Lesson 1 Represent a problem situation in an expression in the form of Whole Number $\div$ Decimal Numbers ( 1 decimal place, $300 \div 2.5$ ).
Lesson 2 Think about ways to calculate $300 \div 2.5$ (including the use of the division algorithm).
Lesson 3 Represent a problem situation in an expression in which both the dividend and the divisor are decimal numbers (1 decimal place), and think about ways to calculate.
Lesson 4 Represent a problem situation in an expression in which the dividend is a decimal number with 2 decimal places by a decimal number with 1 decimal place, and think about ways to calculate.
Lesson 5 Represent a problem situation in which the divisor is a decimal number less than 1, and think about ways to calculate.
Lesson 6 Think about the meaning of the remainder whey dividing by decimal numbers.
Lesson 7 Understand how to approximate the quotient when dividing decimal numbers.
Lesson 8 Understand that division of decimal numbers can be used to make multiplicative comparisons.
Lesson 9 Understand that division can be used to make multiplicative comparisons even when the quotients become decimal numbers.
Lesson 10 Understand that we can make comparisons using both subtraction and division. (Today's lesson)

## 3 Proposal in today's lesson

As we examine various achievement test results, we notice that the success rate for problems involving wariai is rather low. There was a problem involving wariai in the 2013 National Assessment, and its success rate was 76.9\%. In each of the 2008, 2009, 2010, and 2012 National Assessments, there was a problem involving wariai, and their success rates were $55.1 \%, 57.1 \%$, $57.8 \%$, and $58.7 \%$, respectively. Although it is difficult to pinpoint the cause for these low success rates, I believe we need to think about teaching that will help students understand the ideas of wariai.

When we compare numbers or quantities, we can do so by either using the difference or bai (the quotient). Wariai is used when we compare using the quotient. For example, think about the following problem. On which day did the
basketball player do better shooting free throws, yesterday when he made 6 of 8 attempts, or today when he made 7 of 10 attempts? If you just consider the number of free throws made, it looks like the player did better today since he made 7 free throws today while he only made 6 free throws yesterday. However, since the player did not make the same number of attempts, we cannot simply compare the number of successful free throws. In this situation, we can use the number of attempts as the base quantity to calculate bai (how many times as much is the number of successful free throws as the number of attempts) for each situation and compare them. Yesterday: $6 \div 8=0.75$ Today: $7 \div 10=0.7$ Therefore, the player was more successful yesterday than he was today. When we compare two situations using the idea of bai, we consider bai (the quotient) as wariai. Therefore, I believe that students will understand the idea of wariai more deeply if they have opportunities to compare situations using the idea of bai before they receive the formal instruction on wariai.

We use bai to make comparisons when the base quantities are not equal. Thus, in the problem situation above, if the player made the same number of free throw attempts yesterday and today, we can just compare the number of successes. For example, suppose the player attempted 10 free throws on both days and he was successful 6 times yesterday and 7 times today. Then, we only need to compare 6 successes with 7 successes, and the comparison using subtraction is sufficient. In children's everyday life, it is rare that they encounter a situation where they must compare using bai. For example, if they want to compare who can ran faster, they simply compare the time. If they want to know how much their test scores improved from the last test, they will just look at the difference in the scores. If students lack experiences to compare using bai in their everyday situations, then I believe it is necessary to intentionally set up learning situations in which they can experience the comparison using bai before the formal study of wariai.
4. About the task in this lesson

The task being used in today's lesson is based on the problem on p. 56 of the textbook published by Tokyo Shoseki. The original problem is as follows.
(Problem) The prices of a notebook and a pen in 1980 and 2005 are as shown below. Which item's price increased more from 1980 to 2005 ?

Notebook: 80-yen in $1980 \rightarrow 120$-yen in 2005;
Pen: 50-yen in $1980 \rightarrow 90$-yen in 2005
Based on this task, I created a new task in which students can more easily consider the two ways of comparison explicitly depending on cases. To do so, as a part of the task, I included a case in which the comparison using the difference is sufficient (that is, a case in which the base quantity will be the same). The following task will be used in today's lesson.
(Problem) Takashi's parents raised the monthly allowances for Takashi and his brothers. Whose allowance can we say was raised most?

|  | Before | After |  |
| :--- | :--- | :--- | :--- |
| Takashi | 500 -yen | 700 -yen | $(-) 200$ <br> $(\times) 1.4$ |
| Younger <br> brother | 500 -yen | 600 -yen | $(-) 100$ <br> $(x) 1.2$ |
| Older <br> brother | 2000 -yen | 2200 -yen | $(-) 200$ <br> $(\times) 1.1$ |

We will start with the comparison of Takashi and younger brother so that students can make use of the comparison using the difference. Then, the information about their older brother will be presented, and students will be asked to think about whose allowance, Takashi or his older brother, was raised most. Students will realize that this comparison is difficult because the base quantities are different. I will then guide them to consider the idea of using bai to make comparisons. In this way, students will understand that there are two ways to make comparisons, one based on the difference and another based on bai. By including the case where the use of the difference is sufficient, I believe it becomes easier to examine the cases in which different ways of making comparisons should be used.

## 5. Today's lesson

(1) Goal of the lesson

By considering the two ways of making comparisons, one based on the difference and another based on bai, students will understand that the comparison using bai is more appropriate when the base quantities are different.
(2) Flow of the lesson

| Main Activities ( $\mathrm{T} \rightarrow$ hatsumon; $\mathrm{C} \rightarrow$ examples of student responses) |  |  |
| :---: | :---: | :---: |
| 1. Understand the task <br> T: Takashi's parents raised the monthly allowances for Takashi and his brother. Takashi's allowance was raised from 500 -yen to 700 -yen. His younger brother's allowance was raised from 500 -yen to 600 -yen. Whose allowance can we say was raised most? |  |  |
|  | Before | After |
| Takashi | 500-yen | 700-yen |
| Younger Brother | 500-yen | 600-yen |

O Guide students to realize that the initial amounts are the same. Help students to pay attention to the equality of the base quantity as the condition for using the differences to make comparisons.

2 Understand the way to compare using the difference
C: Takashi received a 200 -yen raise and his younger brother got a 100-yen raise. Since Takashi's raise was greater, his allowance was raised most.
T: How did you figure out 200-yen and 100-yen?
C: I calculated $700-500=200$, and 600 $-500=100$.
T: So, you compared the difference between 700 -yean to 500 -yen and the difference between 600-yen and 500yen, didn't you?
3. Understand the main task

T: Actually, Takashi also has an older brother, and his allowance was raised from 2000-yen to 2200-yen.

| Older <br> Brother | 2000 -yen | 2200 -yen |
| :--- | :---: | :---: |

O Instructional consideration I Assessment
O Make sure students understand that if you keep money in a bank for a while, you will receive both the amount you deposited and the interest you earned.

O If the idea of bai is suggested, we will also discuss it. As we discuss the idea of bai, make sure to have students think about which quantity is compared to which - which is the base quantity - so that they can understand how to determine bai.

O Make sure that students realize that they were using the differences to compare.

T：So，whose allowance can we say was raised most，Takashi or his older brother？
C：I think it is the same since Takashi＇s allowance was raised 200－yen，from 500 －yen to 700 －yen，and his older brother also received a 200 －yen raise from 2000－yen to 2200－yen．So，it＇s the same．
C：But their allowances weren＇t the same at first，500－yen and 2000－yen．So， I don＇t think it is a good idea to compare using the differences．

O If we look at the differences，it may appear that Takashi and his older brother received the same raise． Bring students＇attention to the fact that their initial allowances were different and the comparison based on the differences might not work well in this situation．

## How should we compare if the initial amounts are different？

4．Independent problem solving
C：If we consider the initial allowance amount as the base quantity，we can calculate how many times as much（bai） is the new allowance．

5．Whole class discussion
C：I calculated bai by dividing the allowance after the raise by the original allowance amount．
Takashi： $700 \div 500=1.4$
Older brother： $2200 \div 2000=1.1$
When you compare 1.4 times as much and 1.1 times as much，I know 1.4 times as much will be more．So，I think Takashi＇s raise was greater．

O If a large number of students do not think about using bai，keep the independent problem solving time short．
$\square$ Students can think about the way to make comparison other than using the differences．（Mathematical Reasoning）

O If the idea of bai does not come up， ask students how many times as much is each of the brothers＇allowance is compared to the original allowance amount．
\(\left.$$
\begin{array}{l}\text { C: I used double number lines. } \\
\text { Takashi }\end{array}
$$ \begin{array}{l}O When using bai to make <br>
comparisons, it is better to write <br>
double number line representations <br>
one directly above the other, lining up <br>
1's on the number line for bai. <br>
O To help students more easily <br>
understand that the initial allowance <br>
amounts are the base quantities and <br>
we are comparing the raised <br>
allowances, use double number line <br>

representations to highlight the\end{array}\right]\)| relationships between the base |
| :--- |
| quantity and the compared quantity. |

References: omitted (because all documents are available only in Japanese)

