

Grade 3, Class No. 2, Mathematics Lesson Plan

Instructor: Yuji Ishikawa

School: Showa-machi Oshihara Elementary School

1. **Name of Unit:** “Let’s think about division” (Division with remainders)

2. **About the Unit:**

Up to now, students have learned “division” as a new kind of calculation. There are two different meanings of division—partitive division and quotative division. Students have learned both meanings of division, as well as how to express them in problem situations using math sentences. Since division is the inverse operation of multiplication, the answers to division calculations can be obtained by utilizing the multiplication table of the number in the divisor.

In this unit, students will learn about division with remainders for cases in which the multiplication table is used once. The content of this unit includes the meaning of division with remainders, calculation methods, what remainders are, and how to deal with remainders.

When the size of quotient is maximized, the remainder is what is left from subtracting a product of a divisor and the quotient from a dividend. The division problems that students learned previously in which the remainder was always 0 are actually special cases. In order to help students understand this point, division with remainders needs to be integrated with division without remainders that students learned previously as one concept. It is important to build on what students learned before to help them to learn to do division calculations with remainders. Moreover, students should solve problems involving both partitive and quotative division situations in the same way they did before with division without remainders.

Through the instruction of the lessons in this unit, I would like to help students to understand the meaning of remainder as well as expand their view of remainders. First, at the introduction of the unit, I will pose a task that involves division without a remainder, similar to what students have learned before, and help them to understand that division includes the cases that are both divisible and not divisible. For the introduction, a partitive division problem situation will be used. By dividing objects into several groups equally, students will learn that what is left through this process is the remainder. By utilizing this problem situation, I would like to lead the students to learn how to calculate division with remainders. If we use a quotative division problem situation it is much easier to understand that what is left is the remainder. However, by using the partitive division problem situation students can think about the fact that depending on the objects they are dealing with there are object where they can still divide the remainders and objects where they can’t divide the remainders. This view of looking at remainders can be thought as a connection to division with decimal numbers and fractions. After the introduction, I would like to use a quotative division problem situation and help students to understand the relationship between the size of the remainder and the divisor. Moreover, I would like to help students to understand how to utilize multiplication and addition to check division calculations by using several examples.

Next, in the second half of the unit, which includes this lesson, I would like to help students to understand different ways to treat remainders that are different from what students learned in the first half of the unit (e.g. round up remainders and add 1 to a quotient or round down remainders and find a quotient). By thinking about how to treat remainders differently in situations that match the story problems students are asked to solve, I would like to help them to expand their understanding of remainders. At the end of the unit, students will apply this understanding by solving story problems that involve the size of the remainder. For example, “We are going to make basketball teams named by fruits. Students are lined up and divided into three teams—the apple team, the orange team and the banana team—by assigning students into each group in the following order: apple, orange, and banana. What group will the 25th student in the line belong to?” I would like to student to understand that by thinking about the patterns involved in the problem and utilizing division with remainders, this problem situation can be solved efficiently. In addition, I would like to students to see the merit of doing so.

3. Relationship to the School-Based Professional Development:

(1) About the ability to think in logical steps with foresight and to express

This years’ theme of the School-based professional development at Oshihara Elementary School is “Elementary career education¹ that fosters students’ base for independency: Development of instruction that raises students’ ability to think in logical steps with foresight, and to express.” At this school, we have been trying to clarify the relationship between the career education and mathematics education, and wrestle with raising the abilities that students need for career advancement through mathematical instruction. The reasons for doing this at our school is because we see the concrete connections between the goals of career education and the goals of mathematics education in the following four points: “creative thinking,” “autonomy and independence,” “awareness of value of self,” and “relationship between self and society.”

The sub-theme, “Development of instruction that raises students’ ability to think in logical steps with foresight and to express,” is elaborated in the goals of mathematics education [Chapter 1, Section 1, Number 1(3) in the Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grades 1-6)]. In general, the process of learning in mathematics involves posing problems and solving problems. This process involves students using their prior knowledge and skills, grasping the problem situation, thinking about how to solve the problem, utilizing reasoning, and making inferences. The mathematics curriculum has strong coherence between the content that students learn. Furthermore, the basis needed for thinking is relatively clear. Therefore, the subject is appropriate for developing students’ ability to think in logical steps with foresight.

From the results form last year’s professional development, at this school, we think of “the ability to think in logical steps with foresight and to express” in the following manner:

¹ (Notes added by translator) Career education was introduced formally to the schools in 2008 by the Ministry of Education, Culture, Sports, Science and Technology (MEXT). The goal of the career education is to foster each student’s view or willingness to work and become aware of the responsibilities of their profession. According to the MEXT, “Career education is important in educating children in their views of career and work, and in cultivating the ability to proactively select and decide career paths. For that purpose, MEXT is promoting systematic career education applicable to each school stage through experience in the workplace and so on.”

Abilities to think inductively, analogically and deductively.

In addition:

- **Ability to explore clues to solve problems**
- **Have a basis to solve problems, organize own thinking, and be able to express them**
- **Reflect on and organize own thinking**

In this class, I have observed the following points among the status of students' learning on these three highlighted abilities.

About 70% of the class consists of students who can solve problems on their own and show the process in writing. When they have to discuss problem situations or other students' ideas about half of the students often mumble something that they noticed or have their own opinion. In addition, many students actively communicate their ideas and thoughts in the class. On the other hand, they are not so good at clarifying the commonalities and differences between their own ideas and their friends' ideas, and having the will and patience to try to understand what they don't get. These issues may be part of the developmental stage of 3rd grade students, however, something needs to be done to improve.

Up to now, students have learned to speak in the class using phrases such as, "The reason is ...," "as same as ...," and "different from ...". In addition, they have learned to take notes in order to organize their thoughts such as using headings like "problem," "my idea," "friends' ideas," "summary," and "reflection." Students have been developing these skills for two months since the beginning of the school year in April. Lastly, students have been practicing sharing the thoughts that they mumble and describing own as well as other students' thinking. Through this the students are developing their attitudes for listening and the ability to respond to other students' ideas and thoughts.

Based on these students' state of learning, I would like to show an improvement in students' "ability to think in logical steps with foresight and to express" through the following five learning steps.

- 1.) Grasping goals/tasks
- 2.) Having foresight for solving a problem
- 3.) Solving a problem on their own
- 4.) Discussing methods to solve a problem in class as a whole
- 5.) Summarizing learning

(2) About "the ability to think in logical steps with foresight and to express" in this lesson:

1.) Evidence of "the ability to think in logical steps with foresight and to express"
I am hoping that students can show the following evidence of these abilities:

A. Students are able to find clues for solving problems

- In the case of division with remainder, students are able to think by using the partitive and quotative meanings of division that they learned previously through problem situations.
- **By utilizing the meanings of division that students learned previously, they try to express the problem situation with a math sentence.**
- Students think about a method to check the result of a division calculation by utilizing previously learned knowledge, such as division calculation is the reverse operation of multiplication calculation, and methods for checking the results of addition and subtraction calculations.

- B. Students solve problems based on what they learned and are able to organize and express their own thinking.
- In the case of division with remainders, students are able to express how they can utilize previously learned knowledge in problem solving with math sentences, words, diagrams, and pictures.
 - Students are able to express remainders (what part remains, if it is divisible or not divisible) using diagrams and pictures
 - Students are able to express why they need to carefully examine how to deal with remainders in order to come up with an appropriate answer for a specific problem situation.
- C. Students who reflect and organize their own thoughts
- Students are able to recognize that calculations utilizing multiplication tables are efficient for calculations of division with remainders, and utilize this method to solve problems.
 - **Students compare their own ideas and their friends' ideas to find similarities and differences.**
 - **Students recognize the merits of their friends' ideas and utilize them to solve problems.**
 - **Students record their own progression of learning in their reflections in their notebooks.**

2.) Teacher's role and support for raising students' "ability to think in logical steps with foresight and to express."

- A. Teacher provides *hatsumon* that help students to have foresight for solving problems.

The steps involve "grasping goals/tasks" and "having foresight for solving a problem." The teacher asks *hatsumon* such as, "I wonder if we can use something we learned before," "What should we use to think about the problem?," "What are we finding?," and "What do we know and what do we need to find out?" that help students to be able to have foresight for solving problems. By asking these *hatsumon*, the teacher listens and picks up students' mumblings and helps to connect them to understanding the goals/tasks of the lesson. By facilitating students sharing of their thinking in the class, help each student to have foresight for solving problems and engage them in problem solving.

- B. The teacher asks about the basis of students' thinking and provides opportunities for them to express their thinking.

The steps involve "students solving a problem on their own" and "discussing methods to solve a problem in class as a whole." The teacher asks students, "Why did they think that way?" to bring out there bases for their thinking. As the case may be, the teacher helps students to focus on a few ideas and provides them with opportunities to organize and express their own thinking in their notebooks in many different ways and for them to discuss what they notice and think about.

- C. The teacher provides opportunities for students to exchange and examine ideas and creates *bansho* (board writing) that helps students to see the flow of the lesson and how ideas are connected to build understanding.

Steps involve "students discussing methods to solve a problem in class as a whole" and "summarizing learning." The teacher helps students to be exposed to many different ideas and asks questions that lead to the essence of the learning of the content. The teacher asks students about the merits and expandability of each idea,

the similarities and differences among ideas, and the generalizability and consistency of ideas. As the case may be, help students to clarify the ideas that are easier for the students to understand such as, “a method that helps to solve a problem efficiently” and “a method that is easier for everybody to understand.” Also, the teacher will facilitate students’ thinking so they can move on to better ideas as well as deepen and utilize the ideas. In order to help students to think this way, the teacher will think about *bansho* that shows the flow of the lesson and how students’ ideas and thinking are incorporated to develop understanding of the content.

4. Objectives of the Unit:

Mathematics and Moral Education	Career Education
<p>○ Help students to understand division with remainders, deepen their understanding of meaning of division, and utilize them.</p> <ul style="list-style-type: none"> • Students think about the meaning of and how to calculate division with remainders based on division without remainders, the relationship between division and multiplication that they have learned before, and manipulation of concrete materials. [Interest, motivation, and disposition] • Students are able to explain the meaning of and how to calculate division with remainders (multiplication tables can be utilized to find answers) using concrete materials, diagrams, and math sentences. • Students are able to pay attention to remainders and think about the structure of problems [Mathematical reasoning] • Students are able to calculate division with remainders and check the answers. [Skills and Procedure] • Students are able to understand the meaning of remainders and the relationship of the size of the remainder and the divisor, and how to calculate division with remainders. [Knowledge and understanding] 	<p><Ability to take a part></p> <p>○ Students try and are able to understand each other’s ideas [Formation of human relationship and ability to be a part of community]</p> <p><Ability to find and resolve issues></p> <p>○ Students are able to come up with strategies and solve problems/tasks by clarifying the basis for solving the problem by utilizing what they have learned previously [Ability to cope with problems/tasks]</p> <p><Ability to turn into reality></p> <p>○ Students are able to choose a more efficient way from many different ideas and utilize it to solve problems/tasks. [Ability for career planning]</p>
<p><Moral Education></p> <p>1-(2) Once one decides to do something, persevere to complete the task.</p>	

5. Unit plan (Total of 10 lessons):

Sub-Unit [# of lessons]	Lesson	Main Activities	Evaluation Points of This Lesson	Career Education
1 [6]	1 & 2	<ul style="list-style-type: none"> Think about problem situations that are divisible and not divisible based on a fair share (partitive) division problem situation. Understand remainders and think about problem situations where remainders can be still be divided and not be able to be divided. 	<p>Students think about division with remainders based on the division they have learned previously. [Interest, motivation, and disposition]</p> <p>Students are able to understand how to calculate division that is not divisible and about remainders. [Knowledge and understanding]</p> <p>Students think about problem situations that involve remainders that can be still divided and cannot be divided based on a partitive division problem situation.</p>	<p>Students try and are able to understand each other's ideas [Formation of human relationship and ability to be a part of community]</p>
	3	<ul style="list-style-type: none"> Think about a problem dealing with a quotative division situation and compare division that is divisible and division is not divisible. 	<p>Students are able to explain that both partitive and quotative non-divisible division problem situations can be express as division, the calculation process can be thought as divisible division that they have learned. [Mathematical reasoning]</p>	<p>Students are able to solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks]</p>
	4	<ul style="list-style-type: none"> Investigate the relationship between the size of remainder and divisor. 	<p>Students are able to investigate the relationship between the size of remainder and divisor and understand this relationship. [Mathematical reasoning]</p>	<p>Students are able to solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks]</p>
	5	<ul style="list-style-type: none"> Think about how to check calculation of division with remainders. 	<p>Students are able to think about how to check the result of division with remainders based on understanding that division is the inverse operation of multiplication and knowledge of how to check addition and subtraction answers. [Mathematical reasoning]</p> <p>Students are able to understand how to check calculation results of division with remainders. [Knowledge and understanding]</p>	<p>Students try and are able to understand each other's ideas [Formation of human relationship and ability to be a part of community]</p>
	6	<ul style="list-style-type: none"> Practice and become fluent on calculations of division with remainders and the method for checking the calculation. 	<p>Students are able to check the results of division calculation with remainders. [Skills and procedures]</p>	<p>Students are able to solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks]</p>
2 [3]	1 [This Lesson]	<ul style="list-style-type: none"> Think about how to treat remainders appropriately according to the problem situation (round up the 	<p>Students are able to think about and explain how to treat remainders appropriately according to the problem situation (round up the reminders and</p>	<p>Students try and are able to understand each other's ideas [Formation of human</p>

		remainders and add 1 to the quotient)	add 1 to the quotient) [Mathematical reasoning]	relationship and ability to be a part of community]
	2	<ul style="list-style-type: none"> Think about how to treat remainders appropriately according to the problem situation (round the remainders down, keep the quotient as is) 	Students are able to think and explain how to treat remainders appropriately according to the problem situation (round the remainders down, keep the quotient as is) [Mathematical reasoning]	Students are able to come up with strategies and solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks]
	3	<ul style="list-style-type: none"> Solve application problems that utilize division with remainder efficiently. 	Students are able to think about the meaning of finding remainders in a problem situation. [Mathematical reasoning]	Students are able to choose more efficient ways from many different ideas and utilize them to solve problems/tasks. [Ability for career planning]
3 [1]	1	<ul style="list-style-type: none"> Solve problems to summarize learning 	Students are able to solve problems utilizing what they have learned in the unit. [Skills and procedure] Students are able to develop deep understanding of the content they learned. [Knowledge and understanding]	

6. Learning in this lesson:

(1) Goals of this lesson:

Students are able to think about and explain how to treat remainders appropriately according to the problem situation (round up the remainders and add 1 to the quotient).

(2) Date and time: June 28, 2013 (Friday), 1:50 p.m. to 2:35 p.m. (5th period)


(3) Place: Showa City Oshihara Elementary School, Grade 3, Class 2

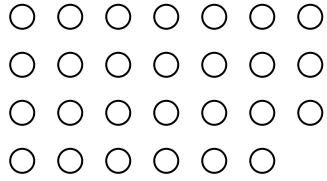
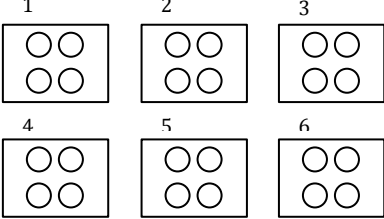
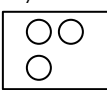
(4) Intention of instruction:

Students have learned the meaning of remainders and how to calculate division with remainders up to the previous lesson. In this lesson, students will learn how to treat remainders (round up the remainders and add 1 to the quotient) that is different than what they have learned. Because of this reason, I am going to provide a problem situation that is close to the students' daily life. Through thinking about the task of this lesson, I would like to students to notice that in some problem situations remainders can be treated differently and thought as a group that is added to the quotient.

Through several problem-solving activities up to the previous lesson, students had to consciously think about situations where they need to find a quotient and a remainder. Because of this reason, students might come up with a wrong answer. Based on a problem situation that involves "all the students need to ride," help students to think about how to treat the remainder. Through learning from this lesson, help students to expand their view of remainders and foster their ability to find an appropriate answer that corresponds with a problem situation.

(5) Flow of the lesson:

Process	Learning activities and content	Instructional points to remember	Evaluation
Grasp the task and have foresight for solving the task 5 min.	<p>1. Grasp the task (1) Understand the task for this lesson.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Grade 3, Class 2 is going to an amusement park. 27 students are divided into groups to ride boats that can hold 4 passengers each. How many boats do we need if everyone rides in the boats?</p> </div> <p>1.) Think about a math sentence for this problem and check it as whole class.</p> <ul style="list-style-type: none"> • $27 \div 4$ 	<ul style="list-style-type: none"> • Write the problem on the board and place the picture. • By facilitating the conversation among students, come up with a math sentence. 	
Solve problem on their own 5min.	<p>2. Solve the problem on their own</p> <p>a. $27 \div 4 = 6$ R3 <u>6 boats</u></p> <p>b. $27 \div 4 = 6$ R3 $6 + 3 = 9$ <u>9 boats</u></p> <p>c. $27 \div 4 = 6$ R3 $6 + 1 = 7$ <u>7 boats</u></p> <p>Because we want all the students to be able to ride the boats, we need to add 1 more boat so the remainder of 3 students can ride in a boat.</p>	<ul style="list-style-type: none"> • For students who are having difficulty calculating, ask them to look at their notes they took up to the last lesson, and utilize them to think about it. • Use a seating-chart to check students' problem solving progress and provide appropriate supports. 	
Discuss solution methods as a whole class 25 min.	<p>3. Discuss solution methods as a whole class</p> <p>(1) Ask students to present math sentence and answer.</p> <p>$27 \div 4 = 6$ R3</p> <p>a. 6 boats</p> <p>b. 9 boats</p> <p>c. 7 boats</p> <p>(2) Students will explain their own thinking for various different answers that are obtained from the same math sentence using their own expression (e.g., using diagrams, words, math sentences, etc.)</p> <p>1.) Compare between one's own idea and friends' ideas and summarize their own idea one more time.</p>	<ul style="list-style-type: none"> • Walk around the classroom and grasp students different solution methods • Write down students' ideas that were presented on the board. • Ask students why there are different answers even though they started with the same math sentence, and help students become conscious about the differences of thinking. • If it is necessary, show students' diagrams and pictures that they wrote in their notebooks using the digital board. 	<p>Students are able to think about and explain how to treat remainders (round up the remainders and add 1 to the quotient) according to the problem situation. [Mathematical reasoning] (Notebook, presentation)</p>

	<p>Examples:</p> <p>< Array Diagram ></p> <p>1 2 3 4 5 6 7</p>  <p>< Picture of boats ></p>  <p>7 (Remainder of 3 students will be riding a boat)</p>  <p>2) Discuss each method as a whole class</p> <ol style="list-style-type: none"> We have 6 boats, 3 students cannot ride. I think those 3 students were asked to get on to each boat (one person per one boat), but the units for students and boats are different so we cannot add them together. If we put the remainder of 3 students on a boat and make the total number of boats 7 boats, all the students can ride. <p>3) Students learn how to treat remainder according to the problem situation</p> <ul style="list-style-type: none"> In order for all the students to ride in the boats, we need to add 1 boat so the remainder of 3 students can ride. So the total number of boats is 7 boats. <p>(3) Students summarize what they have understood from today's task.</p> <ul style="list-style-type: none"> In order for all the students to ride the boats, add 1 boat needed for the remainder of 3 students to the number of boats obtained by the calculation. 	<ul style="list-style-type: none"> Use a student's wrong answer for the class discussion. By paying attention to the commonalities and differences of the presented methods, help student to think about how to treat the remainder. Connect diagrams/pictures and math sentences. Help students understand the meaning of $6 + 1 = 7$ and confirm that if we increase the quotient 1 more, all the students can ride the boats. Ask students to write in their notebooks with their own words. 	
--	--	---	--

Summary 10 min.	4. Summarize the lesson. (1) Students solve an application problem that is similar to the problem they worked on. 1.) “26 students are divided into groups to ride boats that can hold 4 passengers each. How many boats do we need if everyone rides in the boats?” • $26 \div 4 = 6 \text{ R } 2$ $6 + 1 = 7$ <u>7 boats</u> (2) Write reflection in the notebook. • “Depending on what the problem is asking to find, I understand that there is a case where you need to change the remainder to 1 and add to the answer.” • “I was not thinking deeply about what I need to do with the remainder at the beginning, but in order to let all the students to ride the boats it is important to think about what to do with the remainder.” • “I listened to ____’s (a friend’s) idea (or looked at a friend’s diagram/picture) and I understand that we need to add 1, that is for the remainder, to the answer.”		
--------------------	---	--	--

(6) Evaluation:

- Students are able to think about and explain how to treat remainders appropriately according to the problem situation (round up the reminders and add 1 to the quotient). [Mathematical reasoning]

[References]

- 1.新しい算数3上 教師用指導書 指導編 (2010) pp.98-110 東京書籍
- 2.新しい算数3上 教師用指導書 研究編 (2010) pp.198-211 東京書籍
- 3.文部科学省 (2010)「小学校学習指導要領解説 算数編」 東洋館出版
- 4.中村享史 (2008)「数学的な思考力・表現力を伸ばす算数授業」(2010) pp.105-113 明治図書
- 5.杉山吉茂 (2012)「確かな算数教育をもとめて」 東洋館出版

* Translator’s Note

- References 1 & 2 above refers to the textbook. An English translation of this textbook series may be purchased from Global Education Resources (www.globaledresources.com).
- Reference 3 is from *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grades 1 - 6)*, a Ministry of Education document that provides detailed explanations of the Course of Study. It is available online at <http://e-archive.criced.tsukuba.ac.jp/data/doc/pdf/2010/08/201008054956.pdf>
- Other references are from books that are available only in Japanese.