International Math-teacher Professionalization Using Lesson Study

## Grade 3 (Classroom \# 1) Mathematics Lesson Plan

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1. Title of the Unit: Let's think about ways to simplify calculations

## 2. About the Unit:

In this unit, students learn about mental calculations of the sums and the differences of two 2digit numbers (including those that require regrouping).

This unit is constructed to address the following points mentioned in the Elementary School Mathematics Course of Study.

## 3. The Content of Grade 3

A. Numbers and Calculations
(2) Students will be able to add and subtract accurately and reliably and will further enhance their ability to use those operations appropriately.

## Remarks Concerning Content

(2) As for the contents A-(2), (3) students should be able to calculate simple calculations mentally.

In the unit, Better Ways to Calculate, in Grade 2, students learned about calculating 2-digit numbers $\pm 1$-digit numbers by decomposing the numbers using the structures of numbers. In addition, students have already learned how to calculate 2 -digit numbers $\pm 2$-digit numbers using the algorithms. In a 3rd grade unit, Let's Take Another Look at the Multiplication Table, students have explored ways to calculate 2-digit numbers $\times 1$-digit numbers like $12 \times 4$ by the multiplicand, 12, into 8 and 4 or using 10 as the base, 10 and 2 . These units have a common theme, "ways to simplify calculation." In this unit, students will study mental computation based on the same theme.

With respect to mental calculation, Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grade 1-6) makes the following point:

Simple mental calculations indicated in Remarks Concerning Content A-(2) include addition of 2-digit numbers or subtraction that is their inverse. This type of calculation is often used in daily life and is even a necessary part of the process of multiplication and division. Mental calculations are often used to make estimates in daily life. It is important to consider these kinds of applications when teaching. (pp. 90-91)

As noted here, mental addition and subtraction of 2-digit numbers are used in a variety of everyday situations and students' future study. Therefore, it is important for students to understand the needs for mental calculations.

In addition, study of mental calculations may also enrich students' number sense. For example, thinking with approximate numbers ( 38 is "about 40 ") or thinking about a number in relationship to another number ( 38 as " 2 more to make 40 ") are very important parts of fostering number sense. Therefore, we intend to have students deal with compositions and decompositions of a variety of numbers.

The following two points will be kept in mind as we teach this unit.
First, the same calculation may be done in different ways mentally. For example, consider ways to calculate $48+36$ mentally. The following methods are shown in the textbook (New Elementary School Mathematics, 3A, by Tokyo Shoseki, p. 60):


| $48+36$ |
| :---: |
| Think of 48 as 50 |
| Think of 36 as 40 |
| $50+40=90$ |
| $2+4=6$ |
| $90-6=84$ |

There are other ways to calculate the sum. For example, "48 is 2 from 50 , so decompose 36 to 2 and 24 , and calculate: $48+2=50,50+34=84$." The value of the study of estimation is to think about ways to calculate using various properties of operations. Therefore, while acknowledging each student's method, we want to help each student identify more efficient approaches on his or her own while exploring a variety of ideas.

The fact that a variety of methods are possible means that students will have opportunities to encounter many different ideas. Thus, by sharing each other's ideas, students have opportunities to interpret other students' reasoning processes. Through these opportunities, students may discover new ideas of their own or re-examine their own ideas with new insights. Therefore, it is important to establish an opportunity for students to examine and understand a variety of ideas.

Another point to be kept in mind as we teach this unit is to design the lessons so that children can imagine how mental calculations may be utilized in everyday situations. One way to experience the merits of mental calculation is for students to identify situations in their everyday life or in their study where mental calculation is being used and actually use the ideas from this unit in those situations. Therefore, a goal of this unit is for students to develop a disposition to actively use the ideas they learn in this unit in mental calculation they do in their everyday situations while helping them realize mental calculations are used in many situations in their daily living.

## 3. Goals of the Unit:

* As we set up the goals of the unit, we kept the following points in mind so that the goals will reflect mathematics teaching practices that are based on the foundation of the career education.
- In "Goals of the Unit," list both goals of the subject matter (mathematics) and goals of the career education. This is done so that the goals of the unit and the goals of the career education may be compared and linked with each other. Moreover, by listing the goals in parallel, it may be easier to see the relationships to the goals in other subject areas.
- Since Moral Education is the foundation of every subject matter, how it is related is shown.



## 4. About Research Theme:

In order to raise students' "ability to think coherently by anticipating and to represent their ideas," it is helpful to give students tasks in which students experience disequilibrium and experiment with many different ideas. That is because such tasks will generate the process to examine what they have learned previously and investigate ways to use their prior knowledge as a starting point to solve the given tasks.

However, as I look at the students in this particular class, or 3rd grade students in general, many students cannot represent their thoughts. It is not rare that students cannot reach a conclusion because they were unable to summarize their thoughts.

Therefore, in this unit the emphasis are placed on the following. First, students should make explicit their ideas on "for what purpose" and "what strategy will be used." Then, students will be prompted to organize their thoughts by using words such as "first" and "next," or making a numbered list. By doing so, own ideas can be expressed more clearly and others can interpret them more easily. It is also possible that by making one's thought and the thinking process clearer, students' desire for future learning may be heightened.

In studying mental calculation, because mental manipulation of numbers is necessary, it is usually not desirable to have students record their methods in notebooks. However, in order to devise a faster and simpler methods of mental calculation, it is necessary to express one's own thought processes so that they may be compared with other ideas. It is through that examination, students can identify more efficient method. Therefore, in this lesson, we will incorporate an activity
in which students will represent and examine mental calculation processes. The goal is to help students develop more efficient methods of mental calculation through this careful examinations.
(1) Children who "think coherently by anticipating and represent their ideas"

In this unit, it is hoped that students will exhibit some of the following.

- Estimating the sum by approximating one or both of the addends.
- Thinking about different ways to calculate by applying properties of operations.
- Representing their strategies to simplify calculation in equations, diagrams and words.
- Recognizing the merits of other students' ideas and try to use them.
(2) Teaching that will heighten students' "ability to think coherently by anticipating and to represent their ideas"
(1) Consider ways to pose the learning task

In the "Grasp" stage of the lesson, pay close attention to students' ideas, discuss them as a class so that students can understand the task. Ideas like "Can we use what we have learned previously?" "What is different from what we have learned so far?" and "What idea may be useful as the starting point?" can help students anticipate how the solution may be developed. On the other hand, "The answer is about ..." suggest students are anticipating the results. By sharing students' ideas as a whole class, encourage each student to tackle the task on his or her own with clear vision toward a solution.
(2) Make students record their calculation methods in their notebooks

Instead of just writing the results of mental calculations in their notebooks, students will be required to write down their thinking processes. Students will be encouraged to use phrases such as "because $\sim, "$ "in order to $\sim, "$ and "at first I did $\sim$, then $\sim$ " so that the purposes and the processes are clearly recorded. Students should also use equations and expressions to represent their thinking processes, not just in words.
(3) Set up opportunities where students can share their ideas with classmates

When we try to communicate own ideas to others, we reflect on our ideas. In order to help students think about "how can I more effectively communicate my idea to my friends" and "in what order should I describe my ideas," we will set up opportunities where students can share their ideas with each other.
(4) Set up an activity in which students may interpret other's ideas and utilize them

It is intended that students will try to think about the meaning of expressions and equations included in other students' solutions and to represent ideas presented verbally with equations and expressions. In this way, we want students to use multiple representations to express each other's ideas. In addition, in order to help students identify an efficient mental calculation strategy, an activity to use other students' ideas will be incorporated.

## 5. Unit Plan (total of 2 lessons):

|  | Main Task | Evaluation Criteria | Career Education Perspectives |
| :---: | :---: | :---: | :---: |
| 1 | Students will individually think about ways to mentally calculate $48+36$. Students will share and analyze their strategies. | - Students will acknowledge the merits of mental calculation and try to use it in everyday situations and in schools. [Interest, Eagerness, and Attitude] <br> - Students will be able to mentally calculate the sums with two 2digit numbers by making use of the structures of numbers and properties of operations [Mathematical Skill] | Student will be able to select and use an efficient approaches from a variety of ideas. <br> [Selection Ability] <br> Student can reflect on what they have previously learned, explore possible solution approaches, and solve the problem at hand. <br> [Problem Solving Ability] |
| 2 | Students will individually think about ways to mentally calculate 53-28. Students will share and analyze their strategies. <br> [today's lesson] | - Students will be able to think about ways to calculate mentally by looking at numbers flexibly such as decomposing them or using approximations, and they can explain their reasoning processes. [Mathematical Way of Thinking] <br> - Students will be able to mentally calculate the differences of two 2digit numbers. <br> [Mathematical Skill] | Student can listen to other people's ideas and try to understand each other. [Communication Ability] |

## 6. Today's Lesson:

## 1. Goal of the lesson:

Students will be able to think about ways to calculate mentally the differences of two 2-digit numbers and explain their ideas.

## 2. The aim from the career education perspective:

Student can listen to other people's ideas and try to understand each other.
[Communication Ability]
3. Date: Thursday, June 28, 2012, 1:50-2:35 pm (5th period)

## 4. Location:

Oshihara Elementary School, Showa Township Schools, Grade 3 (Room 1)

## 5. Intent of the lesson:

Students can tackle mental calculation of the differences of two 2-digit numbers in this lesson by reflecting on the discussion of mentally calculating the sums of two 2-digit numbers in the previous day's lesson. Students will be thinking about ways to mentally calculate by decomposing numbers based on different properties of operations and using approximate numbers. It is hoped that students will be able to take advantage of their learning in the previous lesson. Therefore, in today's lesson, the emphasis is placed on representing own ideas using equations, expressions, diagrams, and words, and organizing their mental calculation processes clearly. For this purpose, it is necessary that students must
interpret other's ideas and compare and contrast with their own ideas. During the whole class discussion, the activity of interpreting equations and expressions and representing ideas expressed in words using equations and expressions. Then, by having students reflect on their own ideas in light of other's ideas, we hope to help students identify more efficient (faster and simpler) ways of mental calculation.
6. Flow of the lesson:

|  | Contents \& Learning Task | Points of Considerations/ Materials | Evaluation |
| :---: | :---: | :---: | :---: |
|  | 1 Understand the task. $\square$ - 28 <br> (1) Anticipate. <br> (1) Enter one of the numbers, 89, 53, and <br> 68, in the box. <br> - It is easy to calculate with 89 . <br> - With 68 , the numerals in the ones place are the same, so calculation is easy. <br> - With 53 , calculation is more complicated because we must regroup. <br> (2) Check the answer for 53-28 by calculating it using the subtraction algorithm. <br> Let's think about ways to mentally | - Ask for the reasons why calculation is easy or complicated. Help students develop ideas for tackling the learning task. <br> - Confirm that mental calculation of $53-28$ is more complicated. <br> calculate 53-28. |  |
|  | 2 Think about ways to mentally calculate. <br> (1) Find the difference mentally. <br> a) Decompose 53 into 50 and 3 <br> $50-28=22,22+3=25$ <br> b) Decompose 28 into 20 and 8 $53-20=33,33-8=25$ <br> c) Add 5 to 53 <br> $58-28=30,30-5=25$ <br> d) Approximate 28 as 30 <br> $53-30=23,23+2=25$ <br> e) Add 2 to both 53 and 28 $(53+2)-(28+2)=55-30=25$ <br> (2) Record the calculation processes in notebooks. | - Encourage students to think about their strategies for adding 2-digit numbers mentally. <br> - Remind students to use diagrams (arrows, segments to show how numbers are split), words, and equations. | Can students think about ways to simplify the calculation? Can they represent their ideas in words, <br> equations/expressions, and words? (Check students' notebooks) [Mathematical Way of Thinking] |


| 㓪 | 3 Whole class discussion <br> (1) Share mental calculation processes. <br> - Explain ideas represented by equations and expressions. <br> - Represent explanations given verbally using equations and expressions. <br> (2) Think about efficient processes. <br> (3) Use the processes that were judged to be efficient and solve 83-15. | - Acknowledge and support orderly explanations that incorporate words like "first," and "next." <br> - Clarify what idea contributes to the efficiency. <br> - Help students realize that the solutions c, d, and e used properties of operations to eliminate the need for regrouping. | Are students trying to understand each other's ideas? <br> [Communication Ability] |
| :---: | :---: | :---: | :---: |
|  | 4 Summarize <br> (1) Pose a problem in a shopping situation [(amount you have) $-($ price $)=($ amount left)] and have students solve it mentally. (72-48) <br> (2) Have students write a journal entry. <br> - I learned a simple mental calculation process by listening to my classmates' ideas. I want to use it when I go buy something. <br> - I was able to write down and explain my ideas clearly. | - Give a daily situation where mental calculation may be used. <br> - Make suggestions so that students can incorporate the following in their journal entries. <br> - How did your mental calculation processes changed. <br> - What made explanations easier to understand. | Were the students able to do 2-digit minus 2-digit subtraction by mental calculation? <br> [Skills] (Check students' notebooks) |

## 7. Evaluation:

- Were students able to think about and explain ways to simplify mental calculation processes to find the difference of two 2-digit numbers?


## 7. Evaluation from the career education perspective throughout the unit:

In this unit, the evaluation of communication ability, selection ability, and problem solving ability will be primarily through students' notebooks and in-class discussion. Within each lesson, situations will be set up so that students can explicitly think about a specific ability. For example,

- Acknowledge and share with the whole class any student who exhibited the intended career education abilities.
- During the journal writing time, suggest students to reflect on the career education abilities.

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## References

1．新しい算数 3 上 教師用指導書 指導編（2010）pp．92－95 東京書籍
2．新しい算数 3 上 教師用指導書 研究編（2010）pp．66－67，pp．188－195 東京書籍
3．文部科学省（2008）「小学校学習指導要領解説 算数編」 東洋館出版社
4．中村享史（2002）「『書く活動』を通して数学的な考え方を育てる算数授業」pp．11－12 東洋館出版社
5．中村享史（2008）「数学的な思考力•表現力を伸ばす算数授業」pp．72－85 明治図書
6．全国算数授業研究会（1999）「満載 算数的活動」 pp．114－117 東洋館出版社
7．新算数教育研究会（2010）「講座 算数授業の新展開3 第3学年」pp．54－61 東洋館出版社

## Translator＇s Note

－References $1 \& 2$ above refers to the textbook．An English translation of this textbook series may be purchased from Global Education Resources（www．globaledresources．com）．
－Reference 3 is from Elementary School Teaching Guide for the Japanese Course of Study： Mathematics（Grades 1－6），a Ministry of Education document that provides detailed explanations of the Course of Study．It is available online at http：／／e－archive．criced．tsukuba．ac．jp／data／doc／pdf／2010／08／201008054956．pdf
－Other references are from books that are available only in Japanese．

