to understand what would be the features of a live lesson in Japan that would be viewed as reflecting level 3. The one consistent thing has been listening to the children but beyond that...?	I got it not I did it. More about the knowledgeable other.
I enjoyed participating in a big lesson study (in terms of participants). It is good to be able to compare the intent of each lesson study we have had and the actual execution. The Post Lesson Discussion was not at the same level as the previous ones and that has given me an understanding and appreciation of what a 'good' lesson study is compared to a lesson study that needs to be more polished. For example, considering that time is an issue, I wasn't sure why at the Post-Lesson Discussion, the teacher (Mr Yamaguchi Kuniyuki) discussed what he did in the lesson in detail. We all saw the whole lesson and I didn't think he was reflecting. He just summerised what happened which wasn't the best use of time.	Both tasks were based on real life situation which is great as it gives the students a chance to see what they learned in mathematics classroom means in the real world. Whilst the first task had most children engaged, the second one failed to ignite interest. I did not understand why three histograms were used. I think it would have been better if he had used different ways of presenting data and then compare the results, for example pie charts, scatter graphs and so on. This could lead to a discussion on the advantages and disadvantages of each method.	Using manipulatives on the first task helped children understand better what division with remainders meant. Sorting out the histograms in the computer during the second lesson was time consuming and unnecessary.	Hands on tasks are very important! Students collecting data themselves, then recording their results, and presenting them in different ways is something I would like to explore more.

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- the large format for the post discussion in combination with the two lessons being back to back I don't think lended itself to as rich of conversation (also hard to hear translation on this day). Insight - smaller and more focused.	-the histogram lesson- students I think needed to look at whole graph (also brought up by teacher) and have students perhaps collect all of the data to be more vested in the real life problem solving	- emphasis on board work to aide in students understanding- color coding@ and writing students names with their ideas (suggested by comment), space to compare ideas and strengths and weaknesses of those ideas.	Wonderings: we try to get students not to erase to have a better idea of their thinking, and in observing, erasing seems very common practice - keeps notebooks meticulous, but how does teacher get more individual incite into a students thinking since much of notebook is copied from board? - use of manipulatives have only observed once so far, is this common practice? Do students have choice to use or does teacher tell them when to use?
You have to observe lesson study many times to understand how lessons impact students. Often times teachers don't see the gap between what they're teaching and what students are learning.	I appreciated how the teacher started the class discussion with the misconception. Also the teacher asked many open ended questions and questions that pushed students to explain their thinking. How do you know? What do you mean by...?	Students used the counters to explain their mathematical thinking to their peers. Peers also explained each other's thinking.	I would like to purchase the Japanese Math curriculum which includes the large magnetic manipulates for teacher and student use during class discussion.
We need to focus on student engagement and work rather than just the teacher. When we develop an eye for observation, we can critically observe students and see the impact of the lesson on their learning.	Concrete examples are sometimes needed to support student understanding. Students can then build toward abstract concepts.	We need to be aware of student engagement... when an observer is bored, students are definitely bored.	I will be excited to utilize Japanese curriculum when I return.

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With each post lesson discussion, I have noticed the feedback to be critical of the lesson plan, not critical of the teacher that taught the lesson. However, for these two lessons, I thought the feedback was not as in depth and I am wondering if it was because of the 'cross-district' LS approach with so many people in the room.	LS is definitely needed when making attempts at trying new curriculum ideas or implementing new curriculum changes.	Both of these lessons revealed the intention that the teachers had to stay on board the lesson plan as much as possible. However, with both lessons there came a point where the teachers stopped teaching to the students and continued teaching in order to meet the plans/goals they had outlined in the lesson.	During our discussion on June 26th, Dr. Takahashi reiterated the fact that when observing LS, we cannot just rely on video or watching solely what the teacher does, the authentic evidence of what students are learning has to come from observing those actions made by the students during the research lesson. I will share the idea that when we observe with the teacher's actions and board work, it can be very different from what's going on with the struggles and thinking made by the students at their desks.
We need not really discuss lesson study today. I feel like the post lesson discussions have been difficult to understand because of the translation. It is hard to stay engaged. I would like to dive deeper into lesson study components and intentions.	The lesson on division with remainders was helpful. We need to help students understand that they have to pay attention to remainders and grasp the difference between the quotient and remainder.	It was extremely difficult to remained engaged through the organizing data lesson.	We need to increase our content knowledge in order to be comfortable with not following lesson plans exactly.
The importance of not trying to execute a perfect lesson based only on the plan. Sometimes the messy part is where all the learning happens.	the progression of concrete to semi concrete to abstract.	Reading the students and taking them temperature of the room in order to make in the moment adjustments.	
1) math can (and should!) be messy and our lessons need to reflect this!	1) teachers often teach to their plans, not to the students. Sometimes the best move is to veer off course for the sake of student learning, even if it doesn't follow the original plan. 2) it is critical to include time for students to reflect upon misconceptions and apply new learnings in order to assess for understanding.	1) as teachers we get so caught up in our "plan" and making sure things go according to this plan, that we fail to notice if kids aren't responding to it. It is so difficult to abandon a carefully thought out plan in order to address student needs in the moment, but it is a necessity if we are trying to teach to the kids. Otherwise your lesson is for you, it's not about promoting student understanding.	I think it will be important to focus on the idea of deviating from the lesson study plan, when necessary, and what that can and should look like during a lesson study observation lesson to reflect our authentic teaching practices. I also think that it is critical that we build strong systems for collaboration, reflection, and application to maximize student learning through lesson study.
It seems that teachers are "playing it safe" and not taking very many risks to let the math get messy during the research lessons.	Generalize an answer to the problem when you get an answer to the problem (6 boats for 23 people, the teacher charted other values).	I wonder what these lessons would look like if students had more opportunities to share their ideas with a partner or group? There was a turn and talk during the histogram lesson, which is the first that we've seen so far.	Students should be curious about histogram data such that they are compelled to ask questions that dig deeper into the data (e.g. walking vs. bus).

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Professional development should be engaging and exciting. Some of the post-lesson discussions we observed were quite dry, with limited and often inequitable participation by the teachers around the table, and at times with participating teachers losing focus/attention throughout. There is so much incredible content knowledge but I am wondering about the place for humor, equitable participation structures/protocols, and general engagement strategies - throughout adult learning.	In the 6th grade lesson: When making recommendations, students needed to be probed to assess and articulate the strengths and weaknesses of each recommendation. To truly internalize the complexities of the scenario (and the math applications re: data and statistics - specifically the benefits and drawbacks of the mean vs mode vs frequency tables), students needed to articulate the pros/cons of each approach in the class conversation. It it critical the teacher builds in time for students to both grapple with and make meaning of the math, then consolidate their learning for the day, articulating it through the summary process. Without this, students are not reaching the depth of learning possible.	Similar to my comment above about adult learning, I am wondering about the pedagogy of the student lessons. How might teachers bring forward their personalities to build more exciting lessons (use of humor, personal stories, etc.)? How might teachers utilize more engagement strategies to hold student attention and get more participation by all? The content knowledge of the teachers is unbelievable but the pedagogical choices are not always maximizing that incredible knowledge.	How can we build our content knowledge and invest as deeply in the math as teachers here - without allowing the math content to override the strong pedagogical practices already in place.
It was fascinating to see so many educators all studying the 2 lessons.	The remainders lesson was interesting because of the difference between the expression and answering the question and how the boat problem gave real life context to remainders.	Both teachers had nice introductory hooks.	The idea that one class can be 1 problem teased out in many ways as you try to get to the "why"
We need to use students' ideas and work hard to listen to the students during the lesson. More time should be spent on allowing students to come up with their own questions about the problem and/or about one another's strategies for solving the problem.	We need to be very careful and strategic about the numbers we select for a particular problem. In the fourth grade lesson, the teacher deliberately chose to have a remainder of 3 because she didn't warns students to take people from othe boats to try to mark three numbers on the boats more equally distributed. Numbers must be chosen deliberately to fit objective if the lesson.	Students need to feel the value of learning and need to have some compelling reason to solve the problem. They should also have access to a variety of materials and manioulatives so that they may feel free to use them during the problem-solving phase of the lesson.	We need to plan for more lessons on interpreting remainders. This starts to manifest itself as a difficulty in fourth grade, and continues to be an issue in our classrooms in fifth grade. I would also like to think of more ways to incorporate technology into our lessons. The teacher in this lesson sought to use technology as a tool for interpreting data, which I think is an idea that we can try to develop more in our classrooms at Prieto.
The importance of an insightful and related final comment. That 2 lesson studies back to back makes it very difficult to focus on the learning in each.	In order to have evidence of learning it is useful to allow all children to show their learning and progress by giving them a task toward the end of the lesson tat they can record in their books.	How teaching is most effective when done through focused, thoughtful questioning.	To start with smaller more focused LS sessions. To make sure there are always insightful final comments.
That a cross district lesson involves many more teachers. It was impressive to see so many teachers observing the lesson but unfortunatly I was unable to see the student's work. My take away from today is that I need to focus more on the student's work in order to determine how they a thinking about the math.	Make math relevent to the students' lives. This was the division problem with remaiders. How do students interpet a remainder within context. It was interesting to see how the students would justify their answeres for the amount of boats needed. I was great to see students grapple with the idea of a remainder in order to determine how many boats were needed.	I felt like for this particular lesson I learned how a mathematical tool can be used to much. There was a point today in which the teacher spent to much time making a list to show how many boats were needed for specific amounts of people and it got somewhat redundant.	Be careful with diagrams they can be very useful but if the students can make them don't spend to much time writing on the board.
The depth of knowledge the Japanese teachers have about the curriculum and the sequence of mathematical instruction	The importance of understanding progrssions and sequence of learning. What have the students already learned and what do they need to learn.	It's crucial to dedicate time to study the curriculum and the standards deeply.	Taking time to study curriculum deeply. Using lesson study to do this

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During the lesson, I was standing in the back watching the teacher. I should have been standing in the front watching the students. If we are trying to learn how to develop an eye and an ear for student thinking, understanding, and misconceptions in order to best teach them, and if we are trying to learn about what practices are best for students, it is them we should be observing. I need to watch for when they are engaged, when they are disengaged, when they are wanting to talk to one another about math, when they are wanting to talk to one another about something unrelated, when they are looking busy but not actually solving the problem, when they are confused, when they light up with an "aha" moment, etc. These are the moments to watch for and analyze what it was that could have possibly led to those types of student involvement.	One comment during the post lesson discussion was that the problem itself could have been more connected to the students' lives. However I don't agree that it ALWAYS needs to be. Sometimes, problems can be engaging simply because the mathematics itself is interesting and we shouldn't assume that students don't have that inherent curiosity and drive to solve problems.	The themes of this lesson were to improve the teacher's questions and to help students feel the value of learning mathematics. When I went back through my notes, almost every single thing the teacher said was a question. The teacher asked questions like "What does it mean?" "How do you know?" "Do you understand this?" "Can you find out the answer?" "Can someone please explain what he meant?" "Does anyone have a similar idea?" "What do we need to do?" All of these questions put student thinking at the center of the lesson, gave them opportunities to understand each other's ideas and to gain a deep understanding of the math. To take it a step further, I think its important to incorporate more turn and talks and provide opportunities for students to ask questions to each other and discuss among themselves.	Watch the students, not just the teacher, during lesson observations!
The quality of feedback is less when the observers are coming from more disperse communities.	It is worth pushing every day and at every grade level for students to be able to generalize patterns. To generalize patterns, students need to test a theory with more than one number set, as the teacher in the 3rd grade boats lesson did.	Training students to be diligent in looking back at the text of the situation, almost like small lawyers, sets them up to question and test the limits of the mathematical situation. (ie, how many students could fit in one boat and still be acceptable? This is a situation-dependent question, as all mathematical questions are.) It is a sign of strong instruction when students can correct one another by referring back to the specifics of the text.	The flexible meaning of remainders is an important mathematical difficulty, best taught through not only a variety of situations but with a variety of numeric values within each given context.

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I echo Dr. T's statement that seeing live lessons is where the discussion can happen and real change can happen- not the same as watching a video.	I liked the lesson today working toward giving students a reason to have/use irrational numbers.	I'm still wondering about the intro or set-up for the lessons. I think this is important as it gives kids a reason for the mathematics and interest in what they are doing. However, it seems like it might be going longer than is necessary. I want to give enough time for kids to work independently, in groups, and be able to share "whole class."	I'm thinking more about "teacher moves" and having teachers be more reflective about why they are doing certain things in lessons. This is hard to do while you are teaching, but I think it is important. Also, many of the best teachers do great teacher moves instinctively, but if it isn't reflected on, it isn't always repeated by them, and it can't be taught to other teachers.
From our discussion this morning I realised the importance of seeing the children's faces and children's work during the research lessons. I hadn't fully considered previously that this is the only way to truest assess the impact and benefit that the lesson is having. It was also interesting that the statistics lesson (from yesterday) was done due to the change in curriculum (with histograms moving from 7th to 6th grade). Japanese teachers have decided to research this change in order to find the most effective way to implement this. In the U.K. The entire curriculum changed in 2014 yet there was minimal extra training for teachers - it was just expected teachers would know what to do.	What really strikes me is the level of teacher knowledge for the mathematics. Even teachers in the lower grades know exactly what more complex concept each unit underpins and also what came before it. There is very clear progression. Today's lesson about putting irrational numbers into context through studying paper ratios was an example of strong teacher knowledge and understanding of irrational numbers. When I was taught about surds I was simply taught a list of rules and procedures to remember and was never shown the links between this and real life situations. It is interesting how many different ways this could be applied in the professional world. Designers, for example, could use the fact that 1:root2 is aesthetically pleasing to get ahead in the world of design.	Of all the lessons so far, this had the most collaborative group work. I felt that the children were engaged in their groups of four and using one another to solve the problem. The teacher kept reminding them to work together as well by commenting that they could not solve this alone. When one group went up to the board to share the solution with the class, they initially said "so this square has an area of 2" but the teacher proved them to explain further (how they knew this). For me this was a key point as I myself had not realised why the inner square would have an area of 2 but once they explained, it was so clear. I also thought it was fantastic that even in this complex problem, two different groups solved it in different ways and both explained it to the rest of the class. I felt that the start of the lesson went on for too long and that the teacher could have made the sorting of the books quicker - perhaps through collaborative student talk. This would have helped the children be more willing to volunteer answers.	I am going to discuss the importance of teacher subject knowledge with them and give examples of the depth of teacher knowledge I have seen here in Japan. I am also going to give them the paper ratio activity.
Good lessons are engaging, maintain student interest and guide them/push them to solve problems on their own	Group work can be extremely helpful to push student thinking and help them solve a challenging problem	Humor and engagement are extremely effective pedagogy.	The importance of engaging the students in the task, having them construct meaning on their own, and connecting with them through humor.
I am wondering why the teachers are not doing summaries with the students. Some of them have not even left time for reflections. I am wondering if this is a practice they use in Japan when teaching through problem solving. I want Dr. T to discuss the difference between TTP in Japan and U.S..	We have been discussing the idea of a messy lesson. Dr. T said that after a lesson plan, we should forget it so we can be sure to listen to students. Also, as we plan we should come up with different anticipated responses to be prepared but continue to listen and forget the plan if it is not adjusting. I would like Dr. T to discuss more math content. For example, the first day, he gave us a scope for geometry. I really liked that.	The group work seemed to engage the students more. Also, I liked how he connected irrational numbers to real world tangible items.	We cannot get caught up in lesson plans and remember to be flexible.

IMPULS 2017 Reflections Day 6 (Monday June 26th) (Responses)

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When you prepare for one lesson you study ten but when teaching you give up nine and teach one.' Paraphrased but this very much reflects my experience as we adjusted how we were approaching lesson study this year. 'If you watch video it looks beautiful but in the classroom you see the problems children have so lesson study needs to be live.' Again, this reinforces my understanding. When you start teaching forget your lesson plan otherwise you stop seeing the children and focus on the plan. When you are bored as an observor the children will be bored. Lots of people new to lesson study only watch the teacher. Need to watch students' faces and activity. Post-lesson discussion is more than a debrief. Lesson study is about learning from observation and discussion. Moderator 'students role is important - I would like some more guidance on how this should be managed.	The mathematics was revealed through the activity and the children made sense and explained it better than we had managed in the pre-lesson discussion which reflects on the power of the image and context used to explore the mathematics in this lesson.	Good lesson is when the teacher works hard to learn what the children are doing /thinking /understanding. Not a good lesson if the children have to work hard to know what the teacher wants them to do.' Many teachers are uncomfortable with starting from the children if the children do not give the desired response within a short space of time. Today's teacher was comfortable with doing this but it made many of the observers uncomfortable. The activity necessitated group work - when one group had a solution they were primed to share later and the fact a group had found a solution was used to encourage the others.	Many of the thoughts above!
Lesson study is a way to see things through the eyes of children. In order to improve your teaching you must watch lots of research lessons and then make adaptations to your own teaching.	Looking at higher level math that I'm not normally familiar with helped push my content knowledge and thinking.	Not giving up when the students aren't understanding at the beginning of a lesson and continuing to ask the hard questions that forces students to look deeply at the math.	See above about lesson study.
From the discussion reviewing past 3 lesson study, I learned the importance of looking at student work, and that you could watch a video of teaching and have a completely different assessment than if you were in the room observing students.	The teacher today spent a lot of time setting up the problem. It got students thinking and was open ended. It is not always very easy to come up with an open ended way of investigating every day items in all middle school math standards.	Elicit different student thinking or ways of seeing the problem, make subtly suggestions to prompt students to get to the answer.	I have a question of "how can we motivate the problem? How can we guide students to the problem to get their buy in?"
It was really good to see today's lesson as a point of contrast for those we have previously seen. Possibly the biggest difference was there seeming to be less pressure for performance.	Continue working on the introduction to the lesson. It is critical to provide that "hook," but all of the intros we have seen go a bit too long, today's included.	Today's teacher also appeared to have a good rapport with his students. This is an important piece of being able to listen to them and follow where THEY lead.	Get over working in a silo. Open up you teaching to critical eyes. It is hard, necessary work, but important for us as teachers to continue growing.
During our discussions yesterday, Dr. Takahashi explained the point that research lessons provide us an opportunity for us to learn from our students. He encouraged us invite many teachers to the research lesson so they can observe the students' thinking. I agree that during LS lessons, we have insight on student thinking that is difficult to capture from watching a video or by reading the lesson plan.	Today's lesson provided us the chance to see a well prepared and well taught lesson. From the engagement piece, to problem solving, to student interaction, this lesson flowed smoothly and was very effective. It showed how crucial the collaborative team work was in problem solving which had the students learn and solidify mathematical content more effectively than just listening to the teacher or by looking at diagrams. The students took ownership and valued the content that was taught.	Today's lesson showed us the importance of effective planning. During the lesson, the teacher had anticipated the students' thinking and navigated through the material by allowing the student work to lead and develop the rest of the lesson. By deemphasizing board writing and by focusing on manipulatives and group work, the teacher was able to meet the goals of the lesson and I feel that students are very fortunate to be able to learn in this way.	I plan to share the structure of this lesson with my colleagues. To me, this is an excellent structure that can be adapted to other mathematical content areas.

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One key reflection during today's discussion was that despite knowing what isn't working well, in terms of actively supporting pupil learning, these aspects are not necessarily given immediate focus for change. The examples cited today by Dr T and the group were observing children rather than the teacher and secondly, that the teacher must get out into the class and genuinely listen to children's responses and in order for children to be responding, they must be working in groups or pairs and the teacher not be talking. These are fundamental to effective assessment, as agreed in the group, and yet we had evidence this wasn't happening nor apparently changing. I'd like to know about why. I found Dr T 's Japanese quit every helpful here: 'Good lessons are were teachers are working hard to understand what the children are thinking. Bad lessons are where the children are working hard to understand what the teacher is thinking'. Linked to this would be the discussion around teachers following their lesson plans too rigidly. The discussion centred around the thought that perhaps they were conscious of being watched and that's why this occurred, If, however, the point of lesson study is to develop the most effective practice, then a teacher who deviates from the plan and can then explain why in the post lesson feedback would be demonstrating a key process described by Dr T: 'Plan for ten lessons and then forget nine' and 'Plan and then forget your plan'.	I thought the use of real books in today's lesson and paper to connect a very challenging mathematical concept was very simple and effective. I didn't, however, feel the time the teacher spent (20 minutes) questioning the students without response enabled the pupils to think about the mathematics or connect where he was going with their existing knowledge. I feel hands on exploration using the paper and talking to each other would have given all students opportunities to activate their knowledge and pose suggestions around ratio using their own proof and geometric understanding.	There was a great deal of discussion in our group the morning around the conceptual journey from concrete manipulative through to diagrams and then abstract recording. I think this is an area where the three countries represented in the room are at very different places in their experience and understanding. There seems to be a group who see the value of the journey whilst others are focusing more on solving the problem and then explaining thinking using the diagrams. My own studies indicate that this is a very significant area for further exploration internationally and I'd like to know more about how training in Japan is using research from people such as Bruner regarding this approach.	Teacher talk vs. pupil talk with each other coupled with effective teacher listening (I may be repeating myself here). I also appreciated aspects of how Dr T managed our discussions where he didn't offer opinions regarding our contributions, I will use this more in my own work with adults.
1) board work is a significant feature of math instruction in lesson study. It was interesting watching a lesson that did not incorporate carefully-planned board work in the same way. I wonder what the lesson would have looked like of he had followed a more typical TTP lesson structure.	1) group work is a major piece of the learning process, and this has been notably absent in nearly all of the lessons we have seen previously. It is important to include partner talk and work in small groups in order to maximize student understanding because the students can gain so much insight from their peers that they may not receive from solely teacher-driven instruction.	1) I want to be more aware of the ratio of teacher talk to student talk. Here most of the lessons go through the teacher, and I believe that I have a tendency to have a louder voice in the lesson than my students. I want to develop students to think independently and problem solve in their own ways rather than tell them what to do/what I think all of the time. 2) I want to think about the ways that I can boost student engagement through having multiple ways to respond. Think Pair Share and Turn and Talks are great, but I want to utilize other strategies to give students to develop their thinking using multiple modalities.	I think the opportunities for student response needs to be a larger focus this year. We have talked about reducing the ratio of teacher talk to student talk, as a school site, and it might be strategic for us to come at this idea through the lens of lesson study/TTP this year.

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Teacher responsiveness to students during public lesson: Important not to create one pathway in lesson plan, but rather multiple, flexible pathways so teacher is prepared to be responsive to the students in the room and the direction in which the lesson/discussion authentically moves Teacher should begin with his/her lesson plan then throw it out (literally put it down and away) so s/he can be fully responsive to students during the lesson and truly hear student thinking/analysis	The strategic-ness of creating math problems that require groupwork to solve, and thus how groupwork is utilized as a means to an end as well as for the collaborative aspect of learning.	What is a student-centered lesson? If teacher is building off student responses but it is the responses of the quickest students or students with the most understanding, how is this student-centered? Seems critical for the teacher to survey the class, identify patterns and outliers in student work/thinking, then strategically craft the narrative for the class conversation based on whole class data, not the student(s) shouting out or finished first.	Rethinking groupwork (so the intention and purpose is concrete and strategic) and thinking more deeply about what a student-centered lesson looks like in TTP.
From Dr. Takahashi's morning session: - during observation look more at the students actions and reactions, then study the teacher planning and implementation that is occurring; - for deeper conversation in post discussion - what is the breaking point in the lesson? Where did students become disengaged? What was the climax of the lesson? (Thinking through a story analogy here, where the lesson plan is the storyline) - as the planning team you are the bank of content knowledge, while teaching one lesson you are studying 10 lessons. Use this knowledge to establish clear goals and anticipate student responses as much as possible.	- I thought the 4 boys that presented their solution did a nice job working together. One initially asked the others what the answer was but instead of just telling them they explained again their thinking to the other student. - The visual was really needed to see where the the square root of 2 was coming from.	- the energy in the room shifted positively when students were given time to work in their groups. This group work I thought demonstrated the power of planned and purposeful student collaboration. - From Dr. Takahashi's morning discussion: a "good" lesson is one in which the teacher is working hard to figure out what the students are doing, and in a "bad" lesson the students are working to figure out what the teacher is doing.	- see question one and three above for ideas/insights that will be shared. - other thought: in our post discussion conversations I am wondering if there is still confusion about math through problem solving verses the lesson study process? It seems there are a lot of discussion points that are based on observations around teacher action/moves but then not connecting those observations to the lesson plan or even the broader plan which includes the previous learning.
When you do lesson study you study 10 lessons. When you teach, you forget about 9 of them.	When planning a lesson think about what you want students to see, understand, and do. In a good lesson, teachers work hard to understand student thinking. In a bad lesson, students work hard to understand teacher thinking - "what am I supposed to do?" In Japan elementary students have four 45 minute math lessons per week.	Board work isn't always planned out. For example the 9th grade lesson was more about group work and hands on discovery.	
The discussion and the learning came from students. There were moments where the teacher could have interfered but he didn't. It was great to see his confidence in his own abilities and his students and how the lesson progressed to achieve its goal.	The fact that he chose to do this lesson though children had not learned about ratios was challenging and a very brave decision. The fact that children managed to solve the problem by themselves in 50 minutes, shows a deep mathematical thinking in the lesson. I really enjoyed this lesson and have learned a lot from it!	I loved his lesson plan in the sense that there was no real plan. The plan was for students to think, reflect on what they knew and use that information to solve the problem. Being patient and giving the children the confidence to speak in a crowded room was something that I really enjoyed being part of.	Being more creative and asking children to work in groups. Even the hardest of the tasks can become easier if we all work together.the
Na	I appreciated the connections between the books, paper and the characters and how it gave real context to irrational numbers.	I appreciated that there was group work.	The idea that context is important for math so it does not seem so abstract that it feels unimportant in real life.

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Lesson study is much better than watching a lesson on video because it allows for the observer to see the students' reactions to the lesson. These reactions include students' faces, their work in their notebooks, and their conversations in small groups.	A good lesson is one where the teacher is working hard to figure out what the students are thinking. A bad lesson is one where the students are working hard to figure out what the teacher is thinking.	You don't always need a notebook to record your ideas. Sometimes manipulative alone are enough! In this lesson, students did almost no writing, but were able to come up with a variety of ways to solve the problem.	I would like to pay more attention to planning tasks that are more intentional about collaboration. For example, creating tasks that necessitate four pieces of paper to solve the problem, as was carefully planned in this lesson.
It was clear that engaging in lesson study helps teachers to be thoughtful and considered in their everyday practice	A carefully considered task can enable effective group work by making it necessary to work together to complete the task. Importance of snappy and engaging intros.	Putting words in children's mouths vs. building on their ideas/thinking. Good teaching = teacher trying to work out what children are thinking not the other way around.	A goal for group tasks - Task that need collaboration, not just group work for the sake of it.
Not every lesson is a part of lesson study, however the teacher has a plan in mind. In this case the board work was not planned out and was based on what students came up with through their collaborative work.	Students need to justify their their thinking, teacher remains	Grouping students needs to be intentional. They worked in groups of 4 because they needed 4 pieces of paper.	Deepening our content knowledge will be a priority.
Students involved in lesson study are part of a cultural phenomena that they become accustomed to. Without the rituals and prior warning, and even with it, students can easily feel very put on the spot by a research lesson, so it's critical to properly prepare students for the crowds of onlookers, and allow for an icebreaker before diving into the mathematics.	A task that requires all members of a group to solve creates an authentic need for group work. A well designed lesson can jigsaw mathematical content or representations in this way.	I was fascinated by the 32 minutes of wait time given for students to express the characteristic of similar sizes for the various books. The discussion we had about whether this was well spent or wasted time was even more illuminating: We are deeply uncomfortable with discomfort, but the teacher was habituated to it.	All of the above.
The continued idea within lessons to relate mathematics to the students' lives. In this particular lesson the idea of irrational numbers and where students can see them in the real-world	There was an example that the teacher was asking students to visualize 1.5 times as long. I thought this was very clever and it ties back to making math meaningful to students. Just a very simple way to get students to think about multiplication in a tangible manner as opposed to something as abstract as unitless numbers.	There was a point in the lesson where the teacher asked a question and in which the students did not have a response. It was a long time before the teacher got the answer that he was looking for. I like how the teacher did not give a hint or another question untill the students were able to answer the first question. I think it builds a culture in the classroom that when questions get difficult the students need to stuggle to find the answer themselves and that is is respecting the process.	Just the continuing theme of very simple ways I have notices throughtout the trip of how teachers attempt in each lesson to make the math meaningful within their students' lives.

IMPULS 2017 Reflections Day 6 (Monday June 26th) (Responses)

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Through critically observing others as well as taking on the role of the lead teacher, lesson study develops the teacher eye to observe the lesson while simultaneously teaching it. This is no easy task. I am excited to continue my work with lesson study in order to develop that eye in my own classroom.	We have talked a lot about creating a need for group work. Often times teachers put students in groups just to change up the way they are collaborating. However, if there is no need, we have seen that it is often ineffective. What I loved about the ratio lesson was that there was an actual necessity for students to work together in groups in order to solve this math problem.	If teachers do not pay attention to equity in the classroom, the inequities imbedded in a society will be perpetuated. In the ratio lesson, the majority of the participation came from the teacher and a few male students, and much of the validation I saw the teacher give was to the boys. Now I say this with the belief that this teacher honestly has the best intentions, but not being thoughtful about how we create equity in the classroom leaves a lot of room for inequality to persist. I believe it is our job as educators to plan with equity in mind the way we seat students, the way we provide opportunities for all voices to be heard, and the way we foster a sense of competence and confidence in all our students.	Learning is messy! The process of learning entails a productive struggle through the mess of learning and coming out with a clear and comprehensive understanding. It is the teacher's job to guide the students through that mess. I think really listening to students and allowing this mess to occur is scary because we as teachers fear that we don't have the content knowledge or the skills to tease out the learning or guide students through that mess in order to reach clarity and achieve the goal or objective. How can we help teachers become comfortable with the discomfort that comes along with allowing learning to be messy? I would love to make this our new school theme to research during lesson study. Some initial ideas are focusing on developing the skill to anticipate as many student responses as possible so that during the lesson, we are not as caught off guard. Through lesson study, we will also become more comfortable with the content knowledge and pedagogy to be able to deviate from the lesson plan in order to really allow learning to be messy and listen to our students. Another favorite insight from today was that a good lesson is when teachers are working hard to figure out what students are thinking. A bad lesson is when students are working very hard to figure out what the teacher is thinking.

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Through critically observing others as well as taking on the role of the lead teacher, lesson study develops the teacher eye to observe the lesson while simultaneously teaching it. This is no easy task. I am excited to continue my work with lesson study in order to develop that eye in my own classroom.	We have talked a lot about creating a need for group work. Often times teachers put students in groups just to change up the way they are collaborating. However, if there is no need, we have seen that it is often ineffective. What I loved about the ratio lesson was that there was an actual necessity for students to work together in groups in order to solve this math problem.	If teachers do not pay attention to equity in the classroom, the inequities imbedded in a society will be perpetuated. In the ratio lesson, the majority of the participation came from the teacher and a few male students, and much of the validation I saw the teacher give was to the boys. Now I say this with the belief that this teacher honestly has the best intentions, but not being thoughtful about how we create equity in the classroom leaves a lot of room for inequality to persist. I believe it is our job as educators to plan with equity in mind the way we seat students, the way we provide opportunities for all voices to be heard, and the way we foster a sense of competence and confidence in all our students.	Learning is messy! The process of learning entails a productive struggle through the mess of learning and coming out with a clear and comprehensive understanding. It is the teacher's job to guide the students through that mess. I think really listening to students and allowing this mess to occur is scary because we as teachers fear that we don't have the content knowledge or the skills to tease out the learning or guide students through that mess in order to reach clarity and achieve the goal or objective. How can we help teachers become comfortable with the discomfort that comes along with allowing learning to be messy? I would love to make this our new school theme to research during lesson study. Some initial ideas are focusing on developing the skill to anticipate as many student responses as possible so that during the lesson, we are not as caught off guard. Through lesson study, we will also become more comfortable with the content knowledge and pedagogy to be able to deviate from the lesson plan in order to really allow learning to be messy and listen to our students. Another favorite insight from today was that a good lesson is when teachers are working hard to figure out what students are thinking. A bad lesson is when students are working very hard to figure out what the teacher is thinking.

IMPULS 2017 Reflections Day 7 (Tuesday June 27th) (Responses)

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Thinking about the groupwork from yesterday and today, I'm still working on the idea of there needing to be a "need" to work together, which means that problems must be structured to create this need.	I want to explore more how to effectively use a set square.	I want to have my kids do more construction of polygons using appropriate tools.	See above	
The intention behind the teachers grouping students into groups of 4 for the lesson rather than just grouping and the intention to have a well-thought out introduction to engage students even though it ended up being 32 minutes. I think this was not expected and happened because the 9th graders were shy by our presence and their age.	Today the principal knew and understood exactly what was being taught, the goal of the school and what he wanted the students to improve in. He took ownership of student learning. To me, that is shared leadership, collaboration and an investment in his teachers learning and progress.	I had never seen or used the square set before. This was interesting and helped me see the importance of explicitly teaching students how to use tools for math.	I thought the math survey was important to see what students were thinking/feeling about math. I think as teachers we need to survey our students more about how they think/feel across subject areas.	
Dr. Fujii explain that as observers we should be "like air." In other words, we just need to strictly observe. We cannot help or probe students as outside observers.	The lesson from the previous day was very intentional when choosing to have groups of four. The amount of students in each group matters and depends on the goal of the lesson. There is no benefit to group work unless there is a specific purpose for them to work together because then they will just copy each other.	Dr. T explained that a teacher's goal is to figure out the students thinking. If students are spending the whole time trying to figure out the teacher's thinking, then it is an unsuccessful lesson.	I will be sure to discuss the told of the observer as a silent participant. I also liked how the observers made comments on 2 different pieces of paper. One color was things they liked and another color was for questions, comments, or critiques. Maybe our lesson study team could use something like this.	I would like Dr. T to lecture us more. He is an expert. I do not feel like listening to comments/questions from us Impulse participants is an effective use of our time.
The person delivering the Expert Comment is always very well informed and has an exceptionally high level of subject knowledge. This was evident today when he mentioned other methods for completing the parallelogram that I wouldn't have considered (e.g. Splitting off the right angle triangle and using point symmetry).	In my opinion the way in which geometric construction is taught has far more depth than in the UK. In the U.K. It is typical for children to be presented with a single list of instructions which they then have to follow to create a specific shape. By allowing children to explore and come up with their own ways, linked to the properties of the shape, it enables children to "create" maths and to view maths as a creative subject rather than lists of procedures to memorise.	This lesson was the most collaborative that I've seen so far. The children begun by working had independently but then had to share their ideas in their groups. It was a shame that we didn't get to see the next lesson as I would be interested to know how well their group discussions went in terms of determining the method they saw as "best" and how the children defined "best". From what I could gather some groups were arguing with each child convinced their own method was best and perhaps not listening to others. It would have been interesting to have a transcript of each group's discussion. It was also the first time I'd seen a teacher ask children to come forward for further instruction if they were unsure of what to do. This is a strategy I have used in my own classroom which I find effective. I also thought the use of the diagram instructions on the wall to explain how to use set squares was also effective and helped children gain confidence.	I am definitely going to take this learning back and ask my adult students how many ways they can think of to construct a parallelogram. I am also going to introduce the term construct and explain its significance. In addition to this, I am going to get my students to create a giant parallelogram poster (like the one we saw) but with their reflections written on each parallelogram. This will be their directed task to do between lectures.	How long do teachers spend planning typical lessons? Are children with special educational needs included in mainstream schooling?
It is important to have clearly articulated your thinking behind the task and instruction for a research lesson. Without it, it is difficult to show/articulate your goals in action.	Group work can involve procedural tasks (such as students looking at the picture instructions for parallelogram drawings on the board), but then require students to extend or find other pathways together (students were asked to find other ways to draw parallelograms in groups)	Countdown to end student think time by holding up fingers, instead of saying it out loud. Last week, someone in the post lesson discussion said that when the teacher is talking, students cannot think. So I realized that it's worthwhile try to find (even small) ways to minimize my talking.	Teachers do a really great job opening the lesson and introducing the problem, there is sometimes a trade off with the amount of time left for student exploration and thinking towards the end of the lesson.	How should we prepare for the unexpected in a research lesson?

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--from the schools post discussion another accountability tool for observers - the pink and blue paper strips; --in planning cognizant of pacing; --group work needs to be valuable and planned as such (I also mentioned this in reflection 6). --appreciated Dr. Fujii's final comments in recapping what lesson study is...	--compass tool was not widely used by students and more difficult for students to visualize how the tool could help. Seems looking more deeply in to students previous learning of this tool would be beneficial. --want to investigate further how we are teaching use of mathematical tools and with what degree of precision.	- poster use was a great tool for students to accesss prior knowledge about the use of tools, but as was brought up in post discussion wondered if this limited their thinking? -appreciated (and will share) how the teacher spoke to students at beginning of class when asking them to not shout out..." speak to yourself and to your heart" was translation.	-honestly, everything above! Plus details of Dr. Fujii's recap of lesson study. - I've appreciated these reflections and think I'm going to organize all of my "take always" in a similar manner. I'm also going to add a culture piece, wonderings, and next steps...	- specifically from the lesson: ~two students brought up use of grid paper as a tool and wondering why not option? Or at least talked through more. It appeared on group white boards but early told not a tool for the lesson. ~what is/ has been expectation for students writing their thinking in their notebooks? Students seemed to struggle with this. ~in 4 of 8 groups once the white board was introduced not all students in the group participated. Thoughts on this being skipped and moving directly to whole group discussion after the initial group work? ~ would like to know more about students voluntarily going to teacher for extra help - does the teacher feel that this is successful? Data to show? -general question: use of notebook seems to have varied during our observation. Today students were using separate sheet (perhaps to use tools), Thursday's lesson there wasn't a notebook, often students seem to copy everything from board to their notebook and show less of their own thinking...would like to hear more about notebooks and their purposes in Japan.
--from the schools post discussion another accountability tool for observers - the pink and blue paper strips; --in planning cognizant of pacing; --group work needs to be valuable and planned as such (I also mentioned this in reflection 6). --appreciated Dr. Fujii's final comments in recapping what lesson study is...	--compass tool was not widely used by students and more difficult for students to visualize how the tool could help. Seems looking more deeply in to students previous learning of this tool would be beneficial. --want to investigate further how we are teaching use of mathematical tools and with what degree of precision.	- poster use was a great tool for students to accesss prior knowledge about the use of tools, but as was brought up in post discussion wondered if this limited their thinking? -appreciated (and will share) how the teacher spoke to students at beginning of class when asking them to not shout out..." speak to yourself and to your heart" was translation.	-honestly, everything above! Plus details of Dr. Fujii's recap of lesson study. - I've appreciated these reflections and think I'm going to organize all of my "take always" in a similar manner. I'm also going to add a culture piece, wonderings, and next steps...	- specifically from the lesson: ~two students brought up use of grid paper as a tool and wondering why not option? Or at least talked through more. It appeared on group white boards but early told not a tool for the lesson. ~what is/ has been expectation for students writing their thinking in their notebooks? Students seemed to struggle with this. ~in 4 of 8 groups once the white board was introduced not all students in the group participated. Thoughts on this being skipped and moving directly to whole group discussion after the initial group work? ~ would like to know more about students voluntarily going to teacher for extra help - does the teacher feel that this is successful? Data to show? -general question: use of notebook seems to have varied during our observation. Today students were using separate sheet (perhaps to use tools), Thursday's lesson there wasn't a notebook, often students seem to copy everything from board to their notebook and show less of their own thinking...would like to hear more about notebooks and their purposes in Japan.

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This lesson study reiterated what we have learned at Argonne, that group work is good but it is still imperative that you have a way to assess what is happening in those groups.	I was happy to hear what the knowledgeable other had to say about constructing parallel lines with triangles. I want to play with that a bit for better understanding.	I must explicitly teach how to construct with math tools if it has not yet been taught. For example, set squares, or measuring with a ruler.	Emphasize that lesson study can be done across disciplines with a common school theme, it need not be math. Our school needs to continue working on better development of our focus.	
The comment that the knowledgable other made : 'An observer is like air! The data collected is for research purposes and it's important for teachers/observers not to interfere with the data.'	The use of rulers, compass and protractor correctly can be challenging and it's not an easy question for some children. This task highlighted the importance of having similiar tasks from primary throughout secondary.	The teacher asked children that didn't understand the task to go to his table. He explained the task further. One of the children said "Aha..." The task highlighted that children are individuals and different pedagogical strategies may be use different with different children.	Encourage children to use the set of triangles only to draw parallel and perpendicular lines. I feel children should have more 'hands on' activities in geometry.	Is there a reason we have not seen 2-3 lessons on the same topic in lesson studies? E.g. Would it not be useful to see what happens on the next lesson? Having the first lesson study, how will the teacher work on the next lesson? How will he deal with children that did not understand how to draw a parallelogram? What about with those who knew how to draw a parallelogram? Would it not be better for research purposes if this data was collected also?
Teachers observing need to have the plan in time to work on the mathematics themselves prior to the live lesson. When you observe make sure you are like air. Just observe and collect evidence. To collect evidence from the children assign people to a group ; they can then capture exchanges and responses which will enhance the post-lesson discussion. Include everything in the lesson plan that will be part of the lesson e.g. any sheet that will be used. This will aid observers to understand the problems being set which they can then solve for themselves. Pacing is difficult in live lessons.	Construction of shapes is not taught in England at this age but it really makes the children use and apply their understanding of the properties of the shape. Very powerful way to do this. The discussions we are having often combine observations about lesson study, with observations about how the mathematics was taught and I have realised that it is important to separate these out otherwise it is possible that CLR as an approach to PD could be judged by the teaching approaches observed and these are two different things. One of the things I will need to do is separate my thinking for my return to reflect the different elements.	Collaboration is best when there is a necessity in working as part of a group. 'Just in case' support and ' just in time' support - would like a bit more on this to check out my own understanding.	Everything about lesson study above and shape construction.	I am still wondering what the professors are hoping to see in a lesson and what other types of maths lessons take place in schools (fluency) plus what children typically do out of school so that I have a context for what I have seen.
The importance of strategically planning groups. Not just grouping students together to group them, but aligning it with the task and making purposeful groups. Students shouldn't work in groups just to work in groups or they won't see the usefulness in it.	The importance of learning the mathematics and solving problems ahead of time to anticipate all possible solutions.	I liked how in the lesson on Tuesday we saw more student to student interaction. This changed the dynamic of the classroom from being teacher directed to student centered.	Purposeful groupings of students as aligned with the task.	I'm wondering if someone can talk to us more about how research lessons are selected. I have seen a few during this trip where it seems to be an application of knowledge or culminating task and I'm wondering why this is. On planning teams I have been on we have always done something that is "new learning" not just an application of skills. I also am wondering if it is possible (or for next year's participants) to have copies from the Japanese textbook so that we can have a better idea of where the lessons are coming from.
I have been curious about the monthly and yearly summaries of work and so my aha was seeing the book they had made. I took some photos of pages.	I want set squares for our school!	I appreciated how quickly the students transitioned and moved seats around. I appreciated his review and hook.	Set squares, sharing lesson study work...	How do they decide who does the lesson? Only advanced teachers? Any recommendations about using the LS structure with early novice teachers.

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1) the lesson should meet the students where they are at. Rather than rush to get through all of the components to get to the summary isn't supporting student learning. I really appreciated how this teacher opted to stop the lesson in the middle and pick it up the next day, after realizing the students needed more time.	1) group work for the sake of group work is a waste. Students need accountability structures built in to ensure that everyone is actively participating and contributing to the discussion and learning. 2) students may look like they understand and know what they are doing, but it is very easy for a child to manipulate a tool and imitate understanding. Teachers need to give students lots of practice with mathematical tools to develop their understanding of the uses and the need to use them.	1) I want to include more opportunities for students to engage in meaningful discussions with their peers, but need to incorporate structures that will give all students a chance to participate and hold students accountable. 2) I want to be more precise in my instruction about mathematical tools. I want students to see the value in different tools and develop autonomy in knowing when/how to use them.	I love the idea of developing a school-wide norm around mathematical tools. Perhaps we can analyze and assess various tools/what they are used for and create a master list/materials center so teachers can draw on the correct tools to meet their students' needs. We can also monitor which tools are appropriate for students at different levels and provide them opportunities to engage with the tools that will best support their individual learning goals.	I wonder about whether the teacher felt like his students were grasping the mathematical content in today's lesson. If not, what can he be doing to better support their needs?
I'm thinking about my role as supervisor and how to best support strong lesson study practice in SFUSD - I'm considering doing a full run-through of a public research lesson for all new MTs (new MT "retreat"), creating a digital binder of tools to support each step of the process, and providing much clearer and more detailed expectations for each step of the process. I'm also considering the role of the charismatic leader and wondering how dependent lesson study is on the individual leader, and how we might engrain the process into the core fabric of schools and our district without such deep reliance on the charismatic leader - or perhaps we give in and directly pour resources into these select individuals :)	The critical nature of effective pacing, so students can reach the AHA moment (climax) of the lesson, then have time to grapple deeper, then have time to consolidate learning through the summary and reflection. The math content is not fully accessed until this full cycle has occurred, and without effective, strategic pacing students can be left with greater misunderstanding, gaps in understanding, or in frustrated confusion.	The importance of having a rationale for groupwork: why groupwork above individual work? why one group size over another? why particular group roles? what must a group accomplish together? how will the group be expected to do so? what tools and strategies are taught so this time is maximized? Groupwork is a tool like any other requiring strategic thinking and planning, and enough coaching so students are able to utilize the time to extend and push their own thinking beyond what they can accomplish working alone.	I've started working on the resource kit we will provide...	
Through my discussions with other group members I have reflected far more on the purpose of lesson study. This was aided greatly by Dr Fuji's input yesterday. I have learned a great deal about how to improve my own practice by observing the lessons and the way the post lesson discussions are conducted. I'm aware, however, that due to the way observations are conducted traditionally in the UK, I've not always focused enough on the process of the lesson study itself but rather the content of the lesson. Now I'm beginning to really be able to consider what lesson study really is and how it underpins a teacher's philosophy of learning and feedback and I look forward to observing today's lesson with what I've learned in mind.	The level of subject knowledge the teachers have due to their constant research and preparation is what I aspire to for every teacher I support in the UK. The depth of understanding and knowledge of how tiny the steps are to move forwards is so evident in every session we've seen. I've also recognised how well teachers understand the content of the curriculum at other stages of children's learning and therefore they are not merely focusing on their year group but rather the 'bigger picture'.	There is a genuine awareness of how important activating and connecting student's prior knowledge is and yesterday, in the post lesson discussion, the point was raised that children often appear competent when something has just been taught but, given time, often forget. This is a challenge teachers are aware of and the use of the posters was a clear strategy to address this. The teacher was very clear that a difficulty he experienced in yesterday's lesson was accessing the children due to the number of people in the class. He wanted to listen to as many children as he could and was clearly frustrated that he hadn't achieved this as it's a goal for him. Again, this is a practice I'm working hard to support teachers in valuing in the UK as many see talking as teaching but not listening. 'Be the air around the children' Dr T said yesterday.	Putting together clear routes of progress in each area so that all staff (including support staff) understand the whole journey and have a developing depth of knowledge. We have a significant issue with confidently taught inaccurate knowledge which remains unchallenged in the UK. Dr F mentioned that in a session he had observed in England, the students were able to work particularly effectively in small groups. I believe that culturally we have the potential to do this extremely well and I've seen children adapt immediately in many cases to this approach with great results. I would like to reflect with teachers around the many different ways of engaging students and providing rich opportunities for teachers to listen.	How on earth will I begin back at home. Particularly as an independent consultant working alone and dependent upon schools paying for all support, I need to consider carefully how I will offer this approach and ensure that schools have sufficient knowledge a understanding of LS to embrace this very new approach. I'm also aware that many schools use the phrase 'lesson study' and it is a different form of collaborative professional development.

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This research team provided a very well prepared lesson plan. I appreciated their inclusion of providing student surveys in order to help guide their research theme.	This teacher reminded us the importance of making time to be able to assess student work during independent work time. It was helpful to see how his individual conferencing helped his discussion component of his lesson.	During the post lesson discussion, I found it very helpful to keep in mind the idea about if you want students to construct, you need to allow them time to practice.	The flow of the Define, Examine, Construct method.	None
Changing or altering curriculum doesn't always result in a lesson that is better than the text book. Each lesson should have a clear purpose and relatable context.	Students need to be comfortable using tools if they are to select them when solving problems. Teacher should frequently model usage of these tools.	Deep content knowledge is pivotal in defining the purpose of a lesson.		I am interested in lesson study in the primary classrooms. I am curious about the structure and flow.
The use of student surveys that teachers use to gather information on students. The survey for this lesson was to see gauge how interested students were in mathematics and if the found it enjoyable to hear other students ideas.	I liked how the students were using the box set to draw parallelograms. It made me think about the importance of students constructiong geometric figures and how important this is later on when students are working with more abstract geometry ie. proofs.	There were a lot a different ways the teacher used to gauge his students throughout the lesson. He used hand signals many times but he also asked a question and gave a five second process time for students to formulate their thoughts before he took any responses. This is great because it gives more students access to the question and it takes away from the myth that speed is important.	Sometimes group work is not always helpful. From the groups that I observred they were not sharing ideas how to make their parallegrams. I think we need to be very carefull when putting students into groups and make sure that the task is group worthy.	I wonder if this was a group worty task?
N/A	It was an interesting to see how thoughtfully the group task had been designed. Not just for the general benefit of learning to work together but because the task needed students to work together to be successful.	The starter has led me to consider the merits of giving children time to think about things so they can get to where they need to be independently in contrast to using information and questions to guide them.	That group work should only be used if you can justify how it will support learning.	N/A
During this lesson, some observers were helping students. The final commentator brought up the importance of being a neutral observer. Through that observation we can get a clearer picture of what happened and what we can learn from. If we are helping students, that is an unrealistic perspective of classroom learning. Furthermore, every teacher should stay with one group the whole lesson to get a full picture of their learning from start to finish. We need to not only take data on what students are doing, but we need to use that data, otherwise it is not useful.	The problem in this lesson was to make parallelograms for their friends that were coming to visit. The task was to use tools to prove they were accurate parallelograms. However, this problem does not necessitate accuracy. There is no mathematical or situational need to be accurate. Why not just draw some pretty parallelograms for your friends? The only desire to do so would be for the sake of being accurate and to make the gift look nicer. To me, that is not a real need. I believe one of the most important aspects about teaching through problem solving is creating a need to solve the problem to foster persistence in solving.	If there is no need or structure to group work, the group work can waste valuable learning time. In this case, many students in groups reverted back to freehand drawing and working independently rather than trying to work together. Perhaps if they were given roles or if there was a need to work in groups, this time would be more productive. I have had students engage in wasteful group work many times before and I never thought of it through this lens. I am excited to carry this new perspective with me and use group work thoughtfully and intentionally to serve the mathematics or the learning task at hand.	We should not ask students to work in groups without a purpose that will benefit their learning.	I am wondering how to best document the observers insights in order to use that data to inform our learning and our future lessons.
I like how they had the observers write their learning/observations on large "post-its" and then they categorized them up on the board for the post-lesson discussion.	I was struck by the difference between how I teach shapes in my classroom and how little importance I put on drawing things correctly and how necessary that is to truly teach students the properties of shapes.	I thought it was interesting how he had students self select into a small group if they needed help and also how quickly he worked with them. Whenever students felt like they got it they left to get started on their own work. He didn't work with the group for more than 4 minutes.		
Whenever you revise a textbook task, you need to have a strong rational. For example: one of the lessons we observed was worse than the textbook lesson.	Students must have a conceptual understand and then practice the skill. For example: set squares - know the purpose and have a conceptual understanding and then practice drawing set squares.	Students used whiteboards to share their group thinking to be used the next day during class discussion.	Developing a research theme based on our school's mission. Also, really being intentional about which lessons we are revising and the rational behind the revisions.	

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	Effective use of tools for elementary students is a great challenge - protractors, compasses and rulers require a huge investment of time and very explicit instruction in order to provide students with the basic understanding of how the tool was intended to be used. This training seems like it could stand at odds with a discovery or problem solving lesson.	This lesson's research theme and lesson flow more closely resembled the emphasis my Bay Area school community places on partner work in order to promote learning. This lesson did not exemplify productive team work as far as I could tell, and seemed to serve as an example instead of why group work can have many pitfalls, and must not be assumed to be helpful for students.		I'm very interested to know how each tool was introduced initially: discovery or teacher led?
If a teacher deviates from the textbook, there should be a strong rationale for doing so. Observers should stay with the same group of students throughout the entire course of the lesson so that they can track their progress as the lesson unfolds.	Once a mathematics tool is introduced (for example, a number line, a compass, set squares, rulers), teachers should encourage students to continue to use that tool. This helps students to learn through the tool and want to use that tool to help solve the problem in front of them. If students are not comfortable with a tool, they will not use it.	Struggles within a lesson can reveal weaknesses left over from previous grades. If students are not using certain strategies that you want them to use, it is likely because they have not used that particular strategy enough in the past.	Procedural fluency can be used to deepen conceptual understanding. I want to encourage my colleagues to build in a math fluency period somewhere in the day.	

IMPULS 2017 Reflections Day 8 (Wednesday June 28th) (Responses)

What are one or two "Aha's" or insights you had about lesson study?	What are one or two "Aha's" or insights you had about mathematics for teaching	What are one or two "Aha's" or insights you had about pedagogy or classroom instruction?	Please describe any ideas that you would like to share with your colleagues when you return?	Please share any remaining questions or wonderings you have?
Again, the emphasis on carefully choosing the task. The teacher specifically chose the numbers and task to work toward the goal he had for the lesson.	I loved how many different ways there were to think about how to divide fractions.	I'm thinking about when visual models are helpful/unhelpful. There is a lot of emphasis put on double number lines here, which I'm only beginning to understand and incorporate more of into my classroom teaching.	Still thinking....	
A knowledgeable other and a planning team with deep mathematical knowledge are critical for learning from a research lesson. Especially in seeing how skills should or shouldn't be sequenced.	1) Asking students to show their mathematical steps allows for patterns across strategies to better be revealed, which can sometimes lead to new procedures or even properties being hypothesized. 2) Math is the science of numbers: when students notice a pattern that they believe may be a property, it is only a hypothesis until tested on many different scenarios or quantities.	1) Students understanding the context of a problem is foundational to them being able to effectively engage in the mathematics. 2) Double numberlines can support students in transferring their understanding of multiplication or division of whole numbers to fraction or decimal quantities. 3) Double numberlines are also very effective when there is more than one unit or dimension of measurement spoken about in the problem.	All of the above.	I still cannot conceptualize division of a fraction by another fraction myself.
I thought this district wide lesson study was run better than the previous "larger" lesson study we saw due to the fact that there were breaks in the discussion to allow small groups to talk and opportunities for each group to feedback. This created a richer and more diverse range of comments. I also felt that there were more comments from colleagues on the planning team than previously (which made reference back to the planning and decision making involved). I have realised that lesson study is the Japanese form of CPD. I think this is a good thing because it unifies teachers both within schools and across schools both in the local area and further afield. This is in contrast to the UK where CPD courses are run but it is optional to attend meaning not all schools receive the same messages. In addition to this, only one teacher typically attends CPD courses. Here ALL teachers attend which ensures a much more unified approach.	It was fantastic that it was acknowledged several times that division of a fraction by a fraction is a difficult topic and that it combines previous knowledge from all 6 years of elementary school. This shows the depth of subject knowledge held by the Japanese professionals - I'm sure that many U.K teachers think it is relatively straight forward because they simply teach the "trick" for the children to memorise. I also found it interesting that the teacher and the final commentator talked about the importance of recording the interim steps. In this case that was absolutely essential in enabling children to see commonality between the methods. I did, however, agree with the comments in the discussion based on whether the lesson was still quite procedural and questioned whether children had an image in their heads as to what was actually happening. I thought the double number line image shown by the final commentator was excellent and would have been great to incorporate into the lesson.	The comment that struck me most was when it was said that elementary school is not about finding answers but all about creating processes to solve problems. This ability to generalise will enable the children to transfer the skills they have acquired to more difficult problems in secondary school. If children are simply taught to get the correct answer through procedure but they don't understand they process then they will be poor generalisers and will not have the transferable maths skills needed. The emphasis put on the properties of division in previous grades also stuck me as excellent. The children who are secure with these properties can then apply them to the fractions problem. This knowledge will also underpin much of the algebraic work they will encounter later on. I thought that more than any of the other lessons we have seen, this one magnified the importance of prior knowledge the most.	I will give examples to show the importance of process (paying attention to intermediate steps) and generalising.	How long is the training period for teachers? What qualifications do you need to become a teacher on Japan? What level of mathematics have Japanese elementary school teachers studied themselves? (E.g. Equivalent to U.K. GCSE OR A-Level etc).

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considering whether the process if maximized when utilized cross-district or simply different. I see pros and cons to both and am thinking how cross-school work may or may not benefit our SFUSD schools.	As a former 6th grade math teacher, it was mind blowing to see today's lesson and students' ability to access it. The payoff of years of TTP is evident, and students' ability to reason mathematically was far beyond any class I had. I was incredibly impressed with the structure of today's lesson and the teacher's responsiveness to student thinking throughout - what am amazing prototype to witness on the last observation!	I like the approach of "we will fix it together" when a student makes mistakes on the board or in an explanation. I liked students writing their work on the back of the white board then the teacher presenting it so students working independently aren't distracted or tempted to copy.	How we might publish the work of our lesson study teams to support cross-school learning, math department curricular revision, and to strategically "promote" lesson study in SFUSD...	1) Groupwork - In the US and England, we are taught to value group work not simply as a means for students to share ideas, build off ideas, and be pushed/probed about their ideas, but also toward the goals of fostering cooperation/collaboration (negotiating a task together, sharing air time, learning to actively listen, learning to present ideas clearly, etc.) and because we hold a theory that by articulating your ideas orally you will more deeply understand a concept (by putting ideas into words, by sharing them with others, by teaching others - you deepen your understanding). Are these ideas held here as well, or is groupwork seen as useful only when it is the most strategic means of grappling with a mathematical concept? It is clear that in Japan groupwork should be intentional and strategic (not random or thoughtless as often occurs in US classrooms), but it is unclear if it is valued beyond serving as a vehicle for content mastery. 2) Differentiation - we have heard both that tracking does not occur here (same task, varying entry points) and that tracking does occur. We have seen a few schools that separate students into two classes (for a unit or for the year) and some that do not. Given the wide range of academic levels, language levels, and behavioral needs in American schools, many of us are wondering about the role of tracking and differentiation. Can we talk more about re-engagement lessons and Japanese philosophies on approaching differentiation and students missing fundamental math skills?

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So many things today - organisation of the post-lesson discussion when there is a large audience of observers. The most useful lessons seem to be the ones that tackle something difficult as they prompt everyone to think. The observing teachers really need to have grappled with the maths and anticipated responses prior to the lesson; it was clear just in our group that some people thought they understood the diagram (c6) but when asked to fully engage with it by explaining it they realised they didn't understand it. My favourite topic of the knowledgeable other, another view of how they might conduct the final commentary providing direct challenge to the teacher and involving them.	Lots of resonating ideas including: decisions made by the teacher during the lesson are critical; visual images are really useful but sometimes they can be a red herring; the challenge of identifying a meaningful conext; the importance of knowing why you are teaching the mathematics; you have to have the real why you need to teach a topic; if you just teach it because it is in the book the children won't learn anything; students need to make use of the idea of simplifying; if the teachers just use the questions in the book the children will not learn to think; choose numbers that make the learning the mathematics necessary or useful; primary school children need to have the disposition to focus on the process then they will write the intermediate steps and this will allow them to hypothesise.	Again resonating with things we have been working on - listening carefully and choose students who have something important for everyone to hear. Try not to plan lessons that are about what you want to teach! Importance of acting on assessment information in whatever form and supporting children to access the lesson by some intervention before the sequence of work. This fits with our pre-teach project and could be how we take it forwards. If you are just teaching how to do a calculation there is no point in having schools.	I will be sharing everything	I wonder if the intention is that the planning team are responsible for the lesson and not the individual teacher - this has been our focus but it may be that it is necessary in our context and not the intention here. My question from day 7 about what they are hoping to see in a Japanese maths lesson and different types of lessons. What do children typically do out of school for maths. Any more that can be shared about how the knowledgeable other prepares for the final commentary Manipulatives - what do you use? What is meant by 'the most important point from the lesson '?
For my purposes too many teacher in the room makes it difficult to see the learning.	The format of the lesson - Independent, free investigation > carefully selected children sharing their ideas > Drawing out commonalities was great. Even better if their had been some time for application.	It is the job of the teacher to select children to share ideas that will develop learning. This is why in this lesson children were not chosen randomly to share their ideas.	To consider, based on evidence, which children's contributions will further learning (both by sharing misconceptions and ideas)	At what scale to introduce lesson study to my school? How can I adapt the Japanese model for my setting while staying true to the ethos / philosophy of the exercise? What speed / extent do I want teachers to adopt the problem solving model of lessons?

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- during post lesson discussion I appreciated that there was an exchange of ideas between the final commentator and the teacher. Often questions brought up by the final commentator go unanswered. - the plan was really thought out with anticipated responses that seemed to mirror what students were thinking; - I agreed with one of the final comments about the goals of the lesson NOT needing to be the same for the three groups. In discussing this further with colleagues they were surprised that an administrator would agree with this point, thinking that my thoughts would be they all have to be able to solve at the same pace. However, I think providing the specific goals that match the scaffolding needed will build confidence and math knowledge that in the long run will build a deeper understanding and greater "payoff" for the time spent on differentiating the lessons. If the goals are the same, the groups should remain heterogeneous. Most of the time I think heterogeneous groups have the greatest value for student to student learning but if there are the resources to differentiate with fluidity in topics and groups then I would think students could benefit from this format.	- area diagram for fraction divided by fraction seemed very confusing to me and I was relieved that the final commentator also seemed to lean toward the double number line. - importance in knowing why you want to teach topic that you are teaching AND how can students relate (real world application) as well as persevere. - student share outs were very purposeful and lead to a clear connection, or commonality, in how the problems were solved.	- so pleased to see teacher acknowledge incorrect answer and have students work it out together to correct (2nd share out). This I believe is also a testiment to the teachers level of content knowledge and therefore confidence to stray from lesson plan. - teacher made decision not to move forward with the area model at the end of the lesson. I felt this was a strong read of his students and a clear ending point for the lesson today. - In addition, one of the final comments made by a facilitator was around the summary of the lesson, perhaps needing to address the commonalities discovered. I too was leaning in this direction- as personally this was the big aha I had from the discussion between student and teacher.	- so much learning today! I'm so excited to bring back this lesson plan and all of my notes to share with the 5th and 6th Grade teams. This is a topic we frequently discuss and struggle with and I was so pleased to end our observations with today's lesson! Also, had quiet a few flashbacks to the lesson I taught at the conference two years ago :)	- assessment was brought up a good bit during the post discussion and something that we regularly debate and struggle with and I am curious what others think about math journals being the source of individual assessment when studnets have the opportunity to revise thinking and copy friends ideas...
The impact of district wide lesson study	Even when a teacher teaches a great lesson, there is always room for improvement. The teacher's thinking can always be pushed just like the students' thinking can	The importance of turn and talks, summarizing what someone did to solve versus just having them explain, the importance of asking the critical questions to get students to find commonalities between solutions.	See above about pedagogy.	I am wondering if it is possible to see copies of the Japanese curriculum as it relates to the lessons we have seen.
It was interesting to see the set up for a district LS with so many people, the smalll group share out and then the large group share out,	I appreciated the way the teacher had the students explore the in between steps of the 3 example problems to see similarities, I appreciated the walk through of the area diagram with number line.	He gave the students think time and offered time to talk to their neighbor, not all students did it but it made space to talk to someone else before sharing whole group.	The summary of the math of the day should be a hypothesis that gets tested on subsequent days...	How do the cycles of LS on school sites work with LS district wide and how does admin determine who does what? All self selected? Are teachers placed? Does it depend on the schools goals?

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I liked how they facilitated the post-lesson discussion with so many people in the room today. This was the first time they had small break out groups, with facilitators sharing out. From there, the panel is able to pull themes from discussions. I feel that this conversation was then easier to follow and not jumping from one topic to the next. Also, in the final commentary, the speaker asked if teachers had considered different options. I forget that teams spend so much time considering all aspects of the content in preparation for their research lesson that what you see in the actual lesson is only the surface of all the thinking and options that were considered.	Carefully considering the order in which you decide to show student examples can improve lesson flow and clarity. In today's lesson the teacher chose to talk about the third example in the third place because (I assume) he knew he wanted to elaborate more on it.	The final commentator mentioned that you need to listen to students when they do a turn and talk and be very intentional about what you want shared out and who you want to call on. If students don't feel a connection with you (the teacher), they may not let you listen in freely.	I liked how today's lesson tackled a very challenging topic (division by a fraction), which I realize that lesson study should be a vehicle for figuring out how to work through the topics that are most challenging for students.	Why don't post-lesson discussions with smaller groups use the structure we saw today? Are the boys volunteering a lot in the lessons because they feel more comfortable than girls in front of an audience or can we say that it is a reflection of the gender dynamics in the daily classroom culture?
The level of self-reflection expected by the teachers of themselves is at a level I aspire to. Today, the teacher began by saying how much he had to say, as he had so many regrets, but then followed this up by saying he would much rather hear from the observers so he could learn through the feedback. I believe we could tackle the cultural challenge of critical feedback in the UK if we ceased using observations as a judgement and measurement activity and rather times for deeply valued learning and collaborative professional improvement.	At the request of the group, we tried the mathematics ourselves before going into the lesson and this enabled us to not only become aware of the processes we were using but also how the students might approach their learning. The reciprocal relationship was a significant feature of this lesson and for many of us, this was most easily seen through the use of an area diagram (even when this was totally new to some members). I was very interested that this wasn't shown as a possible way of seeing the mathematical relationships and using existing generalisations to make sense of the mathematics involved. The sharing of the diagram at the end of the lesson was a key point in the post lesson discussion.	Each school is very clear about it's focus and then subsequent goals relating to this. Here, listening to children carefully and encouraging pupil talk with each other and the teacher was the focus and very evident in the lesson. My task to support the observing group was to log the number of contributions children made and the balance of male and female; this revealed a very different picture regarding one aspect of the contributions with boys making 21/26 with two individuals contributing 11 of this 21 between them. This showed me that the efforts of the school/teacher can easily mask the reality of the experience for the pupils.	So much! I will focus on the need to do the maths yourself alongside others when planning to both highlight the need to continually invest in subject knowledge but also develop teachers' ability to anticipate student responses.	
The summary of a lesson is the hypothesis to guide future learning.	Students need to show the process of thinking. This is a habit we need to build. The process is the product rather than the answer or calculation.	The teacher should be clear around the purpose of a lesson. Why are students learning this?		How do we pose questions to support learning?

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This lesson study was set in a sports hall. It was the size of the hall that highlighted the size of this lesson study. It also gave me an insight on the different types of lesson studies that happen in Japan and the work and effort that so many people put in them, for them to be successful. Wow!	The task wasn't about teaching a trick or learning a quick method on how to divide fractions. Using the previous knowledge and building on it, the children managed to solve the task in three different ways. Yes, amazing! Again!	The teacher was very confident and you could tell that he was very well prepared. Every minute of the lesson was planned (a magnetic clock was stuck to the white board) and every minute of the lesson was used for children to learn!	To make use of the time, the teacher asked three students (who were completing the task differently) to write their answers on a different sheet (rather than notebook). Then the teacher clipped their ideas to the white board and asked differen students to explain them (rather than ask the three students to get up and write them on the board from scratch). I thought this was a very good use of teaching time.	If there are students who are still unsure on how to do the task, what happens to them? Can they talk to the teacher? What does usually happen in a Japanese school?
Paying attention to what students are saying versus what the teacher wants them to say. Listening to students.	Students will not always say/do what teacher expects or wants, but valuing their mathematical thinking and strategies and acknowledge their ideas.	Creating the time/space for students to demonstrate their mathematical ideas, facilitating the discussion, but not dominating or overtaking it at the sake of the students learning	Importance of having students share their ideas, student lead discussions versus teacher lead discussions	
1) It is important to be highlight solution strategies that meet students where they are but also pushes them to dig deeper into and challenge their understandings. If a problem is too difficult, many students will be left behind. A teacher needs to be strategic about the solutions that he/she highlights and create a learning progression that can catch all students along the way. 2) Students need more than one interaction with a strategy. They cannot generalize from one context, rather they need to see many examples and have many opportunities to apply their new learnings to solidify their understanding.	1) It is important to get students comfortable with sharing their ideas and build routines around student-led interactions. It is also essential to create an environment where students don't feel pressure to always find the "right" answer. Rather, ideally, they would see errors as gifts so they are more likely to share their thinking and use a critical eye when analyzing the solution strategies of their peers. 2) Context is critical. If the students are not engaged in the context and don't feel an intrinsic motivation/desire to solve, then they won't be invested in the problem or their work.	1) I want to be more mindful of the students that I am calling on to share answers. I feel like I tend to gravitate towards the same students who have the "right" answer, when in reality there is so much to be learned from the misconceptions as well. Though we spend time on these, I feel like I could be doing a better job getting students to notice and actually analyze why solution methods do/don't work and make connections between commonalities, as the teacher did in this lesson.	The double number line was a new concept for me, and something that we observed in a few different contexts throughout the various lesson studies. I want to make sure that the staff at my site is familiar with a wide variety of tools and representations, including the double number line. Not only that, but I think it would be good for us to develop norms around how and when to use these tools. It would be ideal for teachers at all grade levels to be exposed to the range of tools, even if it falls beyond their grade-level's content, in order to develop a deeper, shared understanding of mathematical content and the trajectory of concept development.	
That it is very difficult to explain to why we need to divide by fractions. It is important to teachers why students have to divide by fractions.	I like how the double number line was used to divide fractions. The algorythem used to divide fractions is not very helpful when we are looking for a deeper understanding of division by fractions.	Being careful with diagrams. Sometimes diagrams can be limiting students' thinking. Just because a diagram can be used to make sense of a problem there still needs to be multiply ways to reach the learning outcome for students.		

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Today's lesson allowed us to participate in cross-district LS. There was a large amount of interest by educators in this area. Even though the audience of the post-lesson observation was quite large, representatives from select schools provided insightful input that added to the discussion/thinking of the lesson.	Today's lesson allowed us to witness how students can learn from identifying commonalities and how by allowing them to understand this on their own can be a better form of teaching rather than just to lecture to them the three different ways to divide fractions.	Group work was not a primary emphasis on this lesson. This showed me that group work doesn't have to be a part of every lesson, rather it needs to be selectively used when appropriate/useful for the students to benefit from.	The teacher had used student work to help promote key understandings of this lesson. I plan to share how he had the students write out their example and explain their work rather than recording what they say as they explain their thinking. I thought that this was a much better approach of sharing student thinking because it was the student's authentic work.	none.
An effective final commentator is an extremely important part of lesson study. There is so much rich learning that we can gain from one another as teachers, but it is the insight of the more experienced and more knowledgable other that can really push our practice forward in a meaningful and productive way. I have experienced some great final commentators and some not so great final commentators, and I'd like to have them be people we trust with expertise that we value and find meaningful in order to grow as a school or as a district.	During the IMPULS post lesson discussion, someone brought up whether this lesson was really meant to deepen students conceptual understanding or if it was more focused on procedural fluency. In my opinion, it could have been a more effective lesson if the students already had the procedural fluency of working with the tools. However, it did not seem to lend itself well to TTP because there was really no problem or need to solve. I have definitely taught lessons where afterward I thought, "that would have been a better use of time if I had provided some direct instruction on procedural fluency." When the struggle is not productive, then danger can be that the students remain confused. I think lesson study is a great way to learn how to strike a balance between too much and not enough struggle.	My mind keeps coming back to this concept that learning is messy. During my first year of teaching, the learning in my classroom was very messy yet not very productive. During my second year, the learning was not very messy and very productive. However, I have come to a point in my practice where I want to take risks and experiment with how to allow learning to be messy yet ensure it is productive as well. I believe that when we allow learning to be messy, we are actually stepping into the role of the listener who truly wants to gain insight into the minds of our student in order to best guide them, rather than to appear perfect and organized. I am so excited to do my own little research project on this and maybe even bring it to my next school!	I was thinking of getting together with the team to brainstorm about how to start building more relationships with a wider range of people to be our knowledgable other.	I am wondering what knowledge is already out there in terms of allowing learning to be messy yet making it productive and where/how I can access that knowledge. There is no need to completely reinvent the wheel when there are so many people I can learn from.
I thought it was interesting how they had the lesson study observers talk and actually do math in small groups during the post-lesson discussion.	I have never thought about having students develop their own definitions and that is something that I want to implement next year in my own class.	I learned that the summary can be seen as a hypothesis which can then be tested in subsequent lessons and modified or added to.	I will be sharing my insights on definitions with my teaching team and will be planning lessons to develop them with my students.	
District Wide Lesson Study: schools are dismissed 1/2 day. Public lessons are conducted at each school. Each each school is responsible for a different subject. The school we observed had three public lessons: advanced, regular, and basic.	For problem solving lessons: teachers rarely present the problem by using the textbook but it is coming from the text.	Teachers need to listen carefully to pair shares to make sure they call on students who will help move the conversation and say something that all students should hear.	The idea of whole district lesson study: SFUSD should do this to help get into the classrooms, share knowledge, and advance curriculum and student learning.	Why don't students interact more? Student discourse is extremely minimal in Japan. It feels like teachers guide everything.

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You can have only have hypothesis for the next lesson. You cannot plan exactly how you will teach the next lesson without finishing the prior lesson. You have to plan according to what your students need.	The students were only able to see the commonalities between strategies when the teacher guided them to explain every step of their work. He had to help them explain the steps they were doing in their head.	You have to focus on the process of students' thinking and not just the product. I liked how he presented a mistake on the board and had the students correct the work togehter.	I want to share the idea of writing down students' friend's idea with an explanation. I liked how the teacher brought the students back and summarized one idea.	
The knowledgeable other has a very important role because not only do they have a deep understanding of the math concepts, but they also have a strong background in lesson study. This knowledgeable other went further than previous ones we have seen because he actually pressed the teacher during the post-lesson discussion. For example, the teacher was asked "what would happen if you had flipped the numbers in the equation?" The teacher responded that he needed more time to think about that. The knowledgeable other went on to explain that if the teacher and the planning team had reversed the dividend and divisor, there would be an additional strategy that could be represented in the lesson. Even though the lesson to me appeared to be very successful, there is no such thing as a perfect lesson, and there are always improvements and changes that can be made, depending on the process and/or product that the teacher is trying to highlight.	In a problem solving lesson, the teacher should provide students with opportunities to experience the following: - Multiple points of access -active learning experiences - a sense of satisfaction by the end of the lesson -conversational skills (eye contact, explaining your thinking)	Mathematics planning and pedagogy should center around two big ideas/questions: 1. think about what students need to learn - make that the goal 2. think about a sensible context and good numbers to reach this goal Teachers should not simply teach a lesson because it is in the textbook!	I would like to help develop PD around analyzing textbook problems. I would like to study more deeply the CCSSM and work with my colleagues to develop stronger units based on the goals in the CCSSM.	

IMPULS 2017 Reflections Day 9 (Thursday June 29th) (Responses)

What opportunities for teacher learning does Lesson Study afford?	How did your views of lesson study change from your participion in the IMPULS program?	How do you view teaching and learning now?	How will you take this back to your own context?
Lesson Study is a continual process of reflection and growth with a collaborative team. I have had many professional growth opportunities throughout my teaching experience, but LS is unique and ranks with the best.	I don't know that my views have changed, but the IMPULS program has brought those views into better focus. Seeing it in action over a variety of contexts has given me an added direction on how to improve the work we have already been doing.	I already knew, but have confirmed, that my teaching will never be perfect. No matter how well a lesson may go there will always be an opportunity for improvement.	Since I will be away from the context I originally expected when I signed up for the IMPULS program, I am now wondering what opportunities I may encounter as my network hopefully expands. Might I get an opportunity to do some of this work in some international setting? That is truly exciting!
1) Observation of peers and mentors 2) Deep study of the content and goal during the planning process 3) Ability to receive continual feedback on one's teaching practice and develop an orientation of career-long growth.	Cross-site collaboration is a huge aspect of what makes Lesson Study such a powerful practice. As a small school site, I know that seeking cross site collaboration will be critical for us as well.	This experience has shifted my thinking about the positive roll that whole-class discussion can play in holding high expectations and setting clear modelling for all students, if correctly facilitated. As ever, I am committed to the fact that when students persevere to construct or discover solutions to new problem types, they develop critical problem solving skills.	I will participate leading in school-wide PD and decision making on the course of our upcoming years of lesson study and TTP implementation.
It offers opportunity to see new ideas and initiatives put into practice in a culture where it is working. Currently in the U.K. there is a big emphasis on mastery, depth and use of manipulatives. Many teachers have had theoretical CPD on this but many have questions and barriers preventing them from implementing these strategies in the classroom. By being involved in Lesson Study, these teachers could see these pedagogues in practice, could be part of discussions about them or could even be part of the planning team. Maths Leaders could initially teach the research lessons until other practitioners felt confident. This would enable a collaborative, whole staff approach to new ideas meaning that teachers are more invested in them and more likely to try them.	Before I came I knew very little about Lesson study and I definitely didn't understand the scale or structure. I have learnt that it involves a much more coordinated, collaborative approach across schools and districts than I had realised. I also have much more insight into how it is used to improve teaching and learning through providing CPD to ALL staff in school. Additionally I have learnt that research themes are carefully chosen based on areas of perceived weakness or difficulty - it isn't about a "show lesson".	In terms of Maths, it had cemented the importance of teaching conceptually and of enabling children to see the structure and the process involved in the Maths. The statement made, "in elementary school it is the process that is most important" really resonated with me. This is a fundamental idea that is not yet widely recognised in the U.K.	I am going to use what I have learnt in many ways and I have lots of ideas. In terms of the undergraduate students, I am going to implement concepts from Lesson Study to enable them to learn. I am going to start small scale (due to timetabling restrictions) and base the second year undergraduate assignment on lesson study. Students will get together in groups of five to plan a lesson collaboratively, one to teach it and the others to watch, to discuss it and to write up their reflections. In undergraduate first year level I will do the same but students will only be working with small groups of children. In addition to this I will provide the schools hosting these students with an information session and handbook about Lesson Study and its benefits. As well as this I am going to incorporate some of the theory of lesson study and some of the pedagogy I have seen into my lectures. I would like to extend this from the university students out into our partnership schools as well.

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~ deep dive into content standards - builds deeper content knowledge as well as understanding of standards progression from one grade level to the next; ~ need for deeper analysis of assessment - drives lesson choice, anticipated responses, lesson goal, and informs groupings; ~ collaboration w/i teams - provides alignment opportunities across grade levels as well as strength in collective knowledge of the group.	~ process much more inclusive of team members - beyond grade level and content area team members; ~ post lesson - feedback more inclusive of all participants who observed the lesson, and evidence based to develop more meaningful summary; ~ reflection - drives future lesson research as well as team planning beyond lesson study; ~ I can more easily visualize the lesson study process occurring in other content areas.	~on-going ~collaborative ~student focused	~restructuring grade level team meetings for better cross-grade level alignment in planning; ~developing steering committee for math and ELA; ~putting in place more structures/protocols for lesson observations so that post discussions are more inclusive and detailed from all participants ~ensure that a "newsletter" is created after each post lesson discussion to drive future planning and coherency from one grade level to the next.
There are opportunities to improve practice by getting planning together, teaching and receiving feedback and observing, collecting data and discussing data from the lesson. Additionally there are opportunities to learn more about content, pedagogical strategies and gain insight into student thinking.	My views have changed in that before this experience I knew about the shape or lesson study after taking part in 2 cycles this year. I know have a deeper understanding about how LS works at a school level and a beginning of how it works at a district level. I feel like I am beginning to know what I need to learn more about from reading, working and doing,	View now: I am thinking a lot about independent time in the classrooms at our school, giving students quiet time to grapple. I am wondering about group work. I think we see it in SF as a mark of good teaching and I am thinking about how we can track to the extent that it is effective. Dr Fuji said that the lesson is very successful when the teacher does not talk much. What does that look like in whole group in Japan.	This year the K1 will go into year 2 lesson study and we will do an opt in math LS for 4-8th grade. I will share so much of what I have learned in my coaching role. I'm bringing set squares to my school. And I bought set squares for my own kids!
Lesson study offers the most valuable opportunity there is as a teacher; to value your own learning above all else. The approach priorities the need for teachers to deeply understand their subject and continually improve their ability to listen and respond to their students; understanding that this is our primary role as educators. The focus upon the involvement of 'knowledgeable others' and inclusion of all staff/district is a huge strength and one which creates both equity and open-mindedness to the contributions and opportunities to learn from one another.	Having returned to the UK and continued my research, there is still a significant gap between what is being called 'Lesson Study' (and the associated training and publications used in the UK) and the Japanese practice built up for over a century. What I experienced makes complete sense to me and I will work hard to overcome the very significant challenges we face in schools regarding spending and sustained involvement in teachers' CPD. I came away understanding that the shared focus and goals within the school, the central involvement of the senior leadership team and subject experts and the time spent understanding the journey the pupils are on are paramount. We are in great danger in the UK of spending too little time understanding what it is we're teaching and pushing children to 'learn' something because that where we feel they should be (and we'll be judged harshly if they're not). I understand that lesson study isn't about moving children up but deepening their understanding and ability to connect and apply their skills. This approach is about team and school-wide responsibility and understanding that real change takes time and sustained focus.	To be honest, no differently (in a very good way). I have always felt passionately that teaching is about drawing out what pupils think, understand and are curious about and have had a rocky road in maintaining this belief in practice in the UK over the past twenty plus years. Our system wants a 'quick fix' and my very valuable time in Japan has strengthened by resolve to hold on to what I know really matters and ultimately makes the difference to children's lives now and in the future. This experience has given my more knowledge and first-hand experience that I can draw upon and share confidently with those I support and for this, I'm extremely grateful.	This is going to be a great challenge and one that I will have to draw upon my connections with others to begin to implement. As I know that it is ALL of the elements of lesson study which must be present to make a difference, I have to consider how to offer LS to schools very carefully. Understanding the cultural 'norm' here regarding teachers being watched and judged coupled with perceived financial restrictions on training and release time, means I have to consider the strategic planning involved as my first step. I intend to maintain my relationships with others in the group and work on solutions together.

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Massive opportunities - the planning stage is probably the richest because it involve teachers really unpicking their own understanding and thinking about a particular bit of mathematics and how to teach it in such depth that it results in a peeling back of layers, revealing beliefs and understanding that are deeply held and are then in a position to be examined and challenged. Shared observation of a live lesson also exposes assumptions and beliefs and has the potential for allowoing teachers to consider alternatives to their usual view. The final commentary has been a revelation to me in what this offers as a learning opportunity - where the person providing the commentary is particulalry skilled this can add so much to what you can take away from the session in terms of your own thinking/understanding.	I am much clearer on a number of things, particulalry the role of the knowledgeable other and the final commentator. I also have a greater appreciation of how much the Japanese teachers do at the planning stage and how important it is for observing teachers to have engaged with the matheamtics of the lesson in advance. I also have an appreciation of the whole school being involved which will not be easy in our context but is something that I am planning.	Not sure how to answer this - any change here might be much more subtle and probably I have yet to fully appreciate. Much of what I understand, from this experience, about teaching and learning resonates with what i already thought but there are some things about talk in lessons that I am mulling over. There are some lovely things like asking 'what is the mathematical question for today's lesson?' and the board work that will be something I share and draw on. The best question, which sums up what I think was at the heart of what we observed, was 'Are you teching for your students of for yourself?' I shall be using that!	We will be looking to develop further our use of lesson study as part of a project and this will involve whole schools so it will b einteresting to see how this goes. We will look closely at the role of the knowledgeable other and have two involved, one at the lanning stage and one to do the final commentary and we will try to work more closely with the Japanese model, within the restraints of our system.
Planning - Thinking of the task ahead (context, how does it relate to the children, the information given in the task), taking account of the previous learning and future learning, i.e. how can the mathematics used be extended.	I never realised the impact that lesson study had not only on one single lesson level but in the way Mathematics is being learned and taught in schools, districts and generally in Japan.	The Japan trip has made me reflect on my teaching, writing and all the choices I make in my work. I know that change takes time and it is not easy to implement. However, I feel that if we all reflect on why we set the tasks we set and work together to improve, we can make that change happen. Learning should not be a tick box exercise, and discussions around learning should not obsess over self-interest or ego or be a blaming game. It should be a place from where improvement can happen.	I am lucky to work in the profession I do, and even if I can only help shape one mind, and help that mind to open up the endless opportunities inside themselves, then surely this alone is worth its weight in gold. I will of course aim a little higher than just one!!!
Collaboration that allows for growth in content knowledge, pedagogical application, behavioral and SEL considerations - on behalf of all participants. The opportunity to gain direct actionable feedback in real time about a real lesson (contextual application).	I am beginning to understand the impact Lesson Study can have on a school and district as opposed to simply a team. I am blown away by the structured systems in Japan through which lesson study is used as a vehicle for school growth, district learning, and curriculum development.	I am thinking a lot about the variability across US schools and the consistency in practice across Japanese schools. There are clearly pros/cons to both, but what is clear is that the experience of school for Japanese children is far more predictable, and supportive of the range of learners. School benefits a small % of learners here in the US while in Japan, most students appear to gain strong academic and cultural skills.	I am working on redoing our materials for lesson study and creating new systems and structures for next year. I also have a much clearer sense of what is possible, and am ready to begin strategically promoting this work and advocating with the district decision-makers...
Lesson Study gives an oppportunity for teachers to collaborate, work together and learn from each other. The Post Lesson Study offers an opportunity to reflect, listen and take and deepen the knowledge of the maths teaching and learning and pedagogy.	I appreciate better the work that goes in the planning of a lesson study. I wasn't very clear about the post lesson discussion and now I understand it's importance much more than I did before.	Listen, listen and listen more. That's one of the things that I will take with me. The power of listening to the professors, colleagues, students... priceless!	I am writing Problem Solving tasks and books as we speak and I can feel that my writing has changed. My tasks are centered in deepening the mathematical thinking rather than simply finding an answer.

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Lesson study is a collaborative process that helps build and clarify content knowledge within or across grade levels. School based lesson study is centered around a common theme and each grade level plans lesson centered around this theme. All teachers observe and participate in the post lesson discussion, which makes for a inclusive learning environment. This also provides a powerful tool for developing a common vision for the school and consistency in instructional norms. District wide lesson study afford teachers the opportunity to exchange ideas between school and work toward improving learning across the district.	I can see now that lesson study can be used to build content knowledge and teaching expertise. This is a tool to allow teachers to grow in their practice and develop as life long learners. We need to have a growth mindset with regard to content knowledge. It is a gradual process. I see now that lesson study can be inclusive of the whole school, and we should include observers in the post lesson discussion. I also have come to understand that there are different types of lesson stud, and I look forward to participating in both school wide and district wide formats.	Reflection on all aspects of the planning process, lesson implementation, and future steps and implications is key. Reflecting on the levels of teaching provided insight on how we can reflect on a lesson. I also realize that group would must be intentional and purposeful. Integration of group work in a lesson needs to serve a purpose. Teaching and learning can be seen as a collaborative process, rather than one of isolation. Lesson study allows us to be in classrooms and provide open and critical feedback to push our practice. Teaching is an ongoing practice, and we need continued development of content knowledge.	I look forward to integrating many structures I have observe at my site. I want all teachers to have opportunities to observe more and be a part of the post lesson discussion. When individuals have a set purpose for observation, they are able to collect evidence to back their observations. This lends itself to open communication and critical feedback. I also noticed that teachers selected difficult content when planning their research lesson, and this would be a great practice at our site. I look forward to building relationships with nearby universities to grow our network of knowledgable others and build our content knowledge.
Dive deeply into the study of the curriculum, time to look at sequence and progression, collaboration, critical and meaningful feedback	The depth of understanding that Japanese teachers have of math and lesson study. The critical piece of the knowledgeable other. The collaboration and feedback from the curriculum developers/proffessors. Lessons are constantly changing/evolving based off of what students know.	I continue to view lesson study as meaningful professional development that creates the space for teacher collaboration and teacher learning. This has a direct impact on students. The extreme importance of student voice and student thinking. The art of facilitating mathematical discussions that precipitate student learning and help students learn something new. The importance of watching each other teach, learning from our mistakes and changing our own teaching and lessons whenever necessary.	As a coach, I will work with teachers in lesson study groups and encourage teachers to join lesson study teams at their school sites. I will provide resources, help model example lessons and be a resource to answer any questions teachers may have about the lesson study cycle.

What opportunities for teacher learning does Lesson Study afford?	How did your views of lesson study change from your participion in the IMPULS program?	How do you view teaching and learning now?	How will you take this back to your own context?
I continue to be blown away by the depth and complexity of Lesson Study. Just when I think I have it figured out, a new element comes to light that makes me analyze my own practices as well as math instruction as a whole. I love the collaborative nature of Lesson Study, particularly the levels of collaboration we witnessed in Japan. The idea of doing a district-wide Lesson Study is so intriguing to me! It would be so great to come together as a district with a common learning/teaching goal. Educators are doing so many amazing things in isolation, and it would be so powerful if there was a way to capture these things and give teachers in other schools an opportunity to learn from them. In our district especially, there are so many variations on how we teach math and what curriculums/tools we use. Clearly this method is not working, as student math performance has been on the decline for years. If we could come together and problem-solve as a unit, like we do on the smaller-scale at our school-site, I truly believe that we can make some amazing and impactful changes.	I don't think I really grasped the immensity of the professional development piece. I formerly associated lesson study with a way of teaching math - teaching through problem-solving and thought that was the bulk of what comprised this idea of "lesson study." Though we come together to discuss as a staff afterwards, there were still a lot of pieces that were invisible to me. After participating in this program, I now realize that the professional development piece is at the heart of lesson study. The faculty coming together to plan and later dissect/analyze a lesson is where the key learnings come from. I also didn't realize that we can apply this professional development model in all areas of curriculum, not just math, as the Japanese do. I am interested in taking a Lesson Study approach to things like reading and writing to see how it might transform the staff's approach to teaching these subjects.	Teaching and learning are interconnected. We can't be strong teachers without being open to new learnings. Conversely we cannot develop our practice and learn new things without taking risks and testing strategies and practices in our own classrooms. This is the heart of professional development, and the fact that lesson study emphasizes both components equally is what makes it an effective model for teachers.	I think before, I paid close attention to things that were relevant to me and my students in a first grade context, and only peripherally registered elements from the upper grades. What I learned in Japan though, is there is so much to gain from teachers in other grade levels. Though the context of a given problem might be above/below where my students are at, I can still take a lot of key learnings away that will impact my practice at the first grade level. Additionally, developing a better understanding of the trajectory of math instruction throughout the grades helps me to create mathematicians who are equipped with skills/tools to support their later concept development at higher levels.

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Well the main ideas I would like to focus on are the sharing of ideas with teachers and observing student learning. I believe one of the main issues with teaching in the U.S. is that too often teachers are lefy on an island. We are not really used to collaborating with other teachers around lessons and student learning. Also when we do collaborate with other teachers it is often about climate or culture or some kind of comitee not directly tied to content material. I think lesson studies affords teachers to work with other teachers to have the oppurtunity to dive deep into student learning. I personally think it is valuable to talk to more expierenced teachers as well as less expierenced teachers around lessons. Lesson allows teachers the oppurtunity to gain insight from other teachers durring the collaborative process. I als think another understated value of the lesson study process is that the process is based on procedure and that there is a finished product at the end. Besides the planning observing students is without have to teach the lesson is a very rare oppurtunity. To obseve students during the lesson and and seeing what happens in real time is complicated and ivaluable to teachers. Observing students in real time allows us to improve on our instuction in the most benificial way which is through the lens of the student.	I just thought the attention to detail was impressive. I did not realize how much detail goes into lesson study. When I read the lesson plans and get a chance to look into the ideas of teachers and what they are thinkong about in terms of content, how to make it interesting to students, the surveys, pre and post dicussions, and planning time all comming togher is impressive.	I veiw as the greatest tool I have for professional developement. I can think of no better tool that will change the way students learn in my class.	I feel a heavy responsability to improve on the lesson study framework my site has in place already
LS allows multiple entry points for teachers to participate. Teachers can be on planning teams, teach the public lesson, or observe lessons as well as post-lesson commentary. Additionally, participation within IMPULS demonstrated the connection to current research and leaders in the field. This shows me that LS is not a means to an end; it is constantly evolving and allowing educators to learn along the way.	My participation allowed me to see that Japanese teachers were very open to the idea of public observation. The focus of LS is not on how the teacher is teaching; rather it is what students are doing to make sense of what is being taught.	After IMPULS, i view the role of the teacher as to identify the understandings of key students to help further the lesson by allowing those understandings to influence the discussion and understandings of other students in the class. Also, the teacher can allow students to be more independent, both with explaining their understanding but also with taking risks to demonstrate their thinking.	I really look forward to applying what I have observed with IMPULS and applying it to my classroom directly and also by leading a group through LS next school year. I would like to continue to focus on TTP, and explore think time and group work activities. Also, I plan to use the provided lesson plans as framework toward examining similar themes in my classroom. The comprehensive plans will he better improve my teaching and the learning of my students.

What opportunities for teacher learning does Lesson Study afford?	How did your views of lesson study change from your participion in the IMPULS program?	How do you view teaching and learning now?	How will you take this back to your own context?
Lesson study affords the opportunities for teachers to deepen content knowledge and develop a vertical scope and sequence. It provides the platform to develop best practices and improve school-wide, district-wide, or cross-district-wide themes which benefit all students. It affords the opportunity to observe and collaborate with other teachers, develop relationships, and see the areas of growth for yourself and others as well as share strengths so they can have a bigger impact outside of your own classroom. Lesson study affords the opportunity for teachers to inform curriculum decisions based on real research in real classrooms with real students and teachers.	I am now thinking deeper about the value of giving observers the lesson plan beforehand to grapple with the math before the lesson. We as observers can give more critical feedback if we ourselves dive into the mathematics of the lesson before observing. I am also thinking about the importance of actually analyzing and questioning the standards in terms of why we teach them and why they are important for students to learn. Previously, I have not had the brain space to question the standards because my focus has been on many other areas for growth. This seems particularly important though in developing meaningful lessons and if we don't question the standards, how can we expect our students to question and to see things with a critical eye.	Now I view teaching and learning as learning and learning. The teacher and student are both life-long learners and should approach their work as such. I also now see the critical importance of having deep content knowledge in orer to plan the best ways to plan constructivist lessons that draw the mathematics out of the students rather than giving it to them. Furthermore, teaching can not be a one person in their one classroom job. It is the responsibility of each grade level to work together to prepare students for whats to come. Also, I feel it is extremely important to differentiate the entry point rather than the goal. We so often rob students of the opportunity to reach higher expectations by modifying the goal. We are not doing them any favors by assuming that is necessary rather than giving them a different entry point.	At a new school that probably does not participate in lesson study, I will need to do many things. I need to get my principal on board and my staff on board through a possible PD on TTP and on Lesson Study. Once I have that, I will involve Dr. T (as he promised :)). I think it is very important not to rush the process of starting lesson study because I really want the interest to come from the teachers rather than me shoving it down their throats. The first year may feel like a slow start, but I believe that is necessary in order to get full buy-in for meaty and productive lesson study cycles in the future.
Lesson study gives teachers the opportunity to collaborate deeply with colleagues, observe each other teach, and reflect on teaching and learning. These practices allow teachers to deepen their own content knowledge, learn new teaching practices, and increase their unit/lesson planning capabilities.	I saw the power of implementing lesson study on a systemic level and how much that can increase vertical alignment and also how it positions teachers as researchers. My views on post-lesson discussions changed some too. I realized that we need to make our own post-lesson discussions more reflective and not so observational. Also I am now thinking more about how to implement learning after the public lesson and how to share that learning with others.	My big takeaway was that in order to be a better math teacher I need to learn more math content. In order to be as thoughtful around numbers, tasks, and discussions as I saw teachers and experts being in Japan I need to learn more myself.	My team and I will be leading a 2 day lesson study and TTP PD during the summer PD for our whole staff. We will also be leading our own teams at school as we take lesson study school wide this school year.
Lesson study provides opportunity for careful examination of lessons, teacher reflection, collaboration, and advancement in curriculum and lesson study. It allows schools and districts to constantly be examining and reflecting and never being okay with status quo.	I only knew lesson study in terms of our k/1 team - which is very isolated from the rest of the school and the district. I now understand that lesson study is much more - school wide, district wide, research themes, expert commentators, and more.	As a teacher I should constantly be examining, collaborating, and reflecting on my planning and teaching.	Our goal is whole school lesson study within the next two years.
Lesson study allows teachers the ability to dissect lessons, curriculum and pedagogy.	We need to be more thoughtful in the way we plan and design lessons. Lesson study has to be more than a few teachers who volunteer to participate. We need to take lesson study school wide and eventually district wide. Furthermore, to make lesson study even more effective, we need to publish our findings from the public lessons.	We have so much more to learn when it comes to content. Our school needs to build our content knowledge so we can be more thoughtful in our lesson planning and curriculum design.	We are planning a PD for the rest of our teachers so that everyone will participate in lesson study.

IMPULS Post-Program Survey (July 2017)

1. In looking over all the research lessons during the immersion program, name one that was especially meaningful to you, and why:

Response
The final research lesson on dividing fractions. This was especially meaningful because it is a skill that has always been very challenging for me to teach and I was really interested in reading the lesson plan, observing how they decided to present the problem and listen to the debrief of the lesson.
Especially meaningful is the 4th Grade, Oshihara Elementary lesson from June 23, because my colleague and I would be teaching 4th grade this school year and will try using a version of this lesson for an upcoming public lesson day.
I appreciated day five's research lesson. The teacher opened the lesson with an open ended question. In this lesson, students used math manipulatives to explain their thinking and students were asked to explain their classmates thinking through the board work portion of the lesson. This helped me frame how I will advance my board work portion of teaching math.
I found the final research lesson (Grade 6: Let's Think about How to Divide by Fractions) to be one of the most meaningful for me. It was very carefully structured and contained all of the essential components. A lot of the lessons seemed rushed at the end and didn't have a strong summary, but the choices that the teacher made ensured that he had time for all of the pieces. I like how he used student's work to move the students along the progression and used student errors to clear up misconceptions. The way he guided the class to find the common element in each of the models and then used that to come up with the big math of the lesson was really powerful for me.
I have been thinking about an early lesson we saw in the third grade class about the boats. I appreciated many parts: the use of manipulatives on the board, the focus on answering the question, and teaching about remainders.

Response

I need to say that they were all meaningful for me, for different reasons, so I could choose any of them here. I am going to choose the very first lesson we saw because of the beautiful way the start to the lesson was orchestrated and the focus for the lesson which both reflected the problem-solving approach. This was one of the things I was particularly interested in learning more about. The context was a very simple one, buying some ribbon, and the teacher held up a piece of ribbon to start the lesson. As I already knew the numbers in the lesson so I was initially puzzled by why she was holding up what looked like a metre of ribbon. Having established that she needed a metre of ribbon and that someone else had bought the ribbon and paid 300 yen, she revealed that the ribbon she was holding was in fact longer. That moment was brilliant because the children all understood one metre of ribbon and 300 yen and then they realised there was a problem because the ribbon was longer than one m

I particularly enjoyed the lesson around dividing with remainders as it was something I needed to teach to my students. I adapted the lesson in light of my observations and the post lesson discussions and taught it to my students the first week I got back to England. The lesson was enjoyable to teach and really helped the students understanding of remainders.

I really enjoyed each lesson as I felt I learned something from every one of them. The one I enjoyed the most was the lesson with Surds, though it wasn't a lesson study. Though most of the observers in the room, were very nervous as the lesson "took for ever to start", the teacher was very confident in himself and his students. This lesson developed very organically, it didn't follow a particular plan or script and made us all reflect on the way we teach.

I really enjoyed the last lesson on Fractions. I liked how confident the teacher was, every minute of the lesson was planned and the attention to detail was obvious in every aspect of the lesson.

I really enjoyed the lesson on June 28th: Grade 6, Let's think about how to divide by fractions. I enjoyed this lesson because I felt like it most closely aligned to my ideas about the components that exist in a problem solving workshop lesson. The teacher presented a problem, students spent time solving, whole class conversation was the bulk of the lesson, and then they were able to come up with a summary. In all other lesson they were unable to end the lesson with a summary because of time. I was happy to see one lesson go through all the components.

I think all the research lessons were meaningful to me in different ways, but one that is staying with me is actually the lesson that wasn't a research lesson. It was the ninth grade lesson looking at ratios. I was struck by a few things during the lesson. At the beginning, due to a variety of things, the students weren't coming up with some of the answers that the teacher anticipated, and were fairly quiet. But the teacher knew his students and kept pressing them. He knew that they could get to where he wanted to go, and he didn't let the presence of outside observers deter him. Another thing that struck me about that lesson was that he had constructed a task that required students to collaborate- not because the teacher told them to, but because they really needed each other's knowledge and tools. Watching them reexplain things to each other and clearly explain their thinking to their classmates was rewarding. They didn't get all the way through the "lesson", but it was clear

Response

I thought the ninth grade square roots lesson at Kiyose Lower Secondary was meaningful because of the way the teacher led students to the problem and the group work that ensued. In that lesson, the teacher spent about 16 minutes facilitating a discussion about how to sort the set of books. He did not reveal any body language to indicate "right" or "wrong" answers and instead kept asking "what else?" It seemed that he had a trust in students that they would generate the idea in the direction he wanted to go. The task itself was also meaningful because the problem could not be solved alone. I loved how the teacher modeled collaboration very simply by having two students compare two different side lengths of their paper to test for congruency. I'm guessing students may not have known that he was planting a seed of putting each other's papers together, which would later be key. As we content, I feel that we need to motivate why group work is important and this task naturally lent itse

I was most impressed by the division of decimals lesson because a) The math was so challenging for me personally and b) I was very impressed by the way in which the instructor facilitated a process which felt student-led, and used clear board work and intentional questioning to reveal the algorithmic commonality/ new strategy.

I was particularly interested in the 9th grade lesson re: deriving the ratio of the length of the short side to long side of paper. This lesson was fascinating to me for a few reasons: 1) It was incredibly complex yet students persevered and genuinely explored ideas, in order to eventually arrive at a solution 2) It challenged my notion of groupwork, in that in order to solve the problem, 4 students had to collectively share their papers. Thus, groupwork was a means to the end, not simply a structure for the sake of a structure. This pushed my own thinking about when and where groupwork is appropriate, pedagogically/instructionally and as a means of building collaborative culture.

It is very hard to pick just one but I would probably pick the lesson where the children had to construct a parallelogram. For me, this lesson really emphasised the importance of teaching children about the properties involved in a given concept. In this case the children had to use their prior knowledge about the properties of a parallelogram (in other cases other prior knowledge was used - such as knowledge of the properties of division during the 6th grade fractions lesson). When I was taught to construct shapes, I was taught as a list of instructions that had to be remembered. This lesson showed how, if given the correct tools (both physical in terms of set squares etc but also in terms of knowledge), children can work out how to construct shapes for themselves. This allows exploration of a concept and the ability to see that there is more than one way. The children are creating the maths rather than following a set procedure. This lesson also showed children working collaborat

Response

The June 21st lesson at Heisei Elementary School was especially meaningful to me. As the first lesson of the program, it allowed me to witness the hard work and dedication that the planning team applies toward the research lesson. I specifically remember the teacher explaining how she had practiced and memorized the board writing. Although during the lesson, I thought about how great the teacher was doing, it was during the post-lesson debrief that I was able to really make sense of the purpose of LS as the discussion revealed how the teacher taught to the lesson plan in order to get through what she had practiced. However, the key take away from this was that she stopped listening to her students in order to finish her lesson. Overall, this was meaningful to me because at the beginning of the trip I was still new to the LS process. So, this lesson solidified many understandings of what LS seeks to provide for the research team by looking at what students can reveal during a less

The June 26th lesson - Grade 9 Square Roots. The lesson increased my mathematical understanding of square roots when I was able to see where the square root of 2 was coming from! This lesson also highlighted the importance of having purposeful group work. Often we are encouraging group work/conversation and in reflecting I am not always sure why or if these conversations are necessary.

The boat lesson was especially meaningful to me because of the teacher's questions. Although there are always things teachers can do better, what stood out to me was the way she put the cognitive load on the students through her questions that were of varying levels of rigor. Almost every word that came out of her mouth was a question. It made me think deeper about the questions I ask or am afraid to ask when I teach math and how I can improve that practice, truly holding to this constructivist, student-centered approach and fostering a problem solving mindset in my students.

The final lesson was extremely meaningful to me. The teacher had the students analyze the solutions/strategies that were presented and make connections between them. I thought this was an extremely powerful practice, and something that I do not do enough in my own classroom. Building these connections helps students to see how a learning progression builds. They can see how the strategies and solutions are the same, but also where the variations come in.

The first lesson we observed had the greatest impact on me. It was an opportunity to see board work in an organized format. While the teacher did stick too closely to her lesson plan rather than adjust to what was happening in the classroom in that moment, it was an opportunity to see a TTP in real time I appreciated her reflection and the participation of the whole school in the lesson debrief.

Response

The fourth grade lesson at Showa City Oshihara Elementary School on June 23 was the most meaningful to me because it made me think about the importance of diagrams in representing students' authentic ideas. Having students recognize the utility of diagrams is something that I have been struggling with, especially this past year. The goals of the unit, "Pay Attention to Commonalities", focused on students being able to see the usefulness of diagrams in visualizing the relationships between values. This unit seeks to have students express their mathematical thinking processes in clear, rational steps, i.e. in a way that promotes logical thinking. For me, this lesson did not was a great research lesson in that it was the true "testing of a hypothesis". The end of the lesson brought more questions than it did answers, which allowed for rich learning for the post-discussion participants (and we observers, too). Did putting up a diagram toward the beginning of the lesson narrow students'

The last lesson on June 28th was the most meaningful for me. I liked how this teacher incorporated all the parts of the public lesson including a summary and reflection. Most of the previous lessons did not have a summary. I liked how developed the students thinking.

The lesson I found most meaningful was the 6/23 lesson "Pay Attention to Commonalities (Thinking with Diagrams)" because it was the lesson I felt most prepared to observe as a result of Makoto's demo lesson the day before and because the lesson demonstrated how challenging it can be to rely on student input to guide a lesson.

The lesson involving the array investigation was particularly interesting to me as I really enjoyed the exploration of student responses, and how that tied into the lesson plan.

The lesson on 23rd June in Showa City Grade 4. (I also think because we'd discussed this in the university beforehand, it was even more meaningful to me - Reko and Kota). In this lesson we saw diagrams used a great deal and these were a focus for the school and specifically in the lesson. What was very interesting was how the teacher's questions focused the students' thinking very early in the lesson and led to many being drawn to the wrong concept and then shutting out their ability/willingness to reflect on whether their was more to consider. Some students sat back having come up with an answer and waited giving us an insight into the way they viewed the problem and what they'd been asked to do. The post-lesson discussion focused upon how 'neat' the lesson was and the importance of 'messy learning' which reflected the students' thinking and their interpretation of the problem. I found this incredibly interesting as many teachers in the UK are a long way off using diagrams with the le

The lesson that was most meaningful to me was the third grade lesson that we saw Saturday at the conference. I really appreciated watching this lesson because I felt that I struggle with similar things that this teacher also struggled with. It was refreshing to see a research lesson where I could identify with the teacher. For example, the direction she went at one point in the lesson was not what the students needed for this lesson. This teacher also tried to put a lot into one lesson versus breaking the lesson into 2 lessons (which I also have problems with in my own instruction). This lesson was also a grade that I am familiar with and so it was easier for me to understand the math as well as the instructional choices the teacher was making.

Response

The most meaningful lesson was the 9th grade lesson and teaching students how to find the length of the side of the book by using the square root. The teacher took time to create a meaningful introduction, waited for student responses, guided their thinking and gave them time to come up with various solutions. I learned a lot from the lesson. Especially, about the need to create an engaging introduction for students to have buy in, and for them to want to learn the content.

Thursday June 22. It was meaningful for me in two areas content and student observation The content which involed the use of stone patterns is a tool that I use in my classes as well. THis was a seventh grade class and I use these types of patterns but I have never thought about rearranging the patterns in order to predicrt how the pattern is growing. Also It was the only lesson I was able to get up close to the whole class and observe students durring the lesson. During this time I realized I need much more practice observing students and what to look for and how to record the student data.

2. In looking over all the post-lesson discussions during the immersion program, name one that was especially meaningful to you, and why:

Response

I particularly liked to post-lesson discussion for the middle school lesson at the school connected to the Tokyo University we partnered with. I think a big part of why I enjoyed that discussion was because they purposefully went slow for our translator so I could keep up with not only hearing the translation but it gave me time to process what was being said. I also am really interested in using dots to introduce algebra and predictability- so that influenced why I enjoyed the post lesson discussion.

A post lesson discussion that was meaningful to me was the very first one. I was so impressed by so many aspects that I had not yet seen done before. I admired their willingness to be honest and open to critical feedback. It was clear that their first priority was the students and their egos were all pushed by the wayside in order to grow their practice. I also liked the circle set up because it is inherently collaborative and uniting.

Again I could choose several of them but I will choose the discussion after the last lesson, lesson 8. This discussion really investigated the decisions made by teachers at the planning stage and during the lesson, with the diagram coming under scrutiny. The final commentator took a different approach, asking the teacher direct questions during the final part and this added to the understanding of observers. There were a number of key things that came out of this discussion for me: understanding why you are teaching the topic or why students need to learn the topic, 'if teachers just use the questions in the book the children will learn not to think as well', the importance of students simplifying, students attending to the process so that they record intermediate steps/expressions, being careful not to generalise when hypothesising is what is appropriate, the careful building on existing understanding, using a diagram that supports understanding (in this case the double number line).

Again, I think the final post-lesson discussion was the most meaningful to me. I appreciated how open the teacher was to feedback and how he was already thinking about how to use this feedback to better his instruction for the next time. The final commentator was also extremely knowledgeable and posed some interesting questions that really made me question some of my own choices as a math teacher and bot me thinking about different approaches that might be more impactful for more of my students.

Although I personally found it challenging to follow, I appreciated the district wide post-lesson discussion. It showed me that districts in Japan collaborate on a deeper level to improve instruction and what is best for students.

Response

I appreciated the discussions after the first lesson. I liked that we had circled what we thought was most important and discussed if the lesson was levels 1, 2, 3. In addition I appreciated hearing what Dr. Takahashi and Tad thought about the lesson and what they would have focused on in the student work.

I found the discussion in Yamanashi particularly helpful on many levels. Firstly, the way that the observations had been split between the teachers from various year groups so that the feedback covered the whole class and gave us a very wide perspective upon the children's learning. Secondly, the goal of the school was to use diagrams more effectively and there was a great deal of debate as to how well this had contributed to the lesson, the students' use and understanding and how 'led' the pupils were in their thinking due to the teacher's modelling using diagrams. There were some interesting questions from our group too as part of this discussion giving us an insight into the journey many educators are on in researching and using pictorial models to support students' mathematical thinking.

I found the post-lesson discussion of the square roots lesson (6/26) to be most valuable, especially for the reminder that when a teacher is too attached to their lesson, they are no longer listening to their students.

I really enjoyed the post lesson discussion in the lesson with the children of Year 7 - day 3 (Tokyo Gakugei University). Dr Nishimura provided examples that made us all think. The power of interpreting ideas and making connections was very inspiring!

I really enjoyed the post lesson discussion on June 23rd: Grade 4, Pay attention to commonalities. I enjoyed this post lesson discussion because I really liked the structure. It was the only lesson where the observation template included the objectives of the public lesson. I thought this was a really good way to ensure that the post lesson discussion is meaningful and focused. They were able to discuss every objective because the observation template was so structured.

I really enjoyed the post lesson discussion on the 22nd of June. Dr Nishimura gave examples from Primary Mathematics to Secondary which highlighted the power of interpreting ideas by making connections. There was deep mathematical thinking involved by all the participants.

I think one thing that stood out to me in the post-lesson discussions was in several discussions, it was put forth that the teacher, in an effort to stick to the lesson plan and get to the summary by the end of the lesson, stopped listening to their students. As a teacher, that is really hard to hear, but it makes me more mindful of making sure I am listening well to my students. Watching these lessons, it was clear that the students were not understanding the goal of the lesson. So the teachers did get to the end, but without the students being with them. Definitely something to remember.

Response

I thought that the post-lesson discussion at Koganei was particularly interesting because it delved into the relationships amongst multiple representations, which is a big theme in our 8th grade mathematics curriculum. In this lesson, students had a variety of algebraic expressions that related to the pile pattern. The post-lesson discussion covered a range of directions that the teacher could have emphasized. For example, one comment was that the teacher could have led a deeper class discussion of relationship between the table's growth rate of stones and the physical pile pattern, which in turn may have helped students properly identify the 4 in "4n" later on in the lesson. This discussion was meaningful because this is often the last minute thinking I do before I teach a lesson. It is challenging to organize all of the possible routes for student discussion in my head, but I do enjoy the occasional opportunities to explore optimal routes with colleagues.

I thought the discussion after the same lesson mentioned above was excellent, particularly Dr Nishimura's insightful teaching suggestions

I thought the post-lesson discussion at the Oshihara Elementary School on the 23rd was the most meaningful. In it I could clearly see the structures of the post-lesson discussion laid out and their functions within the lesson study cycle. I like how it flowed between the lead teachers reflections of the lesson, then questions about the lesson plan, then moved into the team's focus area, then to each teacher's specific observations of students, then back to reflecting and finally to the expert commentary. Throughout all of the post-lesson discussions I was really struck by how much content knowledge both the expert commentators had and the teachers themselves. They spoke about math and numbers in a way that showed how deeply they know the content and how it progresses throughout the grade levels.

I was interested in the grade 7 post-lesson discussion from Koganei Lower Secondary School. While students (impressively) derived many complex solutions, the instructor spent the vast majority of class showing endless possible solutions, rather than creating space for analysis and synthesis of the various solutions. As a result, students were frantically jotting down solution after solution rather than weighing the merits of each one. In the discussion, this was explored and critiqued, and it became clear the summary itself was never reinforced or clearly articulated. The expert commentary centered on whether the purpose was achieved (interpreting 4n) - and whether the diagrams themselves facilitated this. N was abstract and students did not connect the variable directly to the diagram. The commentator showed how visually, a different diagram would have allowed for a clearer connection between the visual and the variable - and how the lesson could have been more successfully had studen

In general, I appreciate the structure of the discussions and the reflection that was present in the post-lesson discussions. I appreciated the vulnerability present from the teacher perspective, and the critical feedback others provided.

Response

On day 4 a comment was made about Mr. Ohma being good at taking punches during the post-lesson discussion, "the fish on the cutting board." Versions of the comment had been made prior to this day, but it was here it became clear to me that these Japanese teachers ALWAYS expect critical feedback and NEVER expect their lessons were perfect. There is no such thing as a perfect lesson, but somehow I am always hoping for one, and sometimes in our culture when we start the conversation with the positive, because that's what we do, I am not really listening anyway because the positives feel obligatory. Let's cut to what needs work.

The June 28th post discussion - following Grade 6 lesson Division of Fractions. Clarity provided in use of the double number line verses area model when dividing a fraction by a fraction; discussion on why we have students solve problems like the one presented when there is struggle to identify real world context for them; appreciated the amount of time teacher also was contributing to the post discussion - really also showed his level of content knowledge.

The discussion at the university affiliated school was inspiring for the depth of discussion. I was particularly impressed with the way that the final commentator so clearly understood intention of the lesson and the thinking displayed in the discussion to build on points made and further everyone's understanding of effective practice.

The most meaningful post lesson discussion was at Kiyose Lower Secondary School. They focused on the pedagogy. For example, they explained that a bad lesson means students are trying to figure out the teacher thinking, and a good lesson means the teacher is trying to figure out student thinking. They also discussed the idea of "group work." The amount of each group matters per lesson and there is no point of group work unless they need each other because then they will just copy it.

The most meaningful post-lesson discussion, to me, was at Oshihara Elementary School in Kofu. I specifically needed to understand the roles of the members of the discussion panel during the lesson, and the provided document of Observation and Discussion Points did just that. This document outlined the Focal Points of Discussion for each of the panel members and allowed for a very productive discussion by having members share out specifics of what the planning team had sought out to explore with this research lesson.

The post discussion we had on Monday 6/26 there was a quote that I wrote down,"A good lesson the teacher is working hard to understand students. A bad students are working hard to understand the teacher." I wrote that a common theme that I noticed is that I was observing teachers that were struggling to read their students which I understand. It is difficult for a teacher to read students accurratly and it can be frightful to deviate to much from a lesson plan. But I realize to be come a more effective teacher I have to be willing to step away from a lesson when it benifits the students.

Response

The post lesson discussion on our first day struck me because, regardless of how "perfect" the lesson appeared to me, the team, and the teacher who taught the lesson, found plenty to critique - namely that the lesson was TOO perfect: that the teacher did not listen carefully enough to the thinking of the students, and only picked out what suited her plan. The distinction of how to properly choose and sequence student presentations in a way that best moves student thinking foward without just "picking out" what suits my plan appears incredibly daunting.

The post-lesson discussion during our first research lesson observation on Tuesday was meaningful to me because they discussed a lot about the teacher's use of student work on the board and the intentional choices that the teacher made. There was a lot of discussion about student thinking versus what the teacher put on the board. I have trouble linking student responses and displaying authentic student work when I am facilitating a discussion as a teacher and I was able to identify with this same struggle. They also talked about students going back and revising their work and this is something I try and work on in my own classroom.

The post-lesson discussion that was most meaningful to me was the one after the 6th grade fractions lesson. Firstly I liked the fact that participants were asked to discuss things in smaller groups and then feedback. I felt that this led to a wider range of discussion points and that everyone's view was considered (which is difficult to do in a large group). Secondly I thought the final comment was particularly insightful. I liked the fact that the commentator asked the question of why do we teach division of fractions in school when it is rarely used in real life and that this led on to him discussing the importance of the ability to simplify complex situations in order to make sense of them. This is a useful life skill. It was also interesting to hear the discussion about the importance of the numbers chosen and how we have to ensure that children can't get to the answer by "accident" by doing something else with the numbers or by using them in a different way (such as converting

The post-lesson discussion that was the most thought-provoking was the 6th grade Ohta Ward Kojiya Elementary School lesson that we observed on June 28. Firstly, the teacher allowed for a very thorough discussion because his lesson was, in my opinion, so strong. Then, during the post-lesson discussion he began his comments by saying that he had "a lot of regrets". I could not believe this because of how high quality the lesson was. Nevertheless, this comment from the teacher showed me how important it is as a teacher to be reflective and that there is no such thing as "a perfect lesson". Moreover, this post-lesson discussion was most meaningful to me because it made me reflect on my own planning and instructional decisions. We must be purposeful about the numbers and contexts of the problems we select for our students to solve. What is the point of teaching if not to help students become autonomous, critical thinkers in our society? The final comments drew this point home by emphasizin

Response

The very first lesson was meaningful becuase even though the teacher had created this beautiful board and everything looked good on the surface, she was directed students to where she wanted them to go based off of her notes. She was not allowing the students to guide the mathematical discussion with their ideas. This ended up being a common theme in all of the lessons. Teacher lead vs. student lead discussions. Time and time again the knowledgeable other would remind teachers to listen to their students, what are they telling you. This can be very difficult, especially when you are being observed, but the most important. By listening to student thinking we understand what they know and what we need to do to help them learn the new content. I think this takes a great deal of practice and time.

3. In looking over all the lectures during the immersion program, name one that was especially meaningful to you, and why:

Response

I really liked all the lectures- the last day was a bit much when Dr. T tried to answer everyone's questions. I think there could have been a different format for that day's Q&A.

The lecture given by Dr. Takahashi on June 26. One of the most poignant points that I drew from his lecture was that a "good lesson" is one where the teacher is working hard to figure out what students are thinking and a "bad lesson" is one where the students are working hard to figure out what the teacher is thinking. This, for me, encompasses teaching through problem solving in that a teacher should design a lesson where students are able to bridge the gap between what they already know and what they are trying to learn. Students should not be throwing out ideas to the teacher until the teacher latches onto an idea that is desired. The other point that Dr. Takahashi brought up during this lecture was the idea that when you teach one lesson, you study ten. This also speaks to the depth by which teachers prepare in Japan. Further, this preparation and deep understanding of content allow for the teacher to truly listen to students' ideas (whether correct or incorrect) and adapt the le

I appreciated the lecture and follow up about the paint problem. It helped me to understand the simplest way to look at that particular division with fractions problem.

I enjoyed the last lecture about fractions. Fraction is a difficult topic to teach and I thought the teacher was very skillfil in the way he managed his time and conducted the lesson. The lesson highlighted the importance of questioning why children need to learn Mathematics. Furthermore, as educators we need to think of contexts that children can relate to rather than teach rules.

I enjoyed the lecture from Dr. T about the three levels of teaching and how this is used to evaluate and discuss lessons. I can see this being a helpful tool in focusing our post-lesson discussions.

I found Dr. Takahashi's lecture about geometry starting in Kindergarten and moving up the grades to be very helpful. It gave me an understanding of the importance of knowing how student's understanding starts off in the grade levels and how it needs to progress. Also the way he talked about developing definitions with the students changed the way that I think about defining things with my students. I found the model Sort, Define, Examine, Construct to be eye opening.

Response

I greatly appreciated the Q&A/debrief time the last day with Dr. Takahashi. Idea of using lesson study to research effectiveness of group work and best way to differentiate to close gaps will be something that we will investigate this year. In addition, using lesson study to deepen the content knowledge of our staff, as well as following through on our post lesson summaries so that we are closing the loop better in the research process and therefore carrying the learning over to future planning.

I loved the final comment given on June 21st. The speaker not only gave specific, critical feedback, but the way she ended with "math for fun" / 'math without calculation"- this showed a culture of love of math.

I particularly appreciated the lecture exploring the various ways of utilizing lesson study - within a team, within a school, cross-school, and within districts. This expanded my own thinking of how the process itself can be utilized as a whole school structure for PD and curriculum development, and how a systematic means of improving curriculum based on real teaching experience is possible, with the right structures in place.

I really appreciated all of Dr. T's lectures but one specific example that stands out to me is when he was talking about the Japanese scope and sequence for teaching shapes. Although a simple idea, it isn't something that the United States follows. The names of shapes are not introduced until second grade meaning that in first grade shapes are created and manipulated but without names. Vocabulary isn't introduced without deep meaning behind it. This to me has a deeper meaning for how the Japanese teach math - nothing is surface level. Instead, everything is carefully taught with deep comprehensive understanding.

I really liked the Surd lesson. It was a difficult lesson to teach not only mathematically, but also because the students were teenagers. The teacher was very calm and patient in his approach and I feel that I learned a lot during the lesson.

I think these tend to blur together for me, but some of the things that stood out were assumptions about what students know, paying attention to what students are giving you, construction of tasks, choosing numbers carefully, and giving students the opportunity to struggle productively using their own approaches. I think these are great discussion points for any lesson that is taught, and helps me think more productively about the tasks we choose for our research lessons.

I valued all of the lectures we participated in. Of greatest benefit was perhaps the 'pulling together' final day session. The opportunity to share and ask questions, listen to others and contribute our thinking gave me a far deeper understanding of the context of lesson study, the importance of its history and the consistency of its use across Japan. Having had such rich first-hand experiences across the two weeks, I was able to formulate my thinking and begin to synthesise the huge amount of valuable information we'd received. This session focused my thinking upon my actions following the trip and how working with others was essential in taking my practice forwards.

Response

In all of our lectures the ones that I always find most meaningful are those where we do the math that will be taught. When we were doing the division task that the Oshihara 4th-graders would be doing, it was great learning when I had to struggle with the task myself. There was another lecture when I felt like Dr. T. stopped listening to a comment I was making. Ironically, it was during a discussion about not following where our students lead. I know I talk a lot, so I get that, but I felt like he didn't let me finish my comment and he wasn't understanding my question.

It was helpful when Dr. T gave us the sequence of geometry because he provided us with more content knowledge. The last day when we discussed curriculum design was also helpful.

Makoto's demo lesson by far. It allowed to us to understand the thinking of student misconceptions by exploring our own. I realized that when we "got into the math" it was easier to be led by the purpose of the lesson and not the lesson itself.

Not sure which elements are identified as lectures but the second session on day 1 was really useful when we looked at geometry because it gave a real insight into how the progression in geometry is built up, the small but significant steps on the journey through the grades, the focus on understanding (geometric reasoning) rather than memorising labels, building up the understanding through experiences including constructing shapes, definitions being something that you can exemplify (e.g. the definition of parallel) etc. It also gave some insight into how textbooks reflect this understanding and support the journey. Whilst this was in the context of geometry and there were some content specific things that were interesting, it was also illustrative of generic themes which made it particularly useful.

One lecture that I found was important was on Day 7 when student groupings were discussed. One major takeaway I had from this was that students should not be grouped together just to be grouped together but that they should be intentionally grouped together based on the problem they are solving. Students shouldn't just work in groups to work in groups--they should see the usefulness in the group work as it is aligned with the task.

One lecture that was especially meaningful to me was the lecture in which we discussed a level 1, 2, and 3 teacher. Upon first learning about this, I immediately made up my mind that I wanted to be a level 3 teacher at all times. So, when other educators pushed back on that idea, posing the idea that maybe it is beneficial be fluid between those levels based on student needs, I enjoyed how that challenged my thinking. It brought a new light to the idea of really listening to our students. I previously was of the opinion that if we are really basing our instruction on listening to our students, then we should be only providing opportunities for them to develop the knowledge and skills. However, this discussion made me reconstruct my previous idea and think deeper about when it could be useful to flow through the 3 levels. I'm still figuring that part out!

The Geometry lecture was really meaningful. It highlighted the importance of vertical alignment and progressions. It also made it clear to me that foundational knowledge is incredibly important for students. It gave me hope that one day we will have a really clear sense of student learning by utilizing a clear progression.

Response

The final day where we came together for reflection was my favorite day. It was so wonderful to hear Dr. T and his colleagues share more about mathematics in Japan and to learn about how the country has developed teachers who strive to constantly learn and grow. Coming together with my team was very powerful, as it gave us a renewed purpose for next year. It was also really powerful to hear what other schools/groups were taking away and about their goals for the coming year, as we have a lot of the same challenges and goals ourselves.

The lecture at the cross-district lesson study at Kojiya Elementary School was a little disjointed, but the underlying pedagogical philosophy in the speaker's message stuck with me. The speaker asked the teacher a couple of questions, which at one point the teacher admitted that they had not thought about that yet (I don't have notes on what the question was). I thought this was interesting how the lecturer pushed the teacher's thinking off the cuff. The speaker discussed that when planning to teach a topic, ignore the publishers and think for yourself you're your own connection(s) to the topic are. He said, "If the teacher teaches form the textbook, then students will learn to not think." This poses a challenging task of thinking how we can craft a task that builds off of students' understandings of the world, but I liked how the speaker trusted that good instructional decisions can come from teachers' own craft.

The lecture where we had to solve the Riko and Kota paper origami question was most meaningful to me. I enjoyed the fact that we all had to participate in solving the Maths. I was also fascinated by the way our solutions were shared with the group and then grouped together into like solutions based on the underlying structure of the Maths. It really showed me (in practice) how you can look deeply into one problem and spend an entire session discussing it. I have since used this problem on a training morning with ten teachers and they really enjoyed the problem and the discussion that went alongside it.

The most meaningful was learning about the knowledgeable other. This feedback provided by the knowledgeable other is really a crucial element to lesson study and one that we have not understood or taken advantage of in the 6 years I have being doing lesson study at my site. This feedback provides a critical lens and critical perspective about the math content and understanding that teachers may often miss because they are looking so closely at the lesson.

The ones that contextualised the education system at the beginning and end were most imeaningful to me to help in understanding the bigger picture.

The pre-lesson conversation we had on Wednesday 6/28/2017. The problem was about dividing fractions. One participant ask for time to work on the problem and so we began to work on it. I really struggled to find a way to solve the problem without the algorithm. Ir was a very humbling expierence. And it was hunbling to see a room full of educators struggle with the problem. Then we were asked when do we see dividing bt fractions in out lives. I had no clue and I realized I need to to a lot more thinking in general about whrere do my students see math in thier lives.

Response

There was not one lecture that particularly stood out as they all contributed to developing my understanding of Lesson Study and what makes it effective. I will use information and thoughts form all lectures to inform how I will use Lesson Study in my school next year and beyond.

Two things about Dr. Takahashi's lecture early on in the program stuck with me. The first, was the three levels of teaching and how even though the goal is level 3, the observations during IMPULS showed that even veteran teachers were performing at level 2. The second was Dr. T's explanation of the flow of a lesson being to define, examine, and construct. Through this process, students have the proof the need to demonstrate their understanding and to support discussion points or arguments.

4. Was there a conversation among participants during the immersion program that stands out to you? (This might have been an informal conversation, outside the official program.) Please describe, and provide reasons that this stood out for you:

Response
A conversation that was brought up repeatedly was the extent to which teachers in Japan participate in post-lesson discussions, regardless of grade level. A first grade teacher is able to authentically participate in a post-lesson discussion whether it is a second grade lesson or a sixth grade lesson. To me, it seems that this is able to happen because all teachers have deep math content knowledge that extends well above and below the grade level they teach, and also because there are structures in place (one grade level focuses on one particular group of students) to allow for their active involvement in post-lesson discussions.
After the lesson with Surds, there was a lot of discussion. Some of us thought that pedagogically it was crazy to wait for 25 minutes for students to talk, others thought that it was brilliant how the teacher was persistent and encouraged students to express their views even in a classroom full of strangers!
I am grateful that Ms. Johansen was able to attend as well. Having someone else from the same school was very powerful in thinking through how our observations/learnings could impact our greater school community. The time to create an action plan together was also valuable and one we are currently working on implementing for next school year. Also, very inspired to travel to UK and other regions to observe educational settings/cultures!
I appreciated talking with Hillcrest about how they were going to roll out their PD on Lesson Study.
I can't think of a particular conversation, as I participated in so many they kind of all blurred together. One thing that I noticed though is how much of our "free time" was spent talking about math. I honestly have never been one to be super fired up to talk about math after hours, but that changed on this trip. I couldn't wait to dissect the lessons and talk about the intricacies with my colleagues. It was refreshing to be surrounded by like-minded educators who were so passionate about math and to be provided with such rich examples of mathematical instruction and content to discuss outside of our work time.
I enjoyed the opportunities to reflect with my colleagues about taking back structures to our own site. It was interesting to hear about the challenges other sites faced. We look to go whole school with lesson study and feedback from others about challenges they have faced is helpful.

Response

I had a very useful conversation around how to establish Lesson Study where it is new to the school. he advice around starting slowly and ensuring full understanding and buy in from those involved was invaluable.

I had lots of conversations with Rory about starting lesson study as a new initiative in both of our schools. We both are in similar situations where lesson study is either not started or is not a school-wide practice yet. I have experience with being at a school where lesson study was school-wide for a few years and am trying to start lesson study at my new school. I felt like I was able to give Rory good advice about how to make lesson study a successful school-wide practice and he was able to give me good insight into what was/was not working at his school as he was trying to get lesson study started. We are continuing to collaborate about an action plan for both of our schools. I think it's helpful to have someone who is in a similar situation to discuss these things with.

I had many thought provoking conversations with other participants reflecting upon our own practice and what we were seeing in Japan. As the trip progressed we began to shift our opinions away from judging what we thought worked well and what didnt, to why the strategies were being used.

I had so many valuable conversations that it is hard to pick a specific one that stood out. I was interested, however, to hear that in America (in California, at least) a fairly prescribed curriculum is used that dictates the order of teaching in each year group. This is similar to Japan in terms of having a set "book" to follow. In England it is very different, we have a curriculum that simply states objectives using bullet points. There is no further guidance about the order or how to teach...simply that those objectives must be covered within that year. The advantages of this is that it offers teacher flexibility and freedom to chose what to teach when and to chose how to teach it but the disadvantage is that if a teacher has limited subject and pedagogical knowledge then teaching can be weak. I could see that a lot of research had gone into the Japanese curriculum and textbooks and that every number had been carefully thought through. I wasn't sure if this was also the ca

I remember talking about how willing Japanese teachers were to have outside guests come in for observations, and even though they were nervous, they still did it and were able to quickly move into the post-lesson observation. Our conversations outside of the program also talked about how critical the teachers were of themselves. What I gained from these conversations was again, a deeper understanding of LS and how it isn't a reflection of the teacher, but the purpose is to draw out student thinking and if the teacher was critical of themselves, they were most likely referring to how what they did influenced their students.

I spoke with Karen at length about division of decimals. She shared with me her own experience that teachers who are eager to learn but admit to their conceptual gaps are much more likely to grow in their instruction than those who cannot admit their weakness. As with out students, it will take perseverance to overcome our learning challenges.

Response

I think we were all really impressed by the structures that exist in Japan for lesson study. It was inspiring to see entire schools and district staff present at the public lessons. Lesson study is used on a really large scale and it was remarkable to witness systems work so effortlessly.

My colleagues and I had many conversations about student discourse - when students were talking and who was (or wasn't) talking. Although we appreciated the depth of knowledge teachers have in math, we were not always impressed with their pedagogy. Students sit at their own desks, they rarely talk to each other, teacher voice is highly present, and male students tend to dominate the conversation. This raised many conversations around equity issues and student engagement.

One conversation that stood out to me was a conversation with a couple people about the benefits and uses of bar models. I knew almost nothing about them and still know very little. But that conversation led me to see how conducive they are to TTP and helping students develop a deep conceptual understanding of the mathematics. I am eager to learn more about bar models!

One of the "stand outs" about this program is the networking with colleagues that occurs and how impossible it is to put a value on this. I now have better connections with other teachers in my district and new ones from around the globe. Additionally, I felt like I was treated and respected as a professional. This too is invaluable.

The discussions I had regarding the relationship between lesson study and the education of students with special needs was noteworthy. There is a common opinion that lesson study is by nature inclusive of students with special needs, yet there is no clear process for applying TTP techniques to small group discussions or for allowing a variety of ways for students to demonstrate understanding.

The math lessons on the surface all seemed to be really well-done, but it took the eyes and ears of of the stakeholders involved to see the flaws and mistakes in either the delivery or the writing of the lesson. This just re-affirmed the need to have public lessons, and to receive critical feedback, so that teachers are constantly growing and evolving their practice.

There are two conversations I want to mention here. One is the conversation after lesson 6 because it really challenged people to think differently about what 'should' happen in a lesson and many people were uncomfortable with this. There was a big focus on the time taken by the teacher at the start of the lesson, to get some ideas from the students, and many people were of the mind that he should have used some other strategies to get this to happen, mainly involving talking with partners. This really reflected people's own teaching styles and preferences. The problem was the teacher was sure that the students had plenty to contribute; they were acting as teenagers might do anywhere and having looked at all of us (and as this was not a lesson study lesson they were seeing that most of the visitors were NOT Japanese which might have been a factor in what happened) they were reluctant to speak, not reluctant to think. It was suggested that talking to partners would have moved things alo

Response

There were many conversations regarding the huge challenges both countries face due to ineffective policy making and interference but I'd like to focus upon a more positive aspect. Everyone I spoke to acknowledged the Japanese teachers' level of subject knowledge and their continued focus upon anticipating student responses. These are aspects we are working very hard, as consultants and lecturers, to dramatically improve in the UK. The way the program was run facilitated a realisation amongst the group that subject knowledge was generally underdeveloped and needed to become a high priority. This will be extremely helpful, I believe, in alerting us all to maintain a careful focus on this area of our own continued professional development. I also believe that by engaging teachers in subject knowledge development, we can more effectively manage the challenges I listed above.

There were many conversations with David Correa that helped me process all of the research lessons. One that stands out though is when he helped me realize that teachers in Japan were challenged by some of the same content or pedagogical goals that we are challenged by in Oakland/America. Lesson study supports teachers and schools in facing those challenges and in reflecting and learning about their practice. This conversation stood out to me because I was having a hard time envisioning the translation between what I was seeing in these Japanese research lessons to my context in Oakland. This conversation reminded me that teaching is a challenging task regardless of where you come from or where you teach.

There were too many conversations to narrow it down to one. Many of our conversations had to do with what we were seeing, what aspects could be integrated back into our classrooms and how we would do that, what it would look like to try to have whole-school lesson study, and just ways we could learn from the Japanese culture and education system, as well as things we do that might not have been as apparent in Japan.

We continuously explored the question of contrived vs authentic partnering and group work: It is critical to have a rationale for why partner work or group work is included in the lesson. Ideally, these structures should allow for deeper learning, not simply be included for the sake of working in a group. A teacher should consider whether working with others, as opposed to individually, will foster greater student thinking. If groupwork is selected, strategic choices should be made about group size, group roles, the assigned task of the group, and how the group will manage and maximize time. Groupwork is a tool like any other, requiring strategic planning, and enough front-loading and coaching so students are able to utilize the time to extend and push their own thinking beyond what they can accomplish working alone. However, we wonder about the lack of oral participation in many Japanese classrooms when opportunities for pair-sharing and group work are rare. Articulating concepts or

We had a discussion with other people in the bay area and especially in San Francisco about how we can lift lesson study out of our schools and make it have a bigger scope across the district. We also talked about how we could share what we learn in our school-wide lesson study with others schools.

Response

When the Surds lesson finished, I was buzzing with excitement. However there were other participants that were disappointed with the lesson as they thought that pedagogically the lesson could have been taught better. The discussion made me think of the lesson even more and analyse the teaching deeper.

When we were discussing with the larger group, many people spoke about the lessons as if they planned them and offered feedback. The participants did not have the expertise or background in content knowledge to offer that feedback so I did not feel like that was useful.

Yes I was having a discussion we Makota anf he was telling the about a survey for Japanese teachers. The question was, "What is the most important for your student?" THe number one response was that Japanese teachers wanted their students to have a love of mathematics.

5. How did your views about teaching and learning mathematics change as a result of this trip, if at all?

Response

I had two big aha's this trip. One- I am interested in figuring out how to increase my own content knowledge around math. I think that it is necessary if I want to be a more effective math teacher and to develop research lessons. Two- I had a significant shift in the way I think about labeling myself as a master teacher. Japan's decision to consider teachers novice until year ten really resonated with me and I think I would have been more motivated to grow and learn if we had a similar mindset around the profession of teaching here in the US.

I need to deepen my content knowledge above and below my grade level. This is necessary for many reasons, but a new reason is so that I am able to better listen to my students during a lesson. I want to vertically align our grade levels at Prieto so that we can all better understand what knowledge, strategies, and concepts our students are coming to us with, and also to better understand how to support them as they continue to the next grade level. This year, I want to really focus on strengthening my content knowledge.

All of our teachers need to teach through problem solving. Moreover, we need to gain more math content knowledge. I do not feel like we have enough content knowledge to plan the more effective lesson.

As a coach I want to make sure that we are doing math together as adults to help build content knowledge.

From this trip, I realized that teachers are willing to dedicate a decent amount of time to open up and problem and let students arrive at the problem themselves. In our curriculum we have more problems to get through so there isn't that time to carefully consider a situation and what questions we might ask. I realize that teachers must think about their personal connections to the math topic and find ways to help students see the connections to motivate why it is important to engage in a particular math task. Teachers also trust that their students have good ideas worth exploring whether or not they are the most efficient methods or the intended method.

I am much more reflective in my work and the choice of tasks I use. The aim of all my work since participating in the IMPULS project is the development of students' mathematical thinking.

Response

I am not sure I can say how my views about teaching and learning mathematics have changed at this stage - I think this is something that I will be aware of over time. It has reinforced some of my beliefs and given me contexts to use to support these and it has made me question some things. I think that there are features of teaching that we have seen that I will emphasise more, such as anticipating responses and sequences. We have been using the Stein et al paper 'Orchestrating Productive Mathematical Discussions' in our work and most recently with our mathematics leaders so it was reassuring to see this on the first day and I think I started to use it as a tool for reflecting on the teaching in the lessons. I think we can make a lot more of this. It is not something new to me but I have more clarity about the level of understanding of prior knowledge that teachers need to have so that they are clear about what the new learning is building on and this is something that is a real issue

I am not sure how it is possible to change in such a short period of time, but my teaching and learning have definitely changed. I feel that I am thinking more, I am planning better and I think of 'depth' rather than 'surface'.

I don't think I realized how intertwined teaching and learning really are. These Lesson Study observations really illuminated this connection for me. By being exposed to so many contexts of Lesson Study, I really began to get a better sense of what an impactful practice this can be when done faithfully and regularly. As a member of the Math Lead Team at my school next year, I really want to get teachers to recognize this connection and to see how observing and learning from others can impact their own teaching practices, even though the contexts may be different.

I found that in order to be a better math teacher I need to take a deeper look at my own math content knowledge in order to support my students from where they are coming from and take them on a progression to where they need to be. Without this knowledge my teaching is not organized in a way that will help kids gain a deep understanding of math because how I am structuring it it doesn't make sense.

I have a deep appreciation for the depth of content knowledge that teachers can work toward gaining. I can empathize with my students in their learning process.

I have always been an avid proponent of using problem solving and multiple strategies, but it was highly instructive to see TTP in action in Japan. It has raised many questions for me, including how to introduce problems with giving students enough of an entry point without leading them to a particular strategy, how to construct tasks that are worthy of groupwork, and how to choose tasks and tools that will best lend themselves to the goal of the lesson.

I was particularly drawn to the emphasis on processes over answers, which is something that has definitely affected the way in which i teach.

Response

I was struck by the deep content knowledge of the teachers in Japan and am thinking hard about how we can support American teachers to build their mathematical content base. I am curious about the strong school culture in Japan, and what elements are or are not transferrable to American schooling, given the stark differences. I am compelled by the focus on student experience in Japanese classrooms (how students felt in the lesson, their levels of happiness and engagement) and wondering how to move the conversation here toward this, by beginning to validate that a teacher's perception of student experience is itself qualitative data, rather than something to be ignored since it is not quantitative. I left quite impressed with the depth of pedagogical purpose in each lesson, but also at times, unimpressed with the lack of an equity lens in some classrooms (re: gender dynamics, equity of voice within lessons, ability of teachers to bring their personality and humor to the room and allow s

I would not say my views have changed all that much but I will say I have a much greater appriciation for conversations and collaboration with my fellow teachers.

It didn't change my views as such but it did help me to really see how important teacher subject knowledge is and by this I mean teachers who teach in primary school knowing what high school concepts different aspects of maths underpin. It is that knowledge of progression that I think is key. It also showed me the importance of knowing the problem inside-out and being able to anticipate both right and wrong solutions that children could come up with and already having a clear idea about which direction you would like to take them in. I also think it consolidated that idea that looking at the structure and properties of the mathematics is essential and that everything needs to be taught with understanding.

It is so important for teachers to have a deep understanding of math at all grade levels - as opposed to just the grade they teach. Also, referring back to the question above about an important lecture, it is so important for students to develop deep understanding as opposed to just memorizing the facts and process.

It made it clear to me that I need to do better in terms of content knowledge. Most post lesson discussions only discussed content. I feel like in America we do not aspire to be experts in math content in the same way. This is a problem because we can not teach mathematics effectively without understanding what it is that we are teaching.

Last year, our LS team at Lawton began the work for TTP within our mathematics teaching. IMPULS has left me highly motivated to continue this work and reference back to this experience in Japan. My participation within IMPULS has helped shape my understanding of the value and importance of Lesson Study.

Most of my comments above cover this. The big take away is to let go of this idea that somehow, some way I will reach perfection in my teaching. I have never really believed this, but somehow keep striving for it, and it's kind of ridiculous.

Response

My views about teaching and learning changed as a result of this trip because I now see the importance of QUESTIONING. As a teacher with only 2 years of experience under her belt, I have not gotten to a level of mastery to even begin tackling whether or not I agree or understand why we teach each standard. I have been learning the standards and how to effectively teach them. Yet what I hadn't contemplated was the importance of questioning them in order to be effective. It is also important if we want our students to approach their learning with that same critical eye.

My views about teaching and learning mathematics changed dramatically as a result of this trip. I was able to appreciate the math differently because I was pushed outside of my content area and had to really look into the math that I would be observing. I have not observed lessons that are in upper grades very often and so my own math understanding was pushed as a result of this trip. This allowed me to see how important it is to look critically at the math that you are teaching or observing prior to teaching/observing. It also made me realize how little content knowledge we have in the US in comparison with Japanese teachers.

My views are constantly changing as a result of my ongoing learning and this experience contributed on every level to that exciting journey. The trip enabled me to see the impact on teachers as professionals when they don't feel judged but are trusted, work collaboratively and are allowed to be learners themselves. I had many of my own feelings about what is central to effective teaching re-enforced such as the deep subject knowledge, understanding of conceptual journeys across whole schools, use of mathematical terminology and 'professional generosity' in sharing and developing practice together. I also came away from this trip understanding more about how even very positive slow change can be and I'm unsure about whether I'm willing to accept this of challenge myself and those I work with to address this.

My views changed drastically about the need to study curriculum. This can be so hard as a teacher. We have so many demands placed upon us and we teach multiple subjects. Through this experience, I learned that lesson study can provide the platform to study curriculum, standards and content deeply. While I have done lesson study for many years, this was my first year doing math, so sequence and progression of mathematical understanding are concepts that are sticking with me. It's crucial for administrators to set aside time, provide PD and support teachers with this learning.

The biggest change to my understanding was around the group discussion and group work. I will, in the future, consider carefully when to use group work and hen to call on students contributions and do so only to further students understanding - not merely to have students voices heard or because of a vague idea that group work is a good thing.

Thinking of division in terms of equal groups is a less advanced framework than thinking of division in terms of ratios. A double number line is a visual representation of this more advanced usage/conception of division, and is helpful because of how it allows students to flexibly transfer what they know about whole numbers to use with decimal numbers.

Response

This trip transformed my practice. Not only did it give me an important means of assessing my own level of content knowledge, but it deepened my commitment to working in community with peers and "knowledgeable others" to deepen my knowledge as I develop that depth in my students.

What really stood out to me was the importance of the standards and the purposeful approach to which they are taught and referenced across grade levels.

6. How did your views about the essential features of lesson study change as a result of this trip, if at all?

Response
The impact that it can have on a whole school really changed. So, the essential feature would be bringing in all teachers, even if they do not teach math to take part in the public lesson.
After participating in this program I now see that we need to make our post lesson discussions more reflective as opposed to just observational. In the past our post-lesson mostly just was made-up of the team sharing out our student observations and then we waited to here what Dr. T told us. We need to take more of an active role in the post-lesson discussion. Also we need to think carefully about how we are applying what we learn from the lesson study cycle in the future
I appreciate the importance of the post lesson discussion so much more. Prior to the trip I didn't understand what that involved...the criticism, the feedback and all the views were given in such a constructive, respectful way - something that I had never seen before.
I believe we need to focus far more deeply on the unit (rather than simply the lesson) in our planning process. I also am deeply focused on how the debrief/memorializing process can include recommendations and next steps beyond the team - for the school and district - so the work lives on beyond the individuals on the team.
I developed a much better understanding of the different and important roles of the facilitator and of the Koshi. In my experience these roles had been merged to make both less effective. I am now looking for a Koshi for our school and have been in touch with our local teaching university.
I didn't know much about lesson study before the trip so I learnt a huge amount. I realised the depth to lesson study and the amount of time spent at all the different stages. I realised that lesson study isn't about a "showcase" lesson but is about choosing a theme that is something that needs improving and then working collaboratively to improve this aspect. It was also refreshing to see the critical aspect of the post-lesson discussions and to see the teachers embracing this rather than becoming defensive.

Response

I don't know that my views changed exactly, but seeing the different ways that teachers presented student solutions made me more aware of the order and intent with which I share student solution strategies. The summary has always been challenging for me, and it was heartening to see that other teachers struggle with developing this concept authentically during a lesson. This is the component I really want to strengthen in my classroom and I definitely got some ideas from my observations in these classrooms. Finally, I want to incorporate some sort of a toolbox to support struggling students. I know that this is not necessarily an essential feature, but I really liked how several of the teachers provided a variety of tools and options to support students at their varying levels of understanding.

I have a greater appreciation for the post-lesson discussion and the participation of the whole school in the observation and post-lesson discussion. I also see the impact that years of lesson study can bring, as teachers are refining and improving lessons over time.

I have a much clearer understanding of all of the features and in particular: the detail in the research proposal (setting the lesson in the context of the current understanding of the children and a sequence of work) and how it identifies in great detail the anticipated responses from the children, how these will be sequenced and then dealt with; the role of the observers in the lesson and how this may need to be supported; the role of the facilitator in the post-lesson discussion and different ways this can be organised; and the role of the final commentator (I had not understood prior to my visit that there were two knowledgeable others involved although I had understood they should be external to the school) which, when done well, adds significantly to the understanding and learning of all involved. The final commentators were one of the delightful surprises for me because they gave me so much more than I anticipated they would; this all started with the first one who amongst other

I have a much higher appreciation for the process of kyozai kenkyu.

I knew that this was going to be a valuable and important part of my professional development, but I don't think I realized how critical it was for me to see lesson study in Japan, in a system that has been using it for generations. Seeing the different types of lesson study (school-based, district wide) showed me how it can be done differently and serve different purposes. I am also seeing the importance of having a school community be part of lesson study. I'm not sure how to work this out logistically or with reluctant teachers, but I want to try to figure out a way to make this work. I think it could begin to change some of the things we've been talking about (but not making much headway with) for years. I also noticed that each lesson study was different, so that helped me also make more sense of our approach to lesson study. I am now trying to seriously think about the way a problem is set up, as well as the summary, and when and how to use group work, and how to choose an

Response

I knew very little about lesson study before I undertook this study trip. I'd heard 'lesson study' talked about in various schools but felt it probably wasn't representative of the practice so well embedded in Japanese culture. For me, understanding the big picture of lesson study is crucial. It is mad up of parts and each part is crucial to the LS success. The focus upon teams planning together, the involvement of specialists and the high regard the senior leaders and other staff have for the process stands out particularly. I see enormous power in this approach with the respect for and trust in teachers being of greatest importance.

I never realised how much planning was involved and how much collaboration between educators, observers, professors. Also I learned that Lesson Study is not an answer, but a process. Sometimes in UK, quick fixes are required. Lesson Study is not a tick box, but a journey to improvement.

I now better understand that one vital reason Lesson Study exists is so that we can observe the students, not the teacher! I always find myself watching the teacher's moves, but I need to focus on the students' voices, writing, and even their facial expressions more. After observing so many Japanese math lessons, I see how authentically teachers make note of students' ideas. Before this program, I had in mind that a teacher would only observe a lesson in the grade above or below his/her own. In Japan, I saw teachers from all grade levels engaging in observing, taking notes, and participating in post-lesson discussions. I now better understand the benefit of teacher participation in observation and post-lesson discussion across grade levels.

I now see the importance of discussion protocols in the post lesson discussion that allow everyone to focus on just a couple key questions. This helps maintain clarity in order to actually use the learnings moving forward as a school or district.

I want to create better lesson study structures for my school site. I want my administrators to be more involved and I want each lesson study team to incorporate the same structures across the grade levels. In addition, I would like more schools to observe my site's public lessons so that our learning can be shared.

IMPULS did not change my views about essential features; rather it was a very educational and informative opportunity for me to have in order to connect my understandings and my observations of the process of lesson study. It was very helpful to me as a visual learner.

Lesson Study is a much more inclusive practice than what I was envisioning. From planning, to the post discussion, the involvement in multiple grade levels and content areas provided opportunities for building content knowledge as well as coherency in instruction from one grade level to the next.

Lesson Study should be done school wide. This is our goal within the next two or three years. This upcoming year our hope is to have most or all staff participate in the public lesson and post lesson discussion.

Response

Lesson study has always being my favorite work and my favorite type of professional development. This trip solidified my beliefs. I will continue to work in lesson study groups and coach teachers through lesson study cylcles. I am a lesson study advocate.

My views did not change about the essentials.

No I have always beena fan of lesson study I would say now after my trip is that I have a better understanding of lesson study. I look forward to bringing my observations and new learnings back to my site.

Teachers who teach the research lesson are in the spotlight for better or worse, but even if there's considerable flaws in a lesson the criticism in a post-lesson discussion is all in service of everyone's learning. At Oshihara, the administrator opened the post-lesson discussion saying that discussants will share their criticisms, but the lead teacher must remember that everyone has been in that position already. This shared experience I think is key to establishing a culture of taking criticism for the sake of everyone's learning.

The more "buy in," the better, and you can never really have too many critical eyes observing a lesson. There is much we can see when we put our heads together. I have also mentioned before that we are moving our work with lesson study in a positive direction. We have a great deal of work to do, but we are on a positive path.

This trip allowed me to see how important it is for lesson study to be normalized as a part of school-wide professional development. I think when we ingrain it as a part of what we require of teachers it becomes a powerful professional development tool and not just something "extra."

This trip demonstrated to me how crucial the ecosystem of a lesson study community is to its strength as a protocol for ongoing PD. Both the vertical (government/university/primary school) and the horizontal (school to school) connections promote expertise far beyond what a lesson study process inside of a school community can achieve.

We need to be more thoughtful in the way we plan and design lessons. We need to take lesson study school wide and eventually district wide. Also, all teachers have to teach through problem solving. Furthermore, to make lesson study even more effective, we need to publish our findings from the public lessons.

i was unfamiliar with lesson study at the beginning of the trip

7. Please describe the biggest or most crucial changes you are now considering in your lesson study work as a result of this trip, if at all:

Response
Having at least one whole school public lesson each year where all teachers and (someday) all staff can observe and debrief.
All our teachers are going to teach through problem solving and participate in lesson study. We are also going to publish or report our findings to the whole group. We need to develop more math content knowledge so that we can better prepare lessons.
Although we are doing Lesson Study in isolation at our school, this program has demonstrated the need for continued implementation of Lesson Study. We are fortunate in that schools in San Francisco that have been doing Lesson Study for a few years are now motivated to do more cross-school lesson study observations. By doing these types of public lessons, I am hopeful to be able to have more of my colleagues attend in order to grow LS at our school site.
Because I now feel an urgent need to strengthen my content knowledge, especially above (sixth grade) and below (fourth grade) my grade level, I have asked my administration if our fifth grade could semi-departmentalize, where I would focus on teaching only math and science and another teacher on ELA and Social Studies. I feel that semi-departmentalizing would allow me to hone the lesson study skills that I have started to develop. I will also be an active participant in our Prieto's new Research Steering Committee. Lastly, I want to advocate for the use of Lesson Study across content areas, specifically in English Language Arts. Hopefully Prieto can continue to grow its Lesson Study so that all teachers at the school can participate and benefit from engagement with Lesson Study, regardless of whether they teach P.E., art, or 7th grade Social Studies. I will absolutely advocate for this at Prieto this year.
Deep understanding of mathematics content requires a learning community. My biggest shift will be to reach out to a community of other 4th grade teachers to observe more teachers' interpretation of content.
Getting other teachers to buy into the process. I think it will be hard to for some teachers who don't see the benifits of lesson study right away.
I am going to be learning much more about math content and taking a deeper look at math across the grade levels and as a school we are thinking about using our lesson study research theme to study this vertical alignment more.

Response

I am planning on implementing a smaller scale version of lesson study as a way to support my university students learn about how to teach mathematics. I will place second year students into groups of about 5. They will plan a lesson together as a group, one will teach it and the others will observe. They will then have to discuss it and write up a reflective assignment based on this process. I hope to give them session time for the planning so that I can offer support with this. I also hope that the class teacher can observe the lesson and provide a bit of "final comment". For the first year students, they will work together (in groups of about 4) to design a short maths activity for a small group of 4 - 6 children. Each student will then go off into their separate schools to deliver this activity. They will then discuss with each other how it went and prepare a reflective presentation on it to present to their peers. This will form their assessment.

I am redoing all of our materials to bring more clarity to the process and to provide deeper guidance re: best practice. I am designing a lesson study and TTP digital resource kit for our teachers to use. I am working on organizing a cross-school district day for a public research lesson. I am also working to support a number of schools to grow lesson study to a whole school PD model, as a school change design structure.

I am thinking about how to bring in teaching through problem solving into our weekly planning and how it fits in the other initiatives our school takes on like math group work and project based learning. This will be a pilot year for our school for some teachers to take up this work in varying ways and for me to support it.

I am trying to figure out how to incorporate lesson study better into our school goals- how to use it as a vehicle to make the changes we want to see. This is going to require some creative thinking if we want all staff to participate in lesson study (as leaders, planners, or observers.)

I am wondering if our Math Department's lesson study goals are the right ones to help us improve our practice on a daily basis. I walked away form each of these research lessons feeling that my content knowledge deepened. With our team's emphasis on group work, I don't always walk away with the strongest insights on the content, but rather a better understanding of all the challenges and problems with certain types of instructional decisions. Although it is good to be critical, it can be discouraging and I want our team to feel like we learned something that can improve our practice up until the next research cycle.

I am working with the local maths training centre in the UK as they develop lesson study professional development in schools.

I am wrestling with my enthusiasm and belief that Lesson Study is most effective in a whole school context (or wider) and weighing this up with what I have learnt about the need to build Lesson Study into the school culture slowly in order to make it effective.

Response

I have been writing a blog and talking about the lesson study since being in Japan. I am planning to do advocate the importance of the lesson study through writing and talks and consultancy work in schools.
I look forward to helping my school adjust professional development structures so that all can participate in lesson study. I will continue to work with my grade level colleagues to deepen my content knowledge.
I loved the idea of having a Steering Committee. While we have something kind of like this in place at our school, we lack some of the consistency that this model can provide. Since our site is so small, it will need to look different, but I think it is important to establish a core team to ensure the transfer of knowledge between lesson studies.
Keep working to get more of our staff on board and possibly team up with neighboring school/s to extend our work.
Now I'm considering how I will utilize TTP across content areas (reading, writing, etc.).
Plan better, be more reflective.
The biggest change for me is going deep with understanding and questioning each standard. Trying to understand it's real life importance and make sure kids have the opportunity to understand that as well is a new and daunting practice for me, but one I have fully bought into.
The biggest changes I will make are including a knowledgeable other in the public lesson, observing lessons from other schools, and coaching teachers and administrators around the need to make time for curriculum study.
The impact will be three fold. Firstly, the need to develop teacher subject knowledge and accurate use of mathematical language. This already forms a large part of my work and I have seen just how important it is for me to prioritise this even more. Secondly is the opportunity to work alongside colleagues who are aiming for the same impact but work in different contexts where the possibility to create the LS approach will be easier initially. The momentum created by working together and sharing ideas is hugely important and beneficial to sustain my learning and develop my practice. Finally, there is the focus upon 'how' I enable lesson study to happen in the schools I support and ensure it is a true version of what I observed. I've done some reading since I returned and spoken to some schools involved in what is promoted (quite officially) as LS in the UK and I am concerned about what is being recommended. I will work with my fellow UK delegates from this trip to consider how to work w
The most crucial thing for me is starting lesson study at a school that does not currently have school-wide lesson study. This trip allowed me to see different school-wide research themes and allow me time to think through a plan for my new school.

Response

Two things - making it whole school and having two knowledgeable others involved, one as the final commentator. To date we have not done any whole school CLR and we will now be planning this in to our research project next year. We will not be able to shut schools for an afternoon so will be thinking of other ways to make this happen and finding workable solutions will be really important in terms of making this something that English schools will sign up for and invest in, seeing it as a form of PD they can use on a regular basis. Up to now we have not had a final commentator take on the role that we saw during the programme but this is partly because most CLR we have done has been very small group. We did one large session with all the teachers from our project, so over 40 people, and we didn't have the final commentator and it was the missing piece so we will be looking at how we manage this across our team and possibly linking with some of the other UK based participants.

We are now going school wide with lesson study so this trip made it apparent that we need to have clear lesson study structures in place. In addition, this trip has motivated me to learn math content a lot better this next year.

We are revamping our grade level meetings so that after 20 weeks grade level teams will rotate who they are meeting with. For example first 20 weeks K/1st will meet, 2nd 20 weeks, 1st/2nd will meet. In addition, we will be putting concrete observational tools in place so that when observing lessons participants have a clearer roll. We are revising our school wide theme to be more inclusive of all content areas, not just math. We are developing a structure to create our own version of a "newsletter" after each post lesson discussion. Finally, we will implement steering committees.

We have purchased Japan Math for K and 1!

8. How much did you learn about each of the following during the immersion trip to Japan?

(a) Important features of lesson study

Value		Percent	Responses
Some		10.7%	3
Quite a bit		25.0%	7
A lot		64.3%	18
			Totals: 28

(b) How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)

Value		Percent	Responses
Some		7.1%	2
Quite a bit		35.7%	10
A lot		57.1%	16
			Totals: 28

(c) Supporting participants to have powerful and effective lesson study experiences

Value		Percent	Responses
A little		7.1%	2
Some		10.7%	3
Quite a bit		46.4%	13
A lot		35.7%	10

Totals: 28

(d) Writing a good lesson plan

Value		Percent	Responses
A little		10.7%	3
Some		28.6%	8
Quite a bit		35.7%	10
A lot		25.0%	7

Totals: 28

(e) Evaluating the quality of a lesson plan

Value	Percent	Responses
A little	3.6%	1
Some	50.0%	14
Quite a bit	25.0%	7
A lot	21.4%	6

(f) How to observe research lessons

Value	Percent	Responses
Some	10.7%	3
Quite a bit	57.1%	16
A lot	32.1%	9

Totals: 28

(g) Organizing a successful post-lesson discussion

Value	Percent	Responses
Some	28.6%	8
Quite a bit	39.3%	11
A lot	32.1%	9

Totals: 28

(h) The role of the knowledgeable other

Value	Percent	Responses
Some	17.9%	5
Quite a bit	21.4%	6
A lot	60.7%	17

Totals: 28

(i) How to support student problem-solving

Value	Percent	Responses
A little	3.6%	1
Some	32.1%	9
Quite a bit	32.1%	9
A lot	32.1%	9

Totals: 28

(j) How to build students' mathematical habits of mind and practices

Value	Percent	Responses
A little	3.6%	1
Some	39.3%	11
Quite a bit	39.3%	11
A lot	17.9%	5

Totals: 28

(k) How to organize the board

Value		Percent	Responses
Some		21.4%	6
Quite a bit		60.7%	17
A lot		17.9%	5
			Totals: 28

(l) Teacher questioning techniques

Value		Percent	Responses
A little		7.1%	2
Some		53.6%	15
Quite a bit		35.7%	10
A lot		3.6%	1
			Totals: 28

(m) How to summarize a lesson

Value	Percent	Responses
Not at all	3.6%	1
A little	21.4%	6
Some	46.4%	13
Quite a bit	21.4%	6
A lot	7.1%	2

Totals: 28

(n) Anticipating student responses

Value	Percent	Responses
Not at all	3.6%	1
A little	3.6%	1
Some	42.9%	12
Quite a bit	32.1%	9
A lot	17.9%	5

Totals: 28

(o) How teachers support whole class discussion (neriage)

Value	Percent	Responses
A little	7.1%	2
Some	42.9%	12
Quite a bit	42.9%	12
A lot	7.1%	2
		Totals: 28

(p) Strategies for making students' thinking visible

Value	Percent	Responses
Some	42.9%	12
Quite a bit	42.9%	12
A lot	14.3%	4
		Totals: 28

(q) Student note-taking

Value	Percent	Responses
Not at all	3.6%	1
A little	7.1%	2
Some	42.9%	12
Quite a bit	35.7%	10
A lot	10.7%	3
		Totals: 28

(r) Knowledge about the Japanese educational system in general

Value	Percent	Responses
A little	3.6%	1
Some	21.4%	6
Quite a bit	39.3%	11
A lot	35.7%	10
		Totals: 28

(s) Mathematics content

Value	Percent	Responses
A little	7.4%	2
Some	11.1%	3
Quite a bit	51.9%	14
A lot	29.6%	8

(t) Knowledge about Japanese curriculum materials

Value	Percent	Responses
A little	10.7%	3
Some	35.7%	10
Quite a bit	35.7%	10
A lot	17.9%	5

Totals: 28

(u) How to develop a mathematics unit/curriculum

Value	Percent	Responses
Not at all	3.6%	1
A little	17.9%	5
Some	50.0%	14
Quite a bit	21.4%	6
A lot	7.1%	2

Totals: 28

9. If you had other learning experiences (not listed above) during the immersion trip, please describe them below (you can add up to 3) and rate how much you learned about each of them

22. Other Learning Experience 1:

Response

Collaborating with colleagues

Collaboration: Informal discussions to reflect and process with someone who teaches my grade level (middle school math).

How Japanese culture affects school settings

How Lesson Study can be used to meet the goals and vision of a school/district.

How teachers are treated as professional learners in Japan.

Japanese Culture - I greatly appreciated the accessibility of the schools, staying slightly "off the beaten path," being given time to explore...seeing more of the culture really helped to put the education system into a greater context.

Lectures from Dr. Takahashi: -on teacher pedagogy (questioning and listening to students) -on planning caefully and deviating from the textbook in a purposeful way

Self Reflection

The importance of teachers across grade levels knowing well the math content being taught and the vertical alignment of that content.

The role of talk in mathematics lessons in Japan

Understanding of the culture of school in Japan

Other Learning Experience 2:

Response
Teacher training in Japan (in discussions with the graduate students)
The level of parental involvement in children's education (supplying resources as much as supportive attitudes).
The use of manipulatives in mathematics in Japan
Thinking of content used in the tasks.
Touring Oshihara Elementary school: talking with students and teachers in a smaller context (in the classroom during lunch, specifically)
Understanding of student independence in the Japanese learning process

Other Learning Experience 3:

Response

Levels of teaching

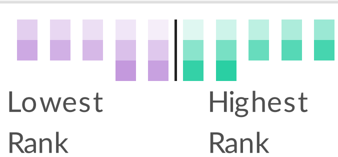
Post-lesson discussion on 6/28 on number selection and the importance of fostering the habit of emphasizing the process over the product

The independence Japanese children are entrusted with and how they are expected to contribute to the running and care of their school.

10. Please select and rank in order of importance the five items from the previous two questions that you believe will be most professionally useful for you within the next year. (Drag and drop your top five from the left-hand list to the right-hand column.)

Item	Overall Rank	Rank Distribution	No. of Rankings
(i) How to support student problem-solving	1		15
(n) Anticipating student responses	2		15
(j) How to build students' mathematical habits of mind and practices	3		11
(g) Organizing a successful post-lesson discussion	4		10
(a) Important features of lesson study	5		10
(k) How to organize the board	6		10
(s) Mathematics content	7		9
(c) Supporting participants to have powerful and effective lesson study experiences	8		8
(o) How teachers support whole class discussion (neriage)	9		8
(h) The role of the knowledgeable other	10		8
(f) How to observe research lessons	11		7
(m) How to summarize a lesson	12		6

Item	Overall Rank	Rank Distribution	No. of Rankings
(p) Strategies for making students' thinking visible	13		5
(b) How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)	14		5
(l) Teacher questioning techniques	15		5
(r) Knowledge about the Japanese educational system in general	16		4
(e) Evaluating the quality of a lesson plan	17		3
(d) Writing a good lesson plan	18		3
(u) How to develop a mathematics unit/curriculum	19		3
(t) Knowledge about Japanese curriculum materials	20		2
(q) Student note-taking	21		2
(v) Additional learning experience 1 from Question 9	22		1
(w) Additional learning experience 2 from Question 9	23		1
(x) Additional learning experience 3 from Question 9	24		1



11. Please comment on the schedule/timetable of the program. Was there enough time for preparation and review? Other issues that would help in planning future programs?

Response

I found the timetable worked extremely well. The level of organisation, the support from the graduate students and the team was excellent. The meals provided for us were wonderful experiences and going to Yamanashi unforgettable. The jet lag was very hard even though I came out a day early. I'd definitely recommend that, where possible, people perhaps travel before the study trip starts to give themselves more time to adjust. We were unclear about when to take/give out our gifts and many of us missed this opportunity; specific advice on this on the daily information would be much appreciated. Hotel: loved staying in Kokubunji area - so Japanese and great to be so well connected but out of the city.

I liked the schedule. Having a later start time was nice - I felt I could then have more time to reflect on the previous day and prepare for the next lesson while still having the ability to explore the culture more in the evening. If possible, adding more time for post discussion amongst the group after observing a lesson would be helpful.

I really appreciated how we were able to observe a lesson and post-lesson discussion within one day. I feel that if trying to do more in one day would be too overwhelming for participants in order to make sense of what was observed that day. Prep time and review was sufficient.

I think it would have been useful to have some time to "have a go" at the Maths for each lesson prior to the lesson (like we did with the Rica and Koto origami question). I also think it would have been useful to have our own post-lesson discussion immediately after the lesson and before observing the next lesson.

I think the programme was fantastic - the variety of experiences built in and all of the wrap around organisation so that we experienced more than just maths lessons. I did find I struggled to have time to process everything and I know that I will still be trying to find time to do this over the coming weeks and months but I think that is the nature of the experience. There was only one place where I felt that it could have been improved and that was when we observed three lessons in a row without having a chance to discuss them until two days later (this was around the Yamanashi visit). I am not sure how this can be avoided; perhaps we needed to discuss lesson 3 on the same day, after the post-lesson discussion and before hitting the hot springs. Then for lessons 4 and 5, if the next day wasn't the free day this would have been better. I would have coped better if there was one lesson which we had to discuss two days later, after the free day, but carrying three across was difficult a

Response

I think the schedule/timetable of the program was extremely effective. The only part I disliked was watching two lessons back to back at the conference. It was hard for me to stay focused and engaged during the second lesson and I wish the post-lesson discussion could have been more focused (ex: just on one lesson).

I think the schedule/timetable was BRILLIANT. It was incredibly organized and effectively executed. I was very impressed that start and end times were honored. When that expectation was clearly stated at the outset, people adhered to it.

I think the timetable was challenging, however manageable.

I thought the schedule was done well, however the day we observed two lessons was extremely difficult. I would have like to have had snacks/water/coffee. It was difficult to sit that long without breaks. Also, on some days the feedback in the microphone made it difficult to hear the lessons.

I thought the schedule was pretty good with plenty of time for preparation and review. I think maybe an additional day off somewhere in the program would have been helpful to feel more refreshed on the other days. I also think it would have been helpful to mix in some different types of activities. Like maybe mixing the lectures days in between the lesson study days so that there weren't so many days in a row of doing the same thing. I also think in the future it would be really helpful to have professional translators. Many of the post-lesson discussions weren't accessible to us because of the translation.

I thought the whole program was excellent and very well planned / executed. I think people would have liked a little more information about the education system and general do's and don'ts in Japanese culture at the beginning.

I would have liked more time and structure in doing the math and analyzing lesson plans before observations.

I would have liked more time to debrief the lessons and get the opinions of the Gakugei professors, and Dr. Takahashi, Dr. Watanabe, Dr. Yoshida. There was sufficient time to preview/prepare for the research lessons though.

I would have loved a final day in the university to complete the lesson report and surveys. I also would really encourage the program to hire professional translators. The lesson plans were translated beautifully but I was unable to learn how teachers lead students to effective whole group conversations because live translation was often lacking or confusing. I have been to a public lesson in Tokyo before and had live translation done by a translator. I felt like translation by an expert was really beneficial. I understand that a translator would lack math/teaching knowledge but this is their profession and I'm sure if provided with the lesson plans they could prepare.

Response

It was exhausting however I do not see another way that it could have gone. Perhaps adding the specifics to when we were previewing and when we were reviewing to the 10 day agenda??

It was great to have all the experiences we had, but it did feel like we needed a bit more time to digest, reflect, and talk about what we were learning. I think the Saturday district-wide lessons could possibly have been eliminated. Many of us were really tired and in need of thinking about what we had been seeing. I think a reflection/Q and A at that time would have been great (since we had another chance to see the district PD model later that week). We needed more time for Q and A, as we clearly ran out of time for that on our last day together. It's great to see a lot of lessons, but what we wonder about and make sense of from them is the most important part. Less is more, in this case.

It was planned perfectly! One lesson a day was ideal. Maybe more time could be spent before the lesson solving the problem on our own/in groups? We did this a few times, but I think it would be a useful exercise before every lesson. This content preview might help participants better absorb the lesson during the observation. Also, I would have loved to see a research lesson and then watch the next day in class, as well. This often gets brought up during post-lesson discussions (i.e. "What did this lesson change for your class in regards to tomorrow's lesson?") maybe in the future, participants could watch the same class two days in a row? The second day wouldn't be a research lesson, but more of an observation and would be helpful as we transition back into our own classrooms.

More time to practice the math prior to observing the lessons.

My reccomendation is that I think it would have been helpful to spend a bit more time working on the math. Also at times it got difficult for me passivly listening to others for extended periods of time.

Overall I feel like there was a good amount of time allotted to prepare for lessons. I think it would have been helpful to have time built into the schedule for lesson reflections though.

Pedagogically, the four types of 'activity' were a)listening to lecture b)observing lessons c) listening to post lesson debrief and d) open discussion of the lessons. It would have been nice if the program had stated what skills it wanted us to leave with, and pushed us to work more narrowly on those skills. For example, was the program interested in honing our observation skills as we watched the lesson? If so, could observation templates or an expectation around other protocols for how we observed be set? Likewise, if becoming more skilled in lesson plan development was a goal, we could have critically analyzed the lesson plans themselves to reveal how they supported or did not support the research goal.

The flow of the program worked well for me. Some days felt very long, with a lot of input and sitting.

Response

The post lesson discussions were hard to follow because the translations were not always clear. Sometimes the students/teachers would say things and the translator was not able to keep up. I would have liked less IMPULS participant input and more expert commentary and lectures from Dr. T and/or other math experts.

The schedule was great - plenty of time for preparation and review.

Timing was impeccable.

While I enjoyed it, not sure that 8 school visits for necessary to get a full picture. More time for debriefing, Q&A, lectures by the IMPULS leaders, and ability to deeply discuss what we were learning would have been useful. I also think waiting for the last day to engage in a Q&A was not the most strategic - had we understood some of the concepts raised on the last day earlier on, we would have better been able to contextualize what we were observing.

Yes, I felt like the program was the perfect balance between observing, listening, and discussion. At times, I wanted to discuss more but I found people that were willing to continue to debrief with me.

12. Are there remaining questions you still have?

Response
Can I do it again?
Can we go to Finland next?
I am just wondering if there would be a way moving forward that the participants could connect in kind of an "exchange" of sorts where we travel to each other's schools for research lessons, etc. This is probably something that we could connect on our own about, but I think it would be nice to have something like this.
I really wish we could have seen at least one primary public lesson. I still have many questions about what board work looks like, summaries, student journals, and the general flow of the lesson. I hope this is something that me and my team get more opportunities to engage with in the near future.
I still have questions about the Japanese curriculum. How exactly do the students use their notes? I saw many students with a single sheet of loose leaf paper instead of a notebook. Do they keep this loose leaf in a binder? Do they keep other things in the binder? What about homework? I would love to see all materials that students use during math over the course of a school year. Maybe participants would benefit from a more explicit sort of workshop to understand how exactly Japanese teachers plan with their texts.
I would still like to see lesson study in action in primary classrooms. I'm looking forward to future immersion opportunities that address this.
I'd like to be part of a wider group who can help me get things going now we're back in the UK. I believe this will happen and I'm grateful for this opportunity. And of course, 'When can I go back!'
I'm wondering if the program can help create regional lesson study networks. It would be really great to receive support from participants of years past and potentially open up our public lessons to one another.
In England we have "Ofsted" who come and make judgements on schools. Is there a similar system in Japan? Do Japan do as well in all the other subjects (such as Science, Geography etc) as they do in Maths?
Many! But I feel empowered to go out and work towards finding the answers.
Many, but I'm still processing most of them.

Response

Of course! I would like to know what they have done in Japan to support teachers in how to observe, how to critique each other and how to design lessons (this was mentioned but without details) as I know this is something we need to do. I am still not clear about whether parents ever attend the public sessions - I think we were told yes and no during the programme. I am also still not clear on what the professors mean by 'the most important point' in a lesson - what does this mean to them because I think it may mean different things to different people. I also have questions about how the final commentators prepare for the final commentary. And that's just for starters!!

Of course. Watching the olympics does not make me an olympic athlete. Watching excellent bansho or reading lesson plans doesn't on its own make me able to design board work or write lesson plans that support TTP.

Structures and tools for how lesson study learnings are shared with the district and ministry of education.

We discussed a little about the articles we were given ahead of time on the first day. I would have liked to talk through a little more the idea of "re-teaching" as discussed in the Fujii article. Also, more information on special education in Japan and the "newer" tracking idea that was touched on in a few of the discussions.