June 20
Ver.2

Grade 3 Mathematics Lesson Plan

How many packages can we make? How many will be left? (Division with remainders)

Friday, June 20, 2014
Grade 3 Room 1 (23 students)
Teacher: Kohko Morita

Research Theme: "I did it! I got it!" Designing mathematics lessons students will be engrossed: Teaching strategies that value students' questions and help students enjoy reasoning and expressing themselves.

1. Name of the Unit: Division with remainders

2. Goals of the Unit

   Students will understand the meaning of division and be able to use it.
   a. Students will learn about division with remainders and think about ways of calculating the answers.
   b. Students will understand the relationship between division and multiplication and subtraction.
   c. Students will be able to calculate accurately division where the divisors and the quotients are both 1-digit numbers.

3. Assessment Standards for the Unit

Interest, Eagerness, and Attitude (IEA)	Students realize that division can be used even when there will be remainders, and they try to use division in various situations.
Mathematical Way of Thinking (MT)	Students can think about the meaning and the ways of calculating division with remainders using manipulatives and drawings while making connections to concrete situations.
Mathematical Skills (MS)	Students can calculate division with remainders, and they can deal with remainders appropriately.
Knowledge and Understanding (KU)	Students understand the meaning and appropriate use of remainders, and they understand how to calculate division with remainders using the division algorithm.

4. About the Unit

(1) Goals

In the previous unit, Division, students learned about the meaning of division and the way to determine the quotients (without remainders) using the basic single-digit multiplication facts. In this unit, the goal is to deepen students' understanding of division by examining division with remainders. Students will think about the meaning of the remainder and think about ways to calculate division with remainders. In addition, they will explore the size relationship between the divisor and the remainder.

(2) About students

Students in this class are generally enthusiastic about mathematics. They are willing to share their ideas freely, and they seem to enjoy tackling problems arising from the tasks given during mathematics lessons.

On the other hand, there are significant individual differences in students' mathematical knowledge, skill mastery, the ability to apply their knowledge, and the ability to express their ideas. During the previous unit, Division, there were students who could represent problem situations using diagrams on their own and explain their ideas logically, while others had difficulty making sense of problem situations and had to work closely with the teacher and with the aid of manipulatives.

(3) Mathematics in the Unit

In this unit, students will think about the meaning of remainders while examining both division without remainders and division with remainders. In addition, students will explore the size relationship between the divisors and the remainders by varying the dividend while keeping the divisor constant. By engaging in those explorations, it is hoped that students can expand the range of numbers in which division can be used.

Moreover, another goal of the unit is to nurture students' ability to think logically and express their ideas clearly. To do so, an emphasis will be placed on activities in which students will devise ways to calculate division with remainders and explain their ideas to other students, utilizing what they have learned up to this point such as multiplication, division, and various diagrams.

5. Scope and Sequence

【Grade 2】 Multiplication

- Meaning of multiplication
- Basic single-digit multiplication facts

【Grade 3】 Multiplication

- Multiplication with 0
- Multiplying multiples of 10 and 100 by 1-digit multiplier
- Multiplication algorithm; mental calculation

【Grade 3】

- Meaning of division
- Division (without remainder) using the basic multiplication facts; dividing 1 and 0

【Grade 3】 Division with

- Division with remainders using the basic single-digit multiplication facts.

【Grade 3】 Division with

- (2- and 3-digit number) × (1-digit number)

【Grade 4】 Division: 1-digit

- Division algorithm
- Dividend = Divisor × Quotient + Remainder
- (1- ~ 3-digit number) ÷ (1-digit number)

【Grade 4】 Division: 2-digit

- Division with 2-digit divisors
- Properties of multiplication and division

6. Unit Plan (Total of 5 lessons)

Sub-Unit	No.	Learning Activity	Assessment			
			IEA	MT	MS	KU
Division with remainders	1	• Explain the meaning of division with remainders using words and diagrams. • Think about and explain ways to calculate division with remainders using diagrams or by applying the reasoning used while calculating division without remainder. • Examine the size of the remainders and develop a new question about the size relationship of the divisor and the remainder.	○	◎		○
	2	• Verify that the remainder is less than the divisor. • Learn the way to check the result of division calculation.		○	○	◎
Problems	3	• Solve word problems involving partitive and quotitive division problems (with remainders). • Write word problems involving division with remainders from a given picture and sample problems.	○		◎	
Mastery	4	• Deepen the understanding of the unit content.			◎	◎
	5	• Consolidate the understanding of the unit content. • Think about how to evenly split juice in two different containers.		◎	◎	

7. Today's Lesson

(1) Goals of the lesson

- Students will understand the meaning of division with remainders.
- Students can think about and explain ways to calculate division with remainders using diagrams or by applying the reasoning used while calculating division without remainder.
- Students will examine the size of the remainders and develop a new question about the size relationship of the divisor and the remainder.

(2) Proposals in the lesson

Up to this point, students have come to understand the meaning of division. They are also able to calculate division without remainders using the basic 1-digit multiplication facts.

Today's lesson is about division with remainders. For the students, this is the first time they encounter division with remainders. Through today's lesson, they will expand the range of numbers with which they can use division. I have devised some strategies to generate questions like "What does it mean to have a remainder?" or "Can we use division for this situation, too?" as students are presented with the problem situation where there will be left overs when items are distributed. Then, by having students explain the remainder using diagrams, I want to clarify the meaning of the remainder, as well as the meaning of division with remainders.

In addition, through the activity of judging "if there is a remainder" as the motivation, I want students to think about ways of calculating division with remainders on their own and explain their ideas to others. From these experiences, I want students to realize that we can use the basic 1-digit multiplication facts to calculate division with remainders just as we did with division without remainders. Moreover, I want to nurture students' ability to express their own ideas logically by incorporating the activity to explain their ideas using not only words and equations but also diagrams.

Then, at the end of the lesson, I want students to generate the question, "What is the relationship between the divisor and the remainder?" by having students think about the size relationship between the divisor and the remainder.

(3) Specific strategies to address the research theme

① Engage students with a problem and draw out questions. (Grasp)

In the introduction, problems that involve division with remainders will be mixed in with those that involve division without remainders that students have already learned. By doing so, I want to draw out comments and questions like "We can't evenly share these" or "There will be left over. What can we do?" I will then inform them that they can still use division in those situations and help students clearly understand the meaning of division equations using diagrams. Furthermore, by having students think about whether or not 21 ÷ 4 and 33 ÷ 4 can be divided evenly, help students have ideas for how to calculate these division problems and make connection to the independent problem solving time.

② Help students experience the joy of solving problems on their own as they solve their questions. (Explore)

During the independent problem solving time, students will think about how to calculate 21 ÷ 4 and 33 ÷ 4 using mathematical expressions, diagrams, and words. For those students who are stuck, I will provide small group mini-lesson to help students get to "I got it!" and "I did it!"

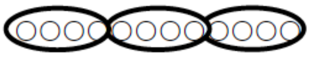
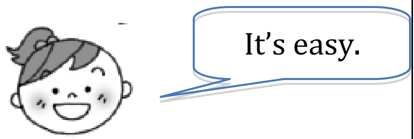
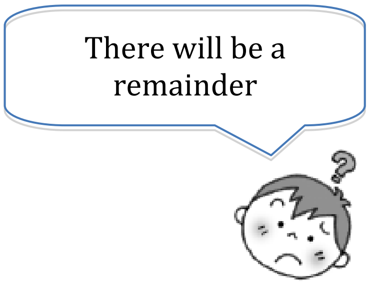
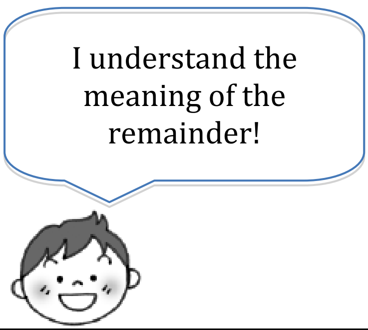
③ Answer their questions and deepen their understanding through the activity to interpret and verify other students' ideas. (Deepen/Heighten)

In this phase, I will have students explain why 21 ÷ 4 = 5 rem. 1 and 33 ÷ 4 = 8 rem. 1 using diagrams. As they do so, instead of simply accepting their words, I will ask "Is it really so?" or "Are you absolutely sure?" to enhance students' ability to explain their ideas logically. Then, I will call on a student who did not use any diagram while we are thinking about ways to calculate 33 ÷ 4 and ask, "How did you figure out 33 ÷ 4 = 8 rem. 1?" Through discussion in pairs and also as a whole class, we will examine ways to calculate division with remainders, and verify the reasoning using diagrams.

④ Draw out new questions through the activity of summarizing and extending. (Summary/Extension)

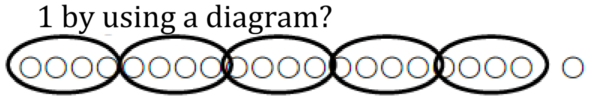
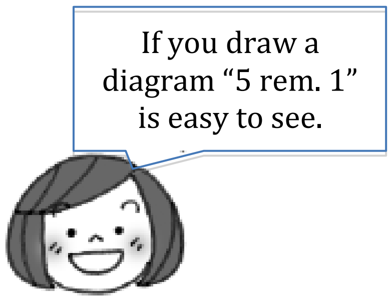
In the summary step of the lesson, we will reflect on the lesson according to students' thinking shared during the lesson. Then, since all division problems discussed in the lesson had the remainder of 1, I want to generate the question, "Is the remainder always 1?" Students should be able to realize that "the remainder can be 2 or 3, too," and I will have them explain their ideas using mathematical expressions and diagrams. Finally, I want to draw out the new question, "Can the remainder be 4 or 5, too?" as the motivation for the next lesson, "I want to investigate the size relationship between the divisor and the remainder."

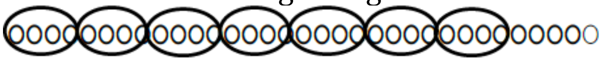
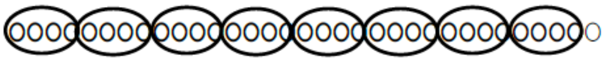
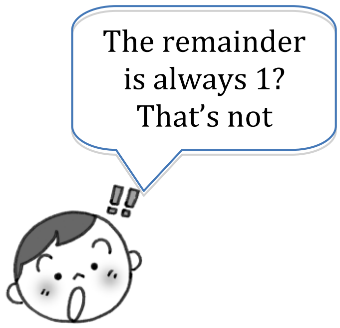
8. Flow of the lesson (Lesson 1 of 5)

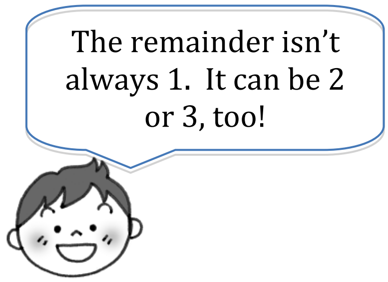
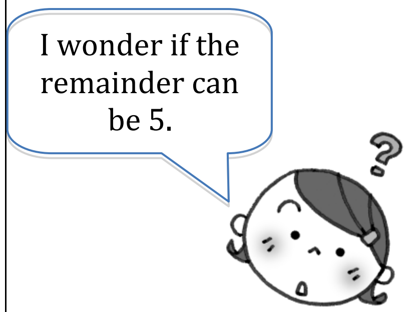
	Learning activities (Main *hatsumon* and anticipated responses)	□ Strategies to address research theme ○ Support and instructional considerations ⊙Assessment
G R A S P	1. Understand the task. There are [ ] pieces of octopus balls. If we put 4 pieces in a pack, how many packs can we make? (If there is any remainder, think about how many will be the remainder.) ← To be written later. T ① What if the number in the [ ] is 12? C We can make 3 packs. C Because 12 ÷ 4 = 3. T If we draw a diagram, it will look like this, right?  T ② What if the number in the [ ] is 20? C Because 20 ÷ 4 = 5, we can make 5 packs. T ③ What if the number in the [ ] is 32? C Because 32 ÷ 4 = 8, we can make 8 packs. T ④ What if the number in the [ ] is 13? C Whoa? Something is wrong. T Why do you say "whoa?" C We can't make 13 exactly. C There will be remainder[1]. T Remainder? What do you mean? Can you draw a picture? C Yes, I can. T Please draw a diagram in your notebook. C I have it.  **3 packs** **remainder** C If there are 13 pieces, we can make 3 groups of 4, and there will be 1 remainder. T I see. If we have 13 pieces, we can make 3 packs, and there will be 1 piece remainder. Even when there is a remainder, like this	□ By discussing division without remainders first, naturally generate the question, "What can we do when we cannot divide evenly?" when the division with remainder is posed as the 4th question. ○ Have a diagram prepared.   □ Have students draw diagrams in their notebooks so that it will be easy to understand. 

[1] It is probably more accurate to translate this statement as "There will be left overs." However, the same Japanese word, *amari*, is used for both "left over" and "remainder." Therefore, the term "remainder" in this lesson plan does not have the significance of formal mathematical term as is the case in English.

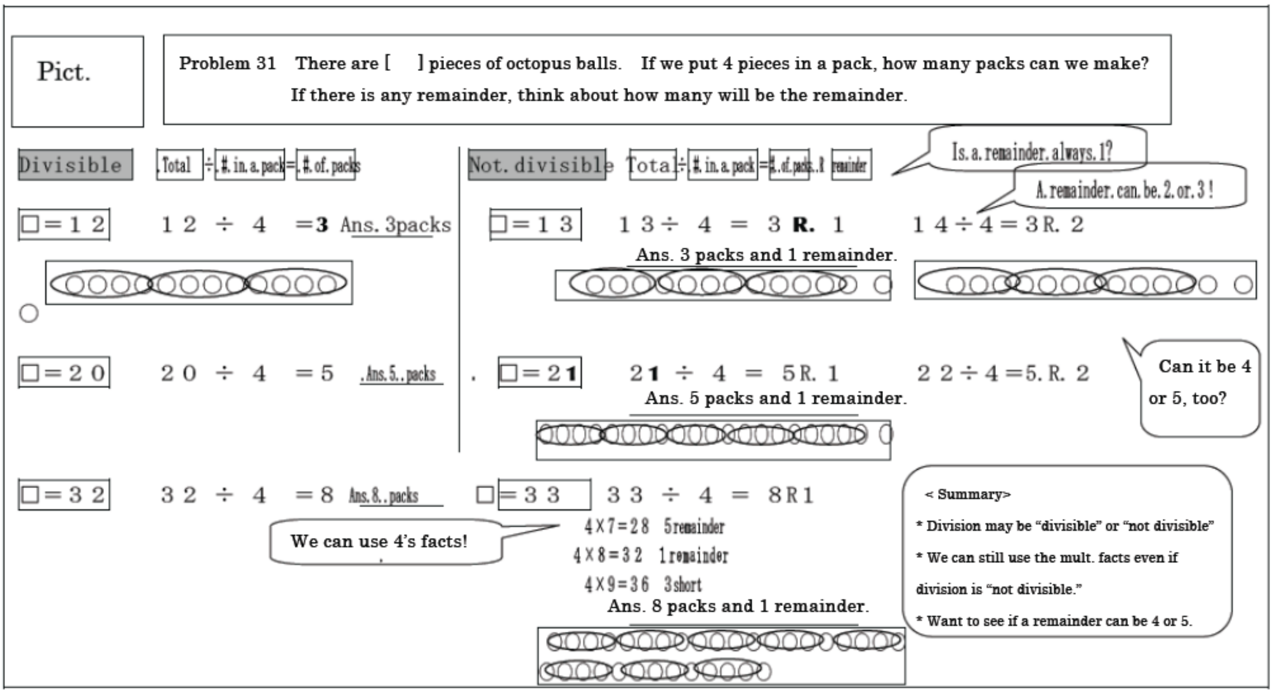
	case, we can still use division. If we represent this situation using a division equation, it will be written as 13 ÷ 4 = 3 rem. 1. Please write it in your notebook. C – Write the division equation below the diagram – T Division we have looked at so far did not have any remainder, and we can always divide equally, can't we? Division like 20 ÷ 4, when there is no remainder, we say "divide evenly." Division like 13 ÷ 4, when there is a remainder, we say "does not divide evenly." Today, let's think about division with remainders. I am going to add something to our problem. Please write, "If there is any remainder, think about how many will be the remainder." T Oh, I have more problems. What if there are 21 pieces of octopus balls? What if there are 33 pieces? How many packs can we make? If we had 21 pieces, do you think there will be a remainder? C Yes. T Do you think there will be a remainder if we have 21 pieces? C Yes. T Are you sure? C Yes, absolutely. T OK, let's find out if there will be a remainder. Please solve 9 ÷ 4 and 21 ÷ 4 using mathematical expressions and diagrams. You can start with either one.	○ Tell students that division can be used even when there is a remainder and show hot it is written. ○ Have students write the additional statement in today's problem. ⊙ Students have their own question and try to solve the problem eagerly. [Interest, Eagerness, and Attitude]
E X P L O R E	2. Represent division with remainders in expressions and think about ways to calculate using diagrams. When there are21 pieces. 21 ÷ 4 = 5 rem. 1  When there are 33 pieces. C: It will be really tedious to draw a diagram.	□ Have students draw diagrams in their notebooks so that it will be easy to understand. □ For students who are stuck, conduct a small group mini-lesson. Hint: 1. What is 20 ÷ 4? 2. Can you draw a diagram to show 20 ÷ 4?

	C: I think we can do it without drawing a diagram. **4× 8 = 32 Since we use 32 octopus balls of 33, there will be 8 packs and 1 remainder.**	3. So, what happens if you have 21 ÷ 4? ⦿ Based on the way to calculate division without remainder they have learned previously, students think about ways to calculate division with remainders using diagrams. [Mathematical Way of Thinking]
D E E P E N	3 Discuss how to calculate division with remainders. T Let's start with the case where there are 21 pieces of octopus balls. Please explain your idea to your neighbor. Please share your equation and answer. 21 ÷ 4 = 5 rem. 1 T How many packs can we make and how many is the remainder? C We can make 5 packs, and there is 1 remainder. T Can someone explain why 21 ÷ 4 = 5 rem. 1 by using a diagram?  C If you look at the diagram, you see there are 5 packs and 1 remainder. So, the equation is 21 ÷ 4 = 5 rem. 1. T Can you see "5 rem. 1" in this diagram? C This part. T I see. I think we can conclude that 21 ÷ 4 = 5 rem. 1. T OK, the other problem with 33 pieces of octopus balls. What is the equation? What do you think is the calculation? C 33 ÷ 4 = 8 rem. 1. So, you can make 8 packs and there will be 1 remainder. T _(name of a student)_ didn't draw a diagram in her notebook. How do you think _____ thought about this calculation? Discuss it with your neighbor.	By incorporating pair-sharing time, give each student to explain his/her idea logically.  By incorporating pair-sharing time, give each student to explain his/her idea logically.

	C I think we can use the 4's facts. T Even when we cannot divide evenly, can we still use the 4's facts? C 4 × 7 = 28, and there will be 5 more pieces. 4 × 8 = 32, and there is 1 remainder. 4 × 9 = 36, and we don't have enough. So, 33 ÷ 4 = 8 rem. 1. T Let's check it using a diagram.  4 × 7 = 28 there are 5 remainders  4 × 8 = 32 there is 1 remainder T We can find the answer for division with remainders using the multiplication facts just as we did with division without remainder, can't we?	
	4 Summarize the lesson and generate a new question about the size of the remainder. T We discussed division without remainder through yesterday. Today, we studied that division sometimes has the remainder. We were able to write equations and diagrams, and we understand the meaning of the remainder. Also, we learned the way to calculate division with remainders. Now we know that we can either divide evenly or we will have the remainder of 1. C No, that's not true. The remainder can be 2. C The remainder can be 3, too. T What? The remainder can be 2 or 3? But look at what we did today. The remainder is always 1. C Yes, but if we change the total amount, we can get the remainder of 2. T OK, please discuss with your neighbor how we need to change the total number so that the remainder will be 2. T If your neighbor's explanation was easy to understand, please raise your hand. OK, __________, could you explain for when we will have a remainder of 2?	By intentionally making a false claim, generate a new question in students and have them express their ideas eagerly.  By incorporating pair-sharing time, give each student to explain his/her idea logically.

	C If you draw a diagram for the case when there are 14 octopus balls. 	
	T OK, so what number sentence should we write? C It will be 14 ÷ 4 = 3 rem. 2. C Because you get 14 when you add 2 to 4 × 3 = 12, the remainder will be 2.	○ Students can explain their own ideas or their friends' ideas. [Mathematical Way of Thinking]
	T I see. We can have a reaminder of 2, too. C It can be 3, too. T What? The remainder can be 3, too? C If we add 1 more, the remainder will be 3. T I see. The remainder isn't always 1. It can be 2 or 3, too. I wonder if there is any division where the remainder will be 4 or 5? C I don't think that's possible … T The remainder cannot be 4 or 5? C Absolutely not. T OK, then let's think about if there is any division where the remainder is 4 or 5 tomorrow.	 
	T Let's write the summary of the lesson. C With division, we sometimes we have the remainder and other times there is no remainder. But, the way to calculate is the same for both cases. C The remainder is not always 1. It seems like a remainder can be 2 or 3. I wonder if it can be 5.	○ Students have a new question and eagerly trying to solve it. [Interest, Eagerness, and Attitude]

9. Board writing plan



10. Observation points for the lesson

1. In order to have students generate their own questions, division without remainders and division with remainders were posed together in the beginning of the lesson. Was it effective?
2. Instead of simply accepting students' responses, the teacher posed follow-up questions and had students discuss their ideas in pairs. Did that strategy lead to the activity of students explaining and expressing their ideas logically?
3. Was the activity of having students draw diagrams in their own notebooks a useful strategy to help students independently organize their own thinking and express their ideas?
4. Other