Mathematics Department Research Lesson
Writing an Equation for a Recurrence Relationship
Mathematics Lesson Plan

Tokyo Gakugei University International Secondary School
Date: Thursday, June 25, 2015
Time: 15:05 - 15:550
Class: Home Room 2, Grade 7
26 students (8 boys & 18 girls)
Teacher: KOBAYASHI, Ren
Name of Unit: How to observe phenomena

1. Overview of the lesson

(1) Problem Statement

There is no question about the importance of developing students' ability to grasp changes. There are at least two facets in one's ability to grasp changes: understanding fundamental patterns of changes and their characteristics, and mastering methods for analyzing changes (Odera, 2009; Stewart, 2000).In this research lesson, we will focus on the latter, "mastering methods for analyzing changes."

In Japanese mathematics education, the ability to grasp changes is developed mainly through the learning of functions. However, if we examine the instruction of functions through the lense of "changes and correspondences," the focus is on correspondences. Explicit instruction on representing changes with algebraic equations does not happen until the unit on sequences in Mathematics B of upper secondary school.[1] We belive that is too late. Even without grasping explicit correspondences, if we can express the patterns of changes in algebraic equations, we can capture changes using technology. Furthermore, it is a natural and fundamental strategy for making sense of changes globally to focus on patterns of changes locally. Therefore, we argue that it is more important during the compulsory education stage[2] to focus our instruction on solving problems by expressing changes (local changes) in algebraic equations.

Therefore, in the series of Grade 7 lessons including the current research lesson, we will assign problems that involve recursive relationships. We will focus on solving those problems by representing the relationships in algebraic equations.

(2) Mathematics curriculum at our school and today's lesson

Our school has been authorized to offer the International Baccalaureate programs, and we are currently implementing the Middle Years Programme (MYP). The goals and evaluation standards for the MYP mathematics curriculum are composed from the following four criteria.

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[1] Furthermore, in the unit on sequence, the focus seems to be on "solving recursion equations," even though we can grasp changes generally by using technology. It is important to place more emphasis on "representing changes in recursion equations" instead of solely focusing on "solving recursion equations."

[2] Even in elementary school mathematics, "horizontal relationships" is often utilized in analyzing data presented in tables. However, it has been reported that lessons in which students express such relationships in equations are rarely observed (Takei & Fujii, 2003).

- Criterion A: Knowing and Understanding
- Criterion B: Investigating Patterns
- Criterion C: Communicating
- Criterion D: Applying Mathematics in Real-Life Contexts

The goals of this research lesson are related to Criteria B and D. Here are the level descriptors for Criteria B and D (for the highest achievement level[3]).

B Investigating Patterns
1. Apply mathematical problem-solving techniques to recognize patterns
2. Describe patterns as relationships or general rules consistent with correct findings
3. Verify whether the pattern works for other examples.

D Applying Mathematics in Real-Life Contexts:
1. Identify relevant elements of authentic real-life situations
2. Select appropriate mathematical strategies when solving authentic real-life situations
3. Apply the selected mathematical strategies successfully to reach a solution
4. Explain the degree of accuracy of a solution
5. Describe whether a solution makes sense in the context of the authentic real-life situation.

At our school, we use the mathematics curriculum that was internally developed, keeping in mind the goals of the MYP. The second unit in the Grade 7 curriculum is " How to observe phenomena" (we will discuss the ordering of the units in this curriculum later). As the second section of this unit, reflecting the problem discussed above, we have established "Recurrence Relationships." This research lesson is the first lesson in this section. The aim of this section, Recurrence Relationships, including this research lesson is to develop students' abilities to identify recurrence relationships hidden in various phenomena (B1), to represent the relationships as generalized patterns (using algebraic equations) (B2/D2), to carry out numerical calculation using the patterns (D3), and to interpret the results of the calculation in the context of the phenomena (D4 & 5).

(3) From the perspective of raising the quality of mathematical process

In the Mathematics Education Research Group of Tokyo Gakugei attached secondary schools, we have been conducting lesson study with the theme, "lessons that raise the quality of mathematical process." In this section, we will discuss this research lesson from that perspective.

As discussed above, the aim of this section is to develop students' abilities to identify recurrence relationships in phenomena, to represent the patterns in algebraic equations, to carry out numerical calculations using the equations, and to interpret the results of the calculation in the contexts. In particular, in this research lesson, the focus is raising the quality of mathematical process of identifying a recurrence relationship and representing it in algebraic equations as a generalized pattern.

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[3] In the MYP, each assessment criterion is evaluated on a 8-point scale. The standards listed here correspond to the highest level, the score of 7 ~ 8 points.

Prior to this unit, students have learned to examine how much dependent quantity varies when the independent quantity increases 1 unit in Grade 4 unit, "Investigating Changes," and Grade 6 unit, "Direct and Inverse Proportional Relationships." However, students have expressed those relationships verbally, and it is uncertain whether or not they recognize those changes as recurrence relationships. In contrast, this unit aims for students to recognize recurrence relationships explicitly and represent them using algebraic equations. Because we believe the formal recursion equations used to represent sequences are too difficult for Grade 7 students, we will introduce a notation to represent recurrence relationships, "NOW-NEXT," for example, NEXT = NOW × 2. In other words, the aim is to raise the students levels from the one in which they can only express patterns of changes verbally to the level in which they can clearly recognize recurrence relationships and represent them using algebraic equations.

However, it is obvious that students will not be able to represent recurrence relationships using NOW-NEXT equations at the end of this lesson which is the first lesson in the section. We believe it is necessary for the teacher to keep the following 3 points of considerations in mind as he teaches this lesson. First, it is important to set up a task in which students can truly experience recurrence relationships. Second, students must be able to consider the task as an object of mathematical process. Finally, it is important that students can experience merits of representing recurrence relationships in algebraic equations. It is when students experience merits of expressing recurrence relationships in algebraic equations as they investigate several phenomena that they might want to use a particular format. The NOW-NEXT equations will be introduced at that point. In this lesson, our goal is for students to us equations with words that match the phenomenon. However, although we focus on words that match the phenomenon, we also want to aim for words that can be extended to describe consecutive terms in sequences.

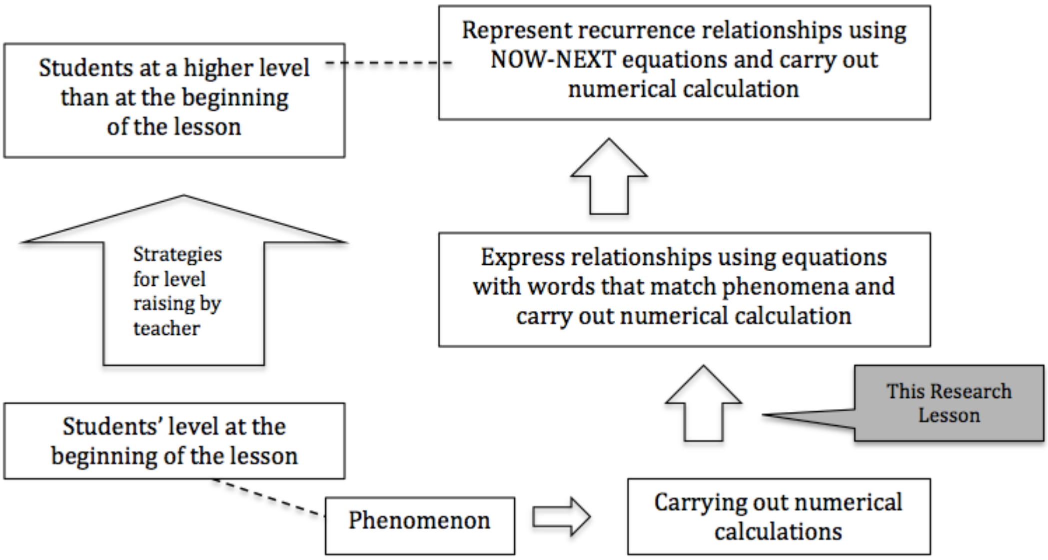

Figure 1 The raising of mathematical process aimed at in this lesson

Because instruction on representing recurrence relationships in algebraic equations is not done in Grade 7, there is little known about how 7th grade students will think about, represent, and utilize recurrence relationships. It is hoped that this lesson will provide some insights into how students at this level will deal with recurrence relationships. Furthermore, we want students to experience the benefit that we can grasp changes generally by representing recurrence relationships in algebraic equations. We want students to be able to make use of this experience in their future investigations of other phenomena.

2 Rationale of the unit, " How to observe phenomena," and unit plan

(1) Rationale of the unit, " How to observe phenomena"

The sequence of Grade 7 units in our curriculum is as follows.

Table 1 Grade 7 Sequence of Units

Chapter	Name of Unit	Sections
1	How to observe numbers	Sec. 1 Whole numbers Sec. 2 Integers
2	How to observe phenomena	Sec. 1 Tables and graphs Sec. 2 Recurrence relationships Sec. 3 Equations with letters and linear equations
3	How to observe geometric figures	Sec. 1 Geometric figures in plane and in space Sec. 2 Structures of geometric figures Sec. 3 Geometric measurements
4	Analyzing data	Sec. 1 Solving problems Sec. 2 Grasping trends

The mathematics curriculum at our school is composed of the following 4 domains: Algebra & Functions, Geometry, Probability & Statistics, and Discrete Mathematics. "How to observe phenomena" deals with the Algebra & Function domain. Therefore, in Section 3 of this unit, "Equations with letters and linear equations," we discuss algebraic equations which are absolutely necessary to represent functional relationships. The reasons for discussing "Tables and graphs" and "Recurrence relationships" prior to the study of algebraic equations are as follow. In "Tables and graphs," students learn that some problems involving functional relationships can be solved without using algebraic equations by making use of tables and graphs. However, organizing quantities in tables and graphs is a fundamental strategy to represent functional relationships. Moreover, because students have used these tools in elementary school mathematics, we positioned this section as the first section in the unit. In "Recurrence relationships," students learn to solve problems using recursive relationships. In order to represent recursive relationships, we make use of equations with words, making it possible to position the section before the formal study of algebraic equations. Moreover, as it was stated earlier, it is natural to examine patterns of changes locally before trying to grasp the changes globally. Thus, this section also aims at solidifying ways to observe changes before studying patterns of correspondences. Furthermore, what is studied in this unit will be foundational for the study of every domain in the curriculum, we positioned the unit as the second unit in Grade 7, the first year of the secondary school.

(2) Unit plan for "How to observe phenomena" (Tables and graphs/Recurrence relationships)

<u>Total of 7 lessons</u>

Section 1 Tables and graphs

Lessons	Theme of investigation	Contents
1 ~ 3	Making a box with the largest volume	From a given sheet of paper, make a box with the largest volume. Since the volume will be a cubic function of a linear dimension, students cannot yet represent it using an algebraic equation. However, by actually constructing boxes and calculating their volumes, students can organize the data in tables and/graphs to find the answer.
4	Making graphs/ Interpreting graphs	Students will try to graph the movement of an object. They will also try to figure out the movement from given graphs.

Section 2

Lessons	Theme of investigation	Contents
1	Taking medicine (1)	Students will identify recurrence relationships. They will represent the relationships in simple equations with words and use them to make some numerical calculations.
2	Taking medicine (2)	Students will represent recurrence relationships using the NOW-NEXT equations and make some numerical calculations.
3	Youth population	By assuming there is a recurrence relationship, students will represent a phenomenon using the NOW-NEXT equation and use it in numerical calculations.

3 About mathematics

(1) The learning task for this lesson and rationale

In this research lesson, we will use the following task.

> Yuu has been told to take a certain medicine to treat his illness. The phrarmacist told him to take 400 mg of the medicine every 8 hours. According to the pharmacist, the amount of the medicine in Yuu's body will be a half of the amount immediately after he takes the medicine. Why does the instruction say Yuu must take the medicine every 8 hours, when the amount of medicine in the body is at one-half? Let's investigate this question by figuring out the amount of medicine in Yuu's body immediately after he takes the medicine every 8 hours.

Figure 2 "Taking Medicine" task

There are 3 reasons for choosing this task as the main task for the lesson.

First, we believe that taking medicine is a familiar activity for students, and thus they might feel solving this problem is meaningful. Based on the solution of this problem, students can realize that taking medicine periodically allows the amount of medicine in the body to stabilize. Second reason is that this task allows students to experience a recurrence relationship since taking the medicine at a specified interval is indeed a recurring event. Finally, this task offers a variety of extension possibilities. If a person takes 400 mg of the medicine at the time interval matching the amount it take for the medicine in the body to be halved (i.e., the medicine's half-life in the body), the amount of medicine in the body will stabilize between 400 mg and 800 mg. If we generalize this observation, we can understand that the amount of medicine in the body will stabilize between the amount being taken and twice that amount. It is also possible to change the time interval so that the proportion of the amount of the medicine remaining in the body will be something other than a half. In this way, by investigating a variety of situations, students can deepen their understanding of changes in the amount of medicine remaining in the body. Furthermore, we also believe that students can have deeper sense of the process of representing a recurrence relationship in an algebraic equation and use the equation to carry out numerical calculations. However, in order to understand changes in the amount of medicine in the body when the time interval is varied, students must be able to use a general term which is a little too complex for Grade 7 students.

(2) Points of considerations as the task is assigned

This task may be rather difficult for Grade 7 students to comprehend. Thus, it is important to expand students' image of medicine taking at first. It is possible that students might consider taking medicine "after every meal" and "every 8 hours) are distinct instructions. Therefore, the teacher will explicitly discuss that these two are the same instruction. I plan to use the photo of the prescription medicine I received once that showed both instructions. Second, the changes in medicinal quantity is technically measured with the density of medicine in blood. However, to simplify the problem situation, we will just consider the amount of medicine in the body. Finally, from the beginning, we will make sure that students understand that "taking too much will not be good for your body, and if you take too little, then the medicine will not be effective." This common understanding will suggest that following the given instruction will allow us to avoid these two undesirable cases.

(3) Student ideas to highlight and why

If this task is given in the high school unit on sequences, we can advance the mathematical modeling process by representing the changes in recursion equations. However, even in Grade 7, we can go through a similar process. In particular, I want to highlight students who demonstrate the following two strategies and reasoning.

First type of students to highlight is those who could represent the problem situation visually by drawing pictures or diagrams. Even when this task was implemented with Grade 10 students at our school, some represented the problem situation using pictures shown in Figure 2. At Grade 7, I believe there will be more students who represent the problem situation visually. This strategy help other students in the class to realize that although the total amount of medicine in the body increases, the amount of changes is decreasing. It can also promote students to develop the sense that the change can be represented by recursive calculations.

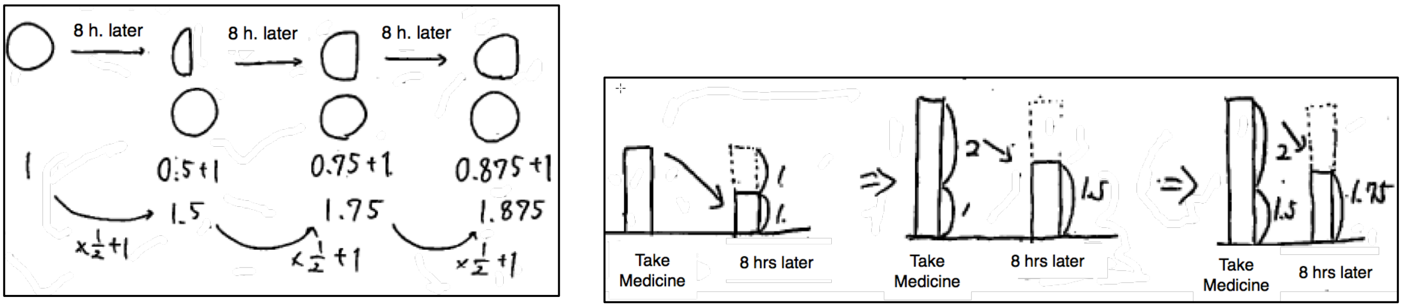

Figure 2 10th Graders' visual representations (different numerical values were used)

The other type of students is those who carry out the calculations, "÷ 2 then + 40," and conclude that the results are approaching 80 (or even if they do not specify the limiting value of 80 but concluding that the amount of increase is gradually decreasing). Later in the lesson, by focusing on this reasoning, we will try to develop an equation with words that match the problem situation, for example, "Immediately after taking medicine" ÷ 2 + 400 = "immediately after the next time Yuu takes medicine."

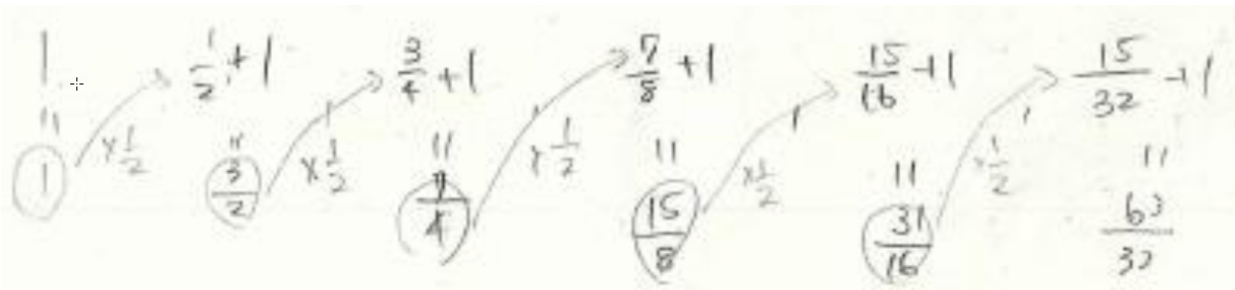

Figure 3 Repeatedly calculating "× 1/2 + 1" (different numerical values were used)

(4) Support for developing equations with words that match the situation

How can the teacher support students develop an equation with words that match the problem situation? In this lesson, after highlighting the students' ideas discussed above, the teacher will first ask, "it appears that the amount of the medicine seems to be approaching 800 mg, but does it reach 800? or is it going to exceed 800?" There are several different ways students might respond to that question, but what is desired are the responses that are based on the recurring calculations, "÷ 2 then + 400." For Grade 7 students, something like the following will be sufficient.

> If we start with 400 and repeatedly calculate ÷ 2 then + 400, the amount of changes will continue to decreases even though the amount of medicine in the body will keep increasing. If the amount in the body reaches 800, after that point, the calculation will always be 800 ÷ 2 + 400 = 800. Therefore, it will not exceed 800. Also, if the amount of medicine in the body immediately after taking medicine is less than 800, the result of ÷ 2 then + 400 will also be less than 800. If we ÷ 2 then + 400 again, that result will also be less than 800. Therefore, we will never reach 800.

Although the initial conclusion, "the amount of medicine in the body will keep increasing," may be instinctive, it is assumed that students will be using the specific values, 400, 600, 750, ... that were shown in the numerical calculations shared earlier. From those numbers, students can infer that the amount of medicine in the body keeps increasing.

When the calculations, "÷ 2 then + 400," are brought up, we will discuss if we can represent those calculations concisely and clearly. We will then try to generate an equation with words that match the context. When the equation is developed, we will teach students how we can use the RUN feature of graphing calculators to carry out the numerical calculations. This is intended so that students can realize the merit of representing recurrence relationships in equations since we can carry out numerical calculations by simply supplying the equations. We also plan to teach the ANSWER function at this point. It should be pointed out that graphing calculators might show "800" because of the limitation on the number of decimal places that can be displayed.

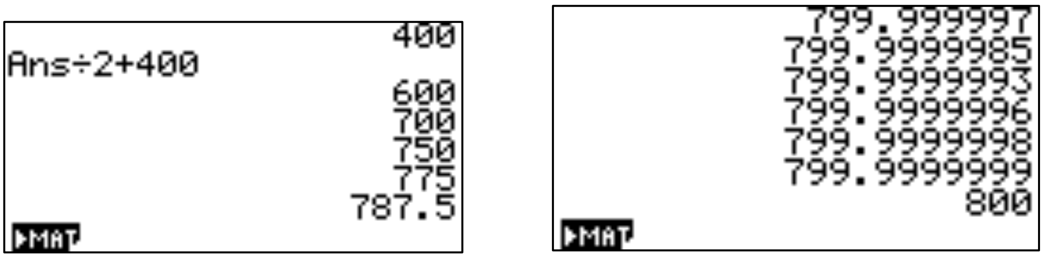

Figure 4 Calculations using the ANSWER feature of graphing calculators.

(5) Conclusions of this lesson and plan for next lessons

In this research lesson, we will observe that the upper bound of the amount of medicine in the body reaches 800 mg. We will also investigate what will happen to the lower bound and interpret the implication of the fact that the amount of medicine will stabilize between 400 mg and 800 mg. At this point, we will re-affirm the idea we discussed earlier, "taking too much will not be good for your body, and if you take too little, then the medicine will not be effective." We will show a graph like the one shown in Figure 5 so that students can understand that if we follow the instruction then the amount of medicine in the body will remain in a particular interval that is the medicine's effective range. This is as far as we try to get in this lesson.

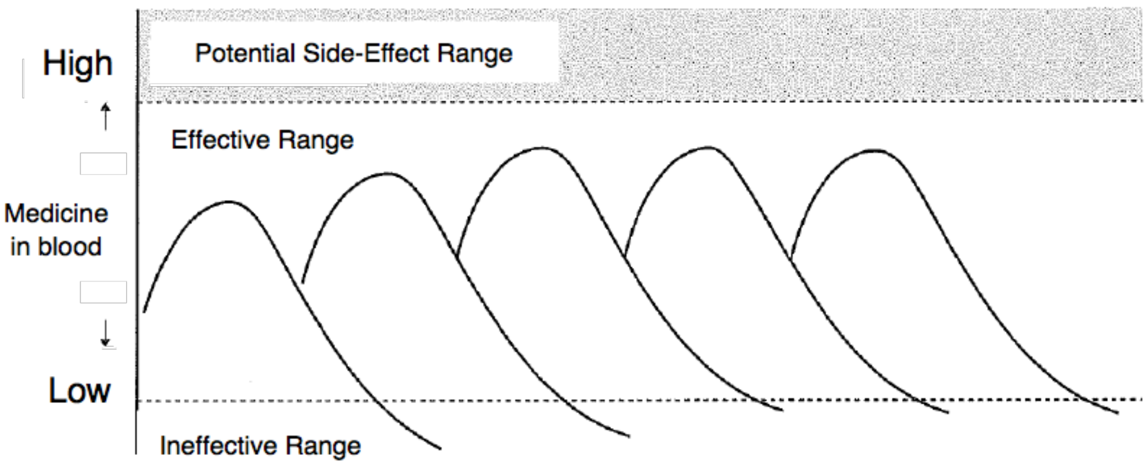

Figure 5 Effective range/Potential Side-Effect Range/Ineffective Range based on the concentration of medicine in blood (Hashimoto et al. 2002)

The result of the Day 1 investigation shows that if Yuu takes 400 mg of medicine every 8 hours, the amount of medicine in the body will stabilize between 400 mg and 800 mg. From this result, students might conjecture that the amount of medicine in the body will stabilize between the amount of medicine taken and twice that amount. In order to verify this conjecture, we will change the amount of medicine taken, and by representing the new situations using equations with words, we want students to carry out numerical calculations. This way, we want to verify that the conjecture appears to be valid, and interpret the result suggesting that knowing this pattern will make it easier for doctors to determine how much medicine to prescribe. Next we will change the interval of medicine taking from the half-life of the medicine in the body. For example, if we take the medicine when the amount of the medicine in the body is 1/3 of the amount immediately after taking the medicine, then the amount of medicine will stabilize with the upper bound that is 3/2 of the amount of medicine taken. At this level (Grade 7), it is not possible to pursue why this is the case. However, students can nevertheless understand that even when the interval is varied, the amount of medicine in the body will stabilize. However, the relationship between the amount of medicine taken and the upper bound amount is rather difficult to observe. Therefore, students might be able to appreciate the simplicity - that is, the amount of medicine in the body will stabilize between the amount taken and twice that amount - when the interval matches the half-life of the medicine in the body.

Representing the situations with different medicine amount or different time intervals with algebraic equations with words matching the situations and carrying out numerical calculations will also serve as an evaluation of Lesson 1. Moreover, we believe those experiences will foster students to experience the merit of representing recurrence relationships using algebraic equations. By reflecting on the series of investigations, and focusing on recurrence relationships, we will introduce the NOW-NEXT equations as a method to represent those relationships. In Lesson 3, we will investigate different types of recurrence relationships and ask students to make use of the NOW-NEXT equations to represent them and solve problems.

4 Instructional Plan for the Lesson

(1) Goal of the lesson

Criterion B: Identify a recurrence relationship in the phenomena and represent it as a general rule using an equation with words that are appropriate for the problem context.

Criterion D: Carry out numerical calculations using the equation and interpret the results in the problem context.

(2) Flow of the lesson

❖ Teacher questions, S student responses, ● instructional strategies

time	Content, main question, & anticipated responses	Points of consideration
10	1. Posing the Task T1: I'm sure you have experiences of taking medicine when you were ill. Here is the picture of the prescription I had to take when I was sick while back. Do you see it says, "Take every 8 hours (after meals)"? I wondered why I have to take the medicine "every 8 hours (after meals)," so I researched a bit. I found out that the amount of medicine in the body keeps decreasing due to metabolism, and after 8 hours it will be only a half of what it is immediately after taking the medicine. It turns out such medicines are to be taken every 8 hours. What do you think about the amount of medicine in the body? S1-1: If the amount reduces so much, it will lose its effectiveness, so we need to keep adding medicine. S1-2: I heard that taking too much medicine is not good. They must decide how much medicine to take. S1-3: I think they are trying to keep the amount of medicine in the body to be closed to a fixed amount. T2: If we don't have enough medicine, it will not work. But, if we take too much, that is probably not good. I wonder if the amount of medicine will be almost constant. Let's investigate that today.	• Show the photo using PowerPoint. • Touch upon how the amount of medicine in the body decreases. • Make sure students can have a good image of metablism affecting the amount of medicine. • Make sure everyone understand that too much medicine is probably harmful, and too little will make the medicine ineffective.
	Yuu has been told to take a certain medicine to treat his illness. The phrarmacist told him to take 400 mg of the medicine every 8 hours. According to the pharmacist, the amount of the medicine in Yuu's body will be a half of the amount immediately after he takes the medicine. Why does the instruction say Yuu must take the medicine every 8 hours, when the amount of medicine in the body is at one-half?	

<table>
<tr>
<td></td>
<td>T3: "Every 8 hours" and "After meals" may not coincide all the time. What should we do?

S3-1: Let's think about every 8 hours.
S3-2: It must be ok if they do not completely coincide.

T4: Let's assume he is taking the medicine every 8 hours. Let's explore by examining how much medicine will be in Yuu's body after each time he takes the medicine..</td>
<td>• Students have not had many experiences of "making an assumption" until now. If necessary, make sure students understand why this is important.</td>
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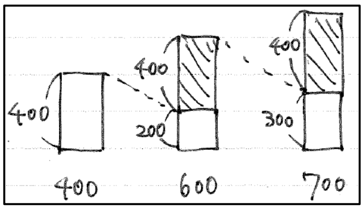
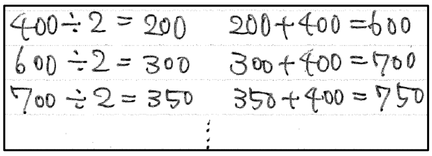
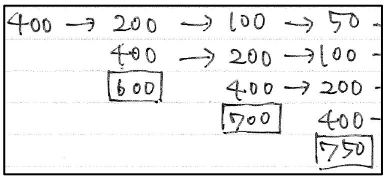
<td>15</td>
<td>2. Independent Problem Solving

S4-1: Draw a picture or diagram to capter the problem situation.


S4-2: Repeatedly ÷ 2 and add 400.


S4-3: Keep halving the amount taken and find the total


S4-4: It's gradually approaching 800.
S4-5: It is not increasing too much.
S4-6: It keeps increasing.
S4-7: The difference of the amount of medicine between 2 consecutive times keeps halving.
S4-8: The difference from 800 keeps halving.
S4-9: Unure what to do.

T5: (Ask students who used drawings of calculations, like S3-1 or S3-2, to reach the conclusions of S3-4 and S3-5 to write their solutions on the board.)</td>
<td>• Diagrams are just examples.

• S4-1 through S4-3 are methods to generate the sequence of the amount of medicine in the body. On the other hand, S4-4 through S4-8 are the analysis of the sequence.

• How many calculations will be repeated may depend on the students' images of medicine taking. If necessary, we will assume Yuu is taking the medicine for 5 days.

[Evaluation] B: Are students somehow representing a recurrence relationship?

• For S4-9, help him/her to imagine the process of taking medicine when the amount in the body becomes a half, and ask him to figure out the specific amounts after each time Yuu takes the medicine.</td>
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<td>15</td>
<td>3 Comparison/Discussion (1) (Discuss whether or not the amount is gradually approaching 800.)

S6: (Explaining S4-2) If we calculate the amount of medicine immediately after taking medicine in order, it will go 400, 600, 700, 750, 775,</td>
<td>● If no students concluded that the value approaches 800, pick on S4-5's idea and ask students to think about what happens to the amount of medicine if Yuu keeps taking the</td>
</tr>
</table>

	786.25, ... It doesn't increase so much, and I thought it was approaching 800. T7: S6 thinks the amount of medicine in the body is approaching 800 (mg), but do you think it will reach 800? Will it ever to over 800? Can someone use S3-2's idea and explain? S7: S4-2 kept doing ÷ 2 then + 400 to the amount of medicine immediately after taking it. As long as the amount immediately after taking the medicine is less than 800, the result of the calculation will also be less than 800. So, no matter how many times you keep doing ÷ 2 then + 400, the result will be less than 800. The amount of medicine in the body will never reach 800. T8: How can we represent the calculation S3-2 did using an equation? S8-1: (immediately after) ÷ 2 + 400 = (immediately after next time) S8-2: Symbols like ○ and □: ○ ÷ 2 + 400= □ S8-3: (First time) ÷ 2 + 400 = (2nd time), (2nd time) ÷ 2 + 400 = (3rd time), ... T9: (Referring to the idea of S8-1) S5 said, "As long as the amount immediately after taking the medicine is less than 800, the result of the calculation will also be less than 800." What does it mean with S6-1's expression? S10: If "immediately after" in the equation is less than 800, the "immediately after next time) will also be less than 800. Since that will become the new value for "immediately after," the next "immediately after next time" will also be less than 800. The same things will keep repeating - they are always less than 800. T10: As we can see from S6's idea, even though the amount of medicine in the body keeps increasing gradually, it will never reach 800, will it? T11: If we can express a set of repeated calculation into an equation like, "immediately after" ÷ 2 + 400 = "immediately after next time," we can	medicine. ● If students are hesitant to share their ideas, give them some time to discuss the question with their neighbors. ● Ideas like S4-7 and S4-8 are also possible. Acknowledge their ideas, but focus the discussion on S4-2's idea. • Words/phrases or what letters/symbols represent is important. Make sure that words/phrases can be used to describe consecutive terms in general. • Make sure that students understand that the result of one calculation will be substituted into the same equation again for the next calculation. [Evaluation]D: Are students checking if the amount of medicine in the body is approaching 800 by making repeated calculations?

	enter the expression into a graphing calculator and calculate the results very quickly. Let's try. S11-1: Wow, it calculates one after another very quickly. S11-2: It reached 800.	• Explain that the limitation on the number of places that can be displayed caused the result to be 800.
5	4 Comparison/Discussion (Interpret what it means to approach 800) T12: Now we know that the amount of medicine immediately after taking the medicine will approach 800 mg. What happens to the amount of medicine immediately before taking it? S12: Since the amount immediately after is approaching 800, the amount immediately before will approach a half of that (or 400 less), so it will approach 400 mg. T13: What does it mean that the amount immediately before approaches 400 mg and the amount immediately after approaches 800 mg? S13-1: The amount of medicine in the body is fluctuate between 400 mg and 800 mg. S13-2: At the beginning of the lesson, we said too little medicine will be ineffective and too much is dangerous. So, perhaps this medicine will not work if there is less than 400 mg in the body, and cause problems if there is more than 800 mg. T14: (Display the diagram showing the effective range of medicine) By taking the medicine at the time when the medicine remaining in the body is one half, we can keep the amount in the body in the effective range, can't we?	[Evaluation]D: Are students trying to interpret the findings that he amount immediately before approaches 400 mg and the amount immediately after approaches 800 mg in the problem context? • Mention that this chart shows the amount of medicine in the body in terms of its concentration in blood.
5	5 Summary T15: Summarize the following points. • It appears that by taking the medicine when the amount remaining in the body is at one half, we can maintain the amount of medicine in the effective range. • By identifying a recurrence relationship and expressing it in an equation, we can examine how the quantity changes. S14: Write their reflections.	

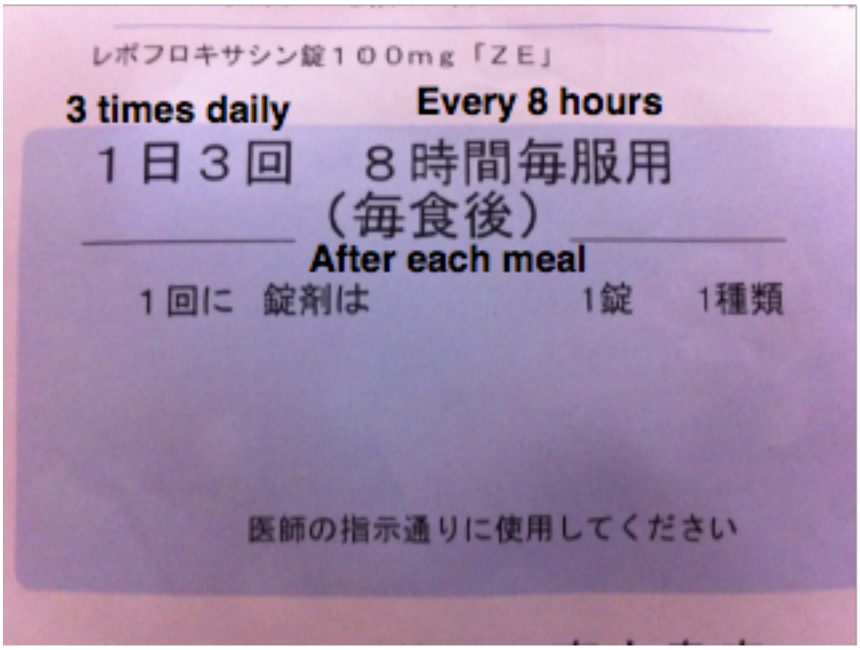
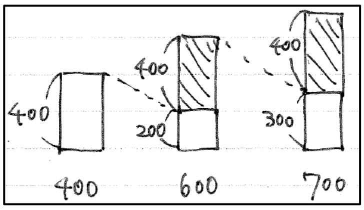


Figure 1: The bag of the prescription I was once given.

5 Board Writing Plan

* In the classroom, there are 2 blackboards. We will set up a screen to the side.

(S4-1's idea)  	(S4-2's idea) 400÷2 = 200 200+400 = 600 600÷2 = 300 300+400 = 700 700÷2 = 350 350+400 = 750 ⋮	Q: Will the amount of medicine in the body approaches 800? Reaches 800? Go over 800? (S7's idea) As long as the amount immediately after taking the medicine is less than 800, the result of the calculation will also be less than 800. It will not reach 800.	S4-2 said, we keep repeating, (immediately after) ÷ 2 + 400 = (immediately after next time) [#] → We can use a graphing calculator! We can see that the amount is approaching 800.

Blackboard 1

Q: What about immediately before taking the medicine? Keep approaching 400 mg. Q: What does it mean that the amount immediately before approaches 400 mg and the amount immediately after approaches 800 mg? • The amount of medicine stabilizes between 400 mg and 800 mg (effective range).	By representing a recurrence relationship in an equation like [#], we can examine how the quantity changes.	

Blackboard 2

Photo of the prescription bag （Keep it displayed during the independent problem solving time） Explanation of graphing calculator Chart showing effective range of medicine

Screen

6 Evaluation Plan

Evaluation in this lesson will be conducted through the analysis of students' worksheets and their reflections. We will also assess students comments from the video recording of the lesson. In the next lesson, students will explore "will the amount of medicine stabilizes between the amount taken and twice that amount?" and "what if we take the medicine when the amount in the body is one third?" I plan to evaluate students' worksheets and reflections, as well as their comments during that lesson, too. Using those data, I plan to evaluate whether or not the goals in Criterion B and D have been achieved. In the third lesson, we will tackle the "youth population" task (not included in the lesson plan). In that lesson, we will make explicit the relationships we explored in the first two lessons as "recurrence relationships," and we will express them using the "NOW-NEXT" equations. In that lesson, I plan to evaluate whether or not the students can represent a recurrence relationship in an equation, use the equations to carry out numerical calculations and interpret the results in the context.

References (omitted)

International Baccalaureate Organization（2014）， *Middle Years Programme Mathematics guide*， www.ibo.org.