June 29
Lesson 1R

Grade 2 Mathematics Lesson Plan

Multiplication (1): I can figure it out without counting them all!

Teacher: Daisuke Tsunoda
Learners: 33 Students from Classroom 1
Lesson Location: Aogiri Hall
Discussion Location: Aogiri Hall

I. About the unit

In Grade 1, students engaged in several activities that are foundational for the study of multiplication. For example, as counting activities, students counted by making groups like in 2's, 5's and 10's. They experienced the benefits of counting by groups such as the ease to insure each item is counted once and only once.

In addition, students have also engaged in the activity of partitioning 12 items into equal groups, which is foundational for the study of division. With the exploration, "Let's make buildings," students worked on the problem: We are going to make a building using 12 color tiles. Pretend each □ is a room and make different buildings." The picture shown on the right is a part of the blackboard from that lesson. Student comments that reflected the multiplicative nature of the problem situation included, "kept on going by 2's" and "4 tiles side by side then another one just like it underneath." We have tried to connect those statements to diagrams and repeated addition expressions.

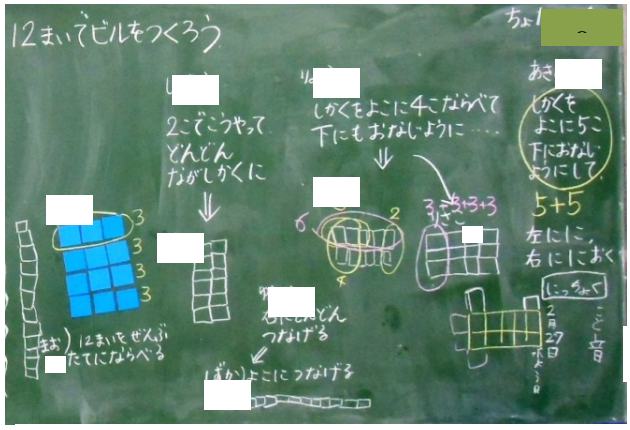


Fig. 1 A part of the blackboard from "Let's make buildings."

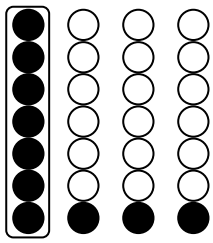


Fig 2 7 × 4

The most important goal of this unit is for students to realize the usefulness of multiplication. A usefulness of multiplication is we can determine the total number of objects without counting all of them once we identify equal grouping. For example, suppose there are 28 balls arranged as shown on the left. If we see groups of 7 balls, we can determine the total number of balls by simply counting those that are shaded in black.

Students who have not yet learned multiplication do not know this particular usefulness of multiplication, that is, we can determine the total number without counting all objects. Therefore, in the beginning of the unit, we plan to include counting activities in which students can make groups of 2, 5, or 10 so that they can use the strategy shown in Figure 2. By making groups of 2, 5, or 10, students can determine the total number by counting "how many in a group" (group size) and "how many groups" (number of groups). We want students to experience the benefit of making use of equal groups. Even though students have yet to learn the basic multiplication facts, if they make equal groups of 2, 5, or 10, they can determine the total number easily by counting by 5 or 10.

However, because they do not know the basic multiplication facts, they cannot determine the total number even if they make equal groups of other sizes. Therefore, what needs to happen with those students who understand the benefit of making equal groups is for them to become aware that if we learn the basic multiplication facts, we can determine the total number easily by making any size equal groups. That awareness will become a bridge to the study of the basic multiplication facts. Moreover, they should be able to experience the benefit of making equal groups in their everyday situations as well as in their study of mathematics.

We will also discuss the properties of multiplication such as the commutative property and the distributive property as well as the relationship between the multiplier and the product. Using these patterns, students will construct the multiplication table on their own. Moreover, we intend to deepen students' understanding by incorporating activities in which students will represent and explain their ideas using expressions/equations, pictures, diagrams, and words. We want to develop students who can re-construct the multiplication table on their own by making use of the patterns of multiplication, not students who simply memorized the multiplication table.

II Goals of the Unit

○ Students will understand the meaning of multiplication and be able to use it.

[Interest, Eagerness, and Attitude]

- Students will recognize the usefulness of multiplication and try to use multiplication to determine the total number of objects.

[Mathematical Way of Thinking]

- Based on the idea of repeated addition, the relationship between the multiplier and the product, the commutative property and other patterns, students can think about and explain ways to construct the multiplication table.

[Mathematical Skills]

- Students will be able to represent situations in which multiplication may be used by using pictures, diagrams, words, or expressions/equations.
- Students will construct the multiplication tables for 5's, 2's, 3's and 4's, and be able to calculate accurately.

[Knowledge and Understanding]

- Students will know the multiplication table and situations in which multiplication may be used, and they understand the meaning of multiplication.
- Students will understand the properties of multiplication (the relationship between the multiplier and the product, commutative property, distributive property).

III Relationship between the research theme and this unit

1. About characters and abilities necessary for learning toward harmonious living

Mathematics is a discipline which is taught systematically. Therefore, it is important that each student develops his or her own questions. Students can then recognize each other's strength from "differences" in their ideas and experience "understanding." We consider this to be the learning toward harmonious living. Moreover, the engine of this process is "questions" students develop. Therefore, we want to help students develop the following creative reasoning ability:

- Ability to think independently and develop own questions in problem solving situations,
- Ability to compare and contrast own idea with those of others in problem solving situations.

In this unit, we intend to organize the lessons based on the properties of multiplication (during the lesson, the phrase, "patterns of multiplication," will be used). We want students to enjoy constructing the multiplication table on their own by using the relationship between the multiplier and the product, the commutative property, or the distributive property, instead of focusing on the memorization of the table. Therefore, we hope to see students with eagerness to find patterns of multiplication themselves. In addition, during the comparing and discussing stage of a lesson, we hope to see students who display their desire to make sense of patterns discovered by other students. The questions relevant to this unit include "What patterns are there?" "Is there another pattern?" and "Which pattern will make the calculation simpler?" Whatever the situation is, we want students to explain the pattern of multiplication they discovered. As they solve their questions, we want students to use various ways of representations to support their explanations.

2. Dispositions for learning toward harmonious living

In teaching mathematics, I aim for a lesson in which students can develop a series of questions by carefully developing problems. Problems should arise from everyday situations, and by putting them on the mathematical stage, students can develop their own "questions." My goal is for students to experience "understanding" by thinking about their questions and comparing and contrasting their ideas with other students' ideas consciously and intentionally. For such lessons, it is essential that students have the disposition to generate and solve their own questions. For students to generate and solve their own questions, it is necessary that they can think logically. And, to think logically, students must be able to express their ideas clearly.

This unit, from the beginning, will be organized based on students' prior learning. The unit will begin with students realizing the benefit of repeated addition. Then, when students realize the expressions/equations for repeated addition are rather long, we will introduce multiplication expressions/equations. Finally, students will know that if they learn the multiplication table, they can determine the total number of objects without using repeated addition even if we have groups other than 2's, 5's or 10's. The construction of the multiplication table with begins with the 5's facts. As we examine the 5's facts, we will help students notice the patterns in the numerals, which is a characteristic of multiplication, and the relationship to the numerals on clock faces so that they can construct the table for the 5's facts on their own. Once students can construct the table for the 5's facts, students should be able to identify patterns in the 2's facts to construct the table on their own as well.

If the focus of the unit is simply memorizing the multiplication table, it will be difficult for students to experience "understanding." However, by organizing the unit in which students will construct the table on their own, they can feel "Even if I forget the multiplication table, I can find the answer on my own." In this unit, the focus of assessment is students' disposition "to construct the multiplication table using the patterns of multiplication." The assessment question will be, "How will you find the answer if you forgot the answer to 4 × 7?"

IV. Unit plan (Total of 24 lessons: 8 lessons shown below + 16 additional lessons)

Sub Unit 1 Multiplication

#	Goals	Learning Activities	Evaluation
1	Students will be able to grasp numbers as "how many in one group" and "how many groups," and try to explain their ideas. **(Today's lesson)**	Make buildings using ○ tiles, then think about simple ways to determine the total number of blocks.	Students are trying to use equal groups. (Interest, Eagerness, and Attitude) Students can determine the total number of tiles by making use of equal groups and explain their ideas. (Mathematical Way of Thinking) [Disposition to build on their prior learning.]
2	Students will learn that multiplication expressions/ equations will be more concise than repeated addition expressions/expressions.	Based on the repeated addition expressions/equations used to determine the total number of tiles, develop multiplication expressions/equations.	Students understand that repeated addition expressions/equations can be written as multiplication expressions/equations. (Knowledge and understanding)
3	Students will grasp "how many in 1 group" (group size) and "how many groups." Represent various situations using multiplication expressions/equations.	From pictures, students will grasp "how many in 1 group" and "how many groups." From various pictures showing equal groups, students write multiplication expressions/ equations to represent them.	Students can grasp "how many in 1 group" in equal group situations and represent the situations using multiplication expressions/equations. (Mathematical Skills)
4 5	Students will translate multiplication expressions into concrete situations, then determine the total number by repeated addition.	Students will represent multiplication expressions using counters. Students will determine the products by using repeated addition.	Students can represent situations in which multiplication is appropriate calculation using counters and multiplication expressions/ equations. (Mathematical Skills)

6	Students will understand the meaning of "times as much."	Students will understand that the length made up of two 3cm segments is "2 times as much as 3 cm." Students will understand that 3 × 2 is used to determine 2 times as much as 3 cm.	Students understand the meaning of "times as much," and they know multiplication can be used to find the amount so many times as much of the given amount. (Knowledge and Understanding)
7	Students will find objects in their surroundings that can be represented using multiplication expressions/equations.	Students will identify situations in their surroundings that can be represented using multiplication expressions/equations.	Students can identify situations in their surroundings in which multiplication can be used and explain them using words and expressions/ equations. (Mathematical Way of Thinking) [Disposition to build on their prior learning.] [Disposition to use manipulation of concrete objects and diagrams/ pictures to represent and communicate their ideas to others.]
8	Summary of the sub-unit.	Students will solve the assessment problem.	Students can solve problems using what they have learned. (Knowledge and Understanding)

Sub Unit 2 Multiplication tables for 5's, 2's, 3's, and 4's (omitted)

V. Today's lesson

(1) Date Saturday, June 29, 2013 (9:00 – 9:45)

(2) Location University of Yamanashi Model Elementary School Aogiri Hall

(3) Goal of the lesson

- Students will be able to grasp numbers as "how many in one group" and "how many groups," and try to explain their ideas.

(4) Rationale of the lesson

This lesson is the introduction of multiplication. The goal is for students to realize the usefulness of multiplication. I would like to communicate to the students the usefulness of multiplication, that is, if we know how many in one group and how many groups, we can determine the total number without counting them all.

The lesson will open by reminding students about a Grade 1 lesson, “Let’s make buildings.” In that lesson, students explored the following task: We are going to make a building using 12 color tiles. Pretend each □ is a room and make different buildings.

Today’s task is “Let’s make buildings with ○○ tiles.” Each group will receive 30 color tiles and they will make different buildings with them. However, students will not be told how many tiles there are. Students can make buildings such as 5 × 6, 3 × 10, or 2 × 15 as shown in Figure 3 without knowing the total number of tiles. Some might make buildings such as those shown in Figure 4. It will be difficult to represent the arrangements like those in Figure 4 using multiplication, but we will utilize these as non-examples of repeated addition situations.

Fig. 3 Anticipated buildings (1)

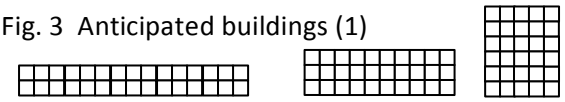
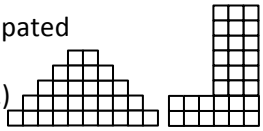


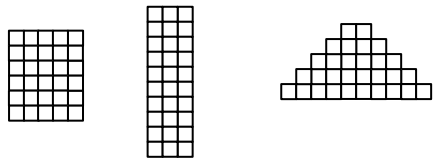
Fig 4 Anticipated buildings (2)



After students make buildings, a follow-up task will be given: Let’s think about ways to describe your buildings to other groups without actually showing what you made. Each group will discuss how they might describe their buildings. Verbal explanations like the following will give others a good idea of the buildings: “It is a 5-story building and there are 6 rooms in each floor,” or “We kept building up 3 rooms at a time.”

At this point, we will have the activity to interpret other groups' descriptions. First, we will discuss those buildings like those shown in Figure 4 which are difficult to express in a repeated addition expression. Then, we discus the buildings like those shown in Figure 3. I will then pose the question, “How many color tiles did Group * use?” and compare and contrast ways to count the total number of tiles. As students translate verbal explanations into diagrams or connect verbal descriptions with mathematical expressions such as “we can use the expression 5 + 5 + 5 + 5 + 5 + 5 to show a 6-story building with 5 rooms on each floor,” we will think about better ways of counting. Although students have not learned multiplication yet, tell them that the experiences they had of counting by making groups of 5 or 10 are leading to the idea of multiplication. After students understand the ideas of “how many in one group” and “how many groups,” I will introduce the multiplication expression: “5 + 5 + 5 + 5 + 5 + 5 can be written as 5 × 6.” I will have students represent other buildings using multiplication expressions, such as 2 × 15 or 3 × 10. It is hoped that students will realize that buildings can be represented by multiplication expressions much more simply than repeated addition expressions.

(5) Flow of the lesson

Min	Content and Tasks	Instructional considerations/Relationship to the research theme
5	1 Introduction Let's Make buildings. Group ○ Group ◇ Group △ 	・Students will be told that each group have different numbers of tiles. They are not supposed to count the number of tiles. There are 8 groups in this class. （Each group actually will receive 30 tiles.） ・Make sure students understand that all tiles must be used, and adjacent tiles must share a side completely.
5	2 Posing the task Let's think about ways to describe your buildings to other groups without actually showing what you made.	・Each group should cover up the building so that others cannot see it. ・During the individual problem solving time, each group will discuss ways to describe the building.
5	3 Individual problem solving I 〈For Group ○〉 C: There are 5 rooms in one floor, and it is a 6-story building. 〈For Group ◇〉 C: It's a 10-story building, and there 3 rooms on a floor. 〈For Group△〉 C: We made it look like a mountain.	・Have students think about a concise way to describe their buildings to others. ・They are not supposed to include the total number of tiles in their description. ・Post each group's description on the blackboard.

20	4 Individual Problem Solving II/ Comparing and discussing solutions Let's draw the building Group Δ made in your notebook. C: It's confusing. Let's draw the building Group ○ made in your notebook. C: They said "there are 5 rooms in one floor, and it is a 6-story building." I think it's easy to draw the building. How many tiles did each of groups Δ and ○ use? C: It's difficult to count the tiles for Group Δ. C: For group ○, the number of tiles is 30 because 5 + 5 + 5 + 5 + 5 + 5. C: It's easier to count when you make groups of 5. C: We can tell the total number of tiles easily without counting them all.	・Instead of comparing and discussing all solutions, select certain ones to carefully examine. ・Discuss the solution like Group Δ's so that the usefulness of equal groups will become apparent. ・Tell students to record how they knew the total number of tiles in their notebooks. If there are students who are using mathematical representations in words, pictures, or expressions/ equations, acknowledge them. ・Based on verbal expressions, develop diagrams and expressions/equations. ・Make connections among different representations and determine the total number of tiles. Highlight the usefulness of expressions/equations. ・Listen for students' comments that suggest making groups of 5 or 10 are much easier to count, even easier than grouping in other numbers. During the process of "learning toward harmonious living" ◎ Abilities to nurture ・Ability to investigate the commonalities and differences between their own buildings and those of other groups'. ・Ability to think about more efficient ways of counting. ◎ Desired students responses ・Students can determine the number of tiles by considering equal groups. ・Students can translate other's ideas into different representations. ・ Students realize that the total number can be determined based by reflecting on various represenations. ◎ Strategy ・Carefully design the learning tasks. ◎ Support ・Have students repeat other students' ideas. ・Encourage students to use mathematical representations used by others.

10	5 Summary of the lesson ・Students will know that repeated addition expressions can be written as multiplication expressions. $5+5+5+5+5+5=5\times6$ C: We can write multiplication expressions for other repeated addition expressions. ・Write a journal entry. C: It's easy to count if we make groups of 5 or 10. C: It's simpler to represent with multiplication expressions.	・Tell students that we will discuss other groups' ideas in the next lesson.

(6) Assessment points

1. Was the "ability to nurture" today's lesson focused in alignment with the dispositions for learning toward harmonious living?
2. Was today's lesson (its organization, the choice of tasks, instructional approach, etc.) effective to nurture characters and abilities necessary for learning toward harmonious living?
3. Through today's lesson, did students "generate new ideas from comparing and contrasting own ideas with others'"? What were some of the new ideas?
4. Were students appropriately assessed and given appropriate support?

(7) References

1 藤井 斉亮・飯高 茂 ほか 40 名 (2011) 「あたらしいさんすう 1」 東京書籍

Translator's Note: This refers to a textbook series. An English translation of this textbook series may be purchased from Global Education Resources (www.globaledresources.com).