July 1

July Combined Research Meeting
Group B Proposal Lesson

Research Theme: Mathematics instruction that will nurture students who can think logically and express themselves

Name of the Unit: Grade 5 Division of Decimal Numbers

1 In response to 4esearch theme

In Group B, we have been focusing our attention on the following two points to address the research theme.

(1) We need to develop teaching plan that carefully consider the coherence of the unit (and the entire curriculum). By doing so, we believe we can nurture students who can think logically by utilizing their prior knowledge from "Multiplication of Decimal Numbers" and "Per Unit Quantities."
(2) We believe that we can address the research theme by having students express their ideas for the purpose of justifying logically how they extended their prior knowledge.

2 Image of students we want to nurture through this unit

In the context of expanding the meaning of operation, students will be able to justify how they extended their prior knowledge.

3 Strategies to nurture students toward the image

(1) **We developed the teaching and evaluation plans by paying close attention to the connection to the unit on multiplication of decimal numbers.**

We believe that we can develop a better teaching plan for the unit, Division of Decimal Numbers, by thinking about the unit, Multiplication of Decimal Numbers.

(2) **Strategies to have students justify their idea logically as they expand the meaning of operations**

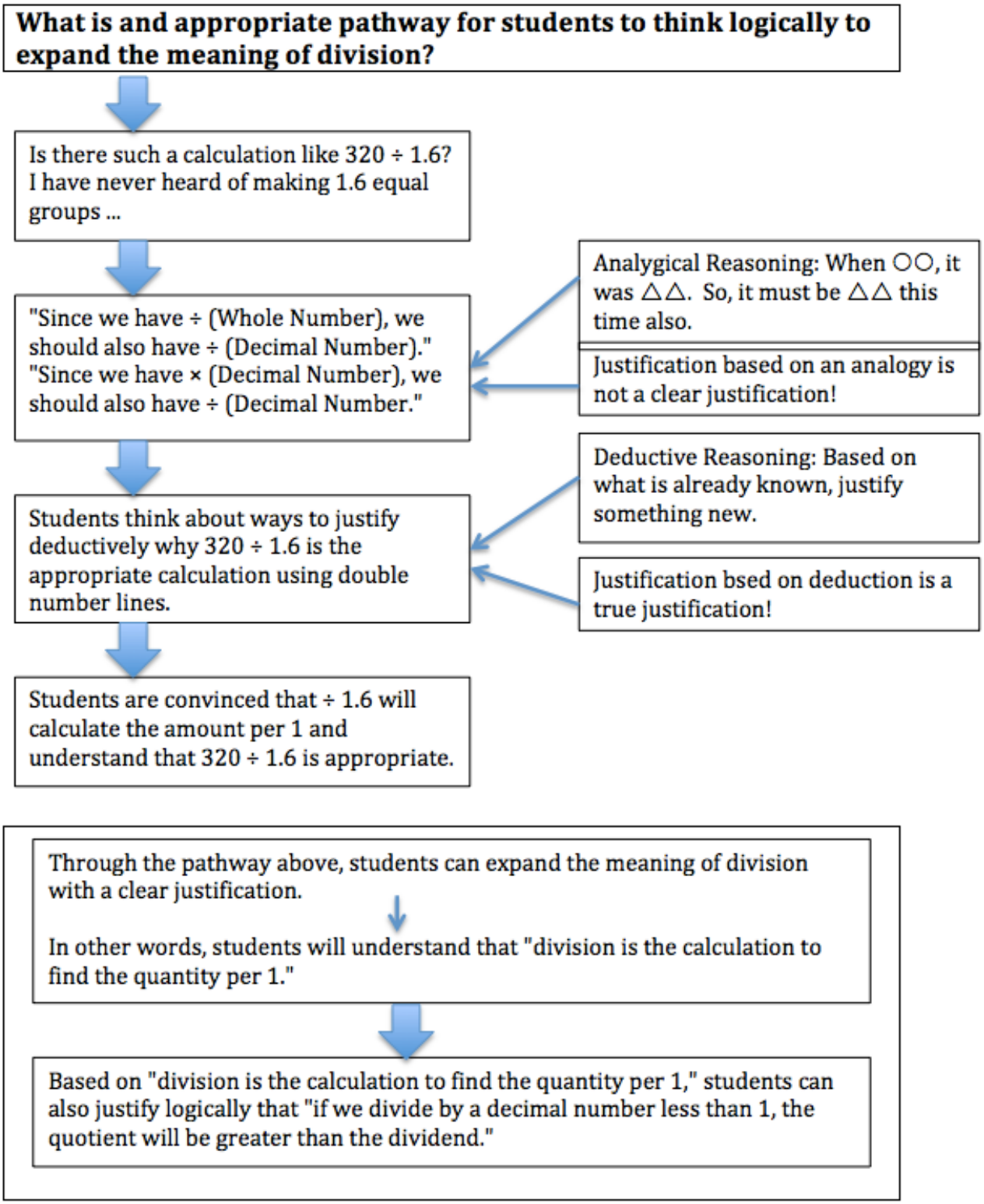
We believed that the true expansion of the meaning of division involves having students justify deductively the process of the expansion by making use of double number lines instead of simply using analogical reasoning based on situations from whole number division.

We believe that the main content of this unit is the "expansion of the meaning of division." In the unit, where students can most easily grapple with the expansion of the meaning is the situation in which the divisor is a decimal number less than 1, which results in the quotient that is greater than the dividend.

In this research lesson, we address the first step of the expansion of the meaning of dividing by decimal numbers. To help students understand the expanded meaning of division that makes it possible to interpret "÷ Decimal Number," we will focus on supporting students justifying this expansion of the meaning of division through the use of double number lines.

(3) Coherent use of double number line

In this unit, double number line plays a key role in justifying the expansion of the meaning of division. In order for students to be able to use double number line as their own reasoning tool, we have intentionally used double number lines extensively in the units, "Per Unit Quantities" and "Multiplication of Decimal Numbers."



Grade 5 Mathematics Lesson Plan

Wednesday, July 1, 2015
Period 5
Sasahara Elementary School
(Setagaya Ward, Tokyo)
Grade 5, 28 students (on level)
Teacher: OOHASHI, Yuuki

1 Name of the Unit

Division of Decimal Numbers: Let's think about dividing by decimal numbers.

2 Goals of the Unit and Evaluation Standards

- Students will understand the meaning of division of decimal numbers and be able to use it appropriately. [A(3)[1]]
  - Students will understand the meaning of dividing by decimal numbers. [A(3) b]
  - Students can think about ways to calculate division by decimal numbers and calculate reliably. [A(3) b]
  - Students will understand the size of remainders. [A(3) b]

Evaluation Standards

Interest, Eagerness, and Attitude	Mathematical Way of Thinking	Mathematical Skills	Knowledge and understanding
• Students will recognize that division by decimal numbers can be thought about in the same way as division by whole numbers, and they will try to use it.	• Students are using the rules and properties of division they learned with division of whole numbers to think about the meaning and ways of dividing by decimal numbers.	• Students can calculate division with decimal numbers to hundredths using the algorithm.	• Students understand the meaning and ways to calculate divisions of decimal numbers. They also understand what to do when there are remainders and can express the quotients rounded to a specified place.

[1] This refers to the specific statement in the National Course of Study for Grade 5 Mathematics.

3 About the Unit

What we want to nurture in students through this unit are "the ability to expand the meaning of division" and "the ability to think about whole numbers and decimal numbers from a unified perspective."

In order to develop those abilities, we first considered the connection to the unit, Multiplication of Decimal Numbers. In Multiplication of Decimal Numbers, we utilized double numbers lines to expand the meaning of multiplication from the repeated addition to multiplication as scaling. In addition, when the multiplier is a decimal number less than 1, we used double number lines to grasp the relationship among quantities to understand that the product will become less than the multiplicand and deepen their understanding of × decimal numbers. Furthermore, we tried to help students understand that multiplication of decimal numbers can be calculated using the idea similar to multiplication of whole numbers while paying attention to the location of the decimal point since decimal numbers are represented in a base-10 notation. We developed our instruction and evaluation plan so that the similar pathway is also visible in Division of Decimal Numbers.

First, "expansion of the meaning of division" is about generalizing partitive division as the operation to find per-unit quantity, i.e., how much for one unit, when the divisors are decimal numbers instead of thinking about it as an operation to find how much for each group when you make *n* equal groups as students have done with whole number divisors. For quotitive division, we want students to understand quotitive division as the inverse operation of multiplication which can be used to find the scale factor (__ times as much) instead of finding how many groups as they did with whole number divisors.

As for "developing a unified perspective on whole numbers and decimal numbers," we want students to truly experience "we can do what we could do with whole numbers even when the numbers become decimal numbers" through activities such as determining the operation necessary to find the missing quantity or checking the validity of properties of operations learned with whole numbers. Through such experiences, we believe that we can not only enrich students' ways of seeing numbers but also develop ways of reasoning that can be used in their study of fractions.

4 About the Students

In the unit, Per Unit Quantities, using double number lines, students learned that the positions on a double number line represented the size relationships of quantities. They have also learned to use □ for unknown and determine the appropriate operation to find the unknown. Moreover, students discovered that as the numbers on the top number line becomes 2, 3, ... times as much, the corresponding numbers on the bottom line also became 2, 3, ... times as much (proportional relationship).

Before the unit, Multiplication of Decimal Numbers, we set up a topic unit, Calculation of Times as Much. In the unit, students learned about times as much with decimal numbers, and the relationship, "if ○ is considered as 1, □ corresponds to △. In this way, students learned the usefulness of double number lines.

In the unit, Multiplication of Decimal Numbers, students learned to think by connecting equations with words and double number lines. ("Because 2.4 meters is 2.4 times as much as 1 meter, the price will also be 2.4 times as much as 80 yen." "The calculation with meters will be the same as calculation with yen.") Moreover, students learned that mathematical expressions represent problem situations and to distinguish mathematical expressions representing situations and mathematical expressions that showed the ways of calculation.

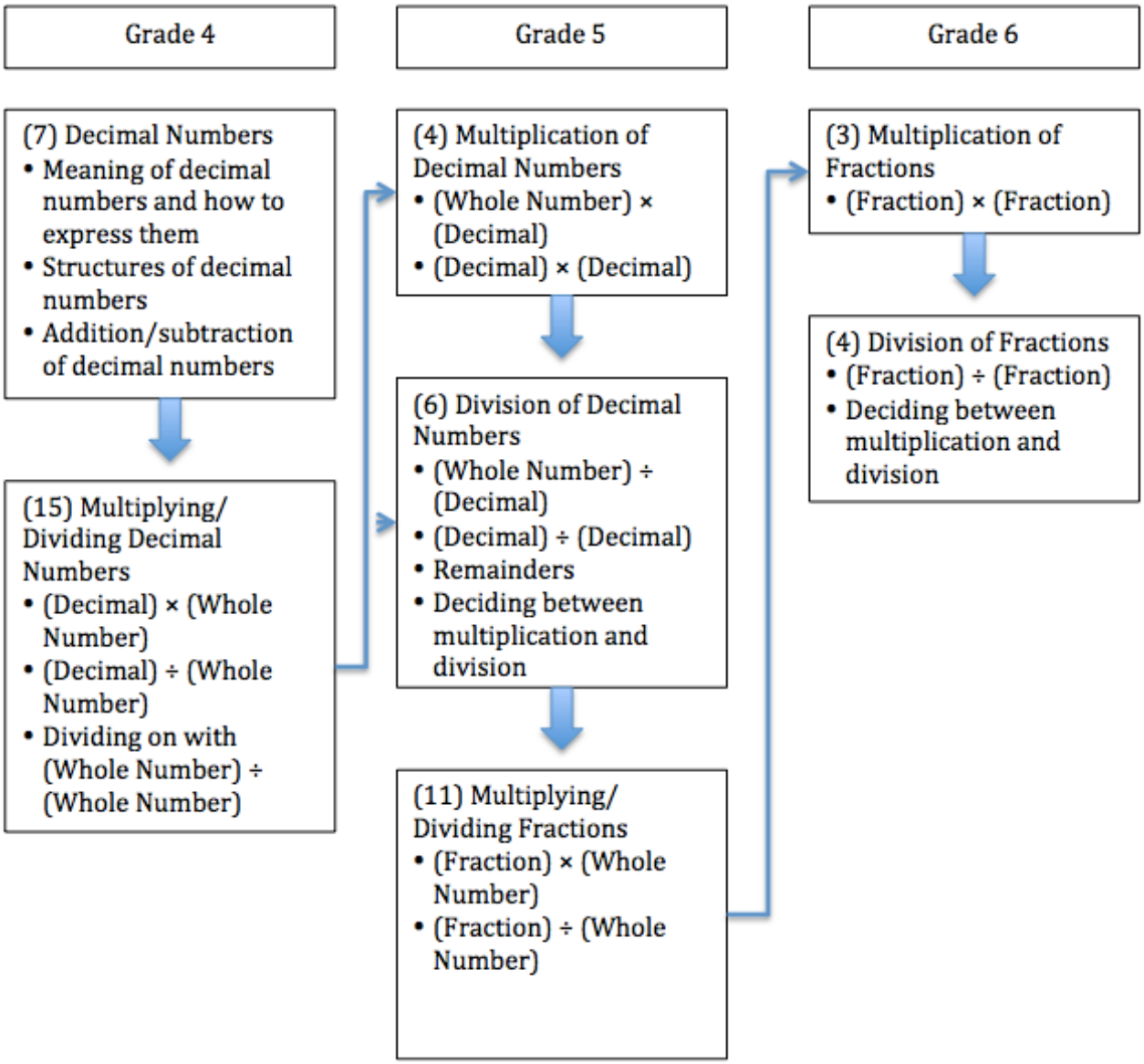
Students began to feel the usefulness of double number lines to justify the choice of appropriate operation as the changes in the number of students who used double number line. While studying (Whole Number) × (Decimal Number), 8 of 28 students used double number lines. While learning (Decimal Number) × (Decimal Number), 19 students used double number lines, and 20 students used double number lines to deal with (Decimal Number) × (Decimal Number < 1). However, while dealing with (Decimal Number) × (Decimal Number < 1), more students represented numbers incorrectly, mistaking the size relationships. Thus, it became clear that we need to continue emphasizing how to draw double number line correctly - the length from 0 represent the quantity (See Seating Chart attached at the end).

To develop students' ability to express themselves, we emphasized the value of using mathematical terms while students are sharing their problem solving strategies or engaged in a whole class discussion. Several students began to notice that by using mathematical terms, they can more easily identify similarities and differences in their reasoning. I have also tried to make it more comfortable for students to share their ideas by encouraging them to explain their ideas using words, mathematical expressions and diagrams. I also promoted students to offer hints to each other and teach each other. I have been trying to develop both students' ability to share their ideas and their ability to listen to other's idea by raising the level of mathematics lessons with special attention to observing students' responses to each other's ideas and making use of their ideas.

In addition to valuing each student's ideas, I also try to focus on students who say "I don't get it" during lessons. I encourage those students to clarify what they do understand and what they do not so that the whole class can share the questions those students have. Then, as students' share their ideas, they are encouraged to address those students' questions and puzzlements. In this way, students' sharing has begun to shift from simply explaining their own ideas to making use of their own ideas to answer other students' questions.

In order to increase student to student interactions, during the whole class comparison and analysis of solution strategies, I often have students sit in front of the blackboard with their own notebooks and pencils. I believe that the students have become much more conscious of others and listen to each other more intently. Some students are now trying to deepen their own ideas and to generate new ideas as they listen to other's ideas.

5. Scope and Sequence



6. Unit Plan and Evaluation Plan (11 lessons)

* To show the relationships to the previous unit, "Multiplication of Decimal Numbers," the plans for both units are shown below. The shaded cells include learning activities that are distinct while other cells show lessons whose learning activities and evaluation points are similar.

Sub-Unit	Lesson	Main Learning Activity	Specific Evaluation Standards	Desired students' reflection
0	1	• Students will understand the meaning of times as much with decimal numbers.	[Mathematical Way of Thinking] Students are thinking about times as much with decimal numbers in the same way with whole numbers using number lines and other diagrams.	• Just like 2 times as much and 3 times as much, we can have 1.5 times as much or 2.5 times as much. • 2.5 times as much is between 2 times as much and 3 times as much. • If you show it on a number line, it is about here. • If we consider ○ as 1, △ corresponds to 2.5. • Decimal numbers can be used with times as much.
1. Calculation of (Whole Number) × (Decimal Number)	1	• Students will investigate if we can multiply by decimal numbers using diagrams and tables. • Using diagrams, students will estimate the price. • Students will think about ways to calculate (Whole Number) × (Decimal Number).	[Interest, Eagerness, and Attitude] Students are estimating the products and try to think about ways to multiply with decimal numbers.	• We can't use the idea that worked for addition. • Double number lines help us decide what calculation to use. • 80 × 2.3 means 2.3 times as much as 80. • On a number line, it is about here. • If we consider 80 as 1, what corresponds to 2.3. • 2.3 times as much is the same as × 2.3. • If we use an expression with words, it will be (price for 1m) × (length)
	2	• Students will explain their ways of calculating (Whole Number) × (Decimal Number) using words, numbers, mathematical expressions, diagrams and number lines.	[Mathematical Way of Thinking] Students are thinking about the meaning of multiplying by decimal numbers that can also be applied to whole numbers, and they are thinking about ways of multiplying by decimal numbers.	• If we pretend to buy 23 m, the total cost is 80 × 23 = 1880 Yen. So, by doing ÷ 10, it will be 188 Yen. • Since the price for 0.1m is 8 Yen, and since 2.3 is 23 pieces of 0.1, 8 × 23 = 188. • If we × 10 then ÷ 10, the answer will be the same. • We are using whole number calculations.
	3	• Students will think about ways to use the algorithm to calculate (Whole Number) × (Decimal Number). • Summarize the algorithm for (Whole Number) × (Decimal Number).	[Mathematical Skills] Students understand how to calculate (Whole Number) × (Decimal Number) using the algorithm, and they can calculate.	• I think we can use the algorithm. • You can multiply by 10 to eliminate the decimal point, then we can ÷ 10. • We can calculate ignoring the decimal point, then drop the decimal point straight down. • If we turn it around it will be 2.3 × 80, and we already know how to do that calculation.

2. Calculation of (Decimal Number) × (Decimal Number)	4	• Students will determine the appropriate mathematical expression using diagrams. • Students will think about ways to calculate (Decimal Number) × (Decimal Number). • Students will explain their ways of calculating (Decimal Number) × (Decimal Number) using words, numbers, mathematical expressions, diagrams and number lines. • Summarize the algorithm for (Decimal Number) × (Decimal Number) in general.	[Mathematical Way of Thinking] Students are thinking about ways to calculate (Decimal Number) × (Decimal Number) using what they have learned.	• We can change to (Whole Number) × (Decimal Number). • If we can change either the multiplicand or the multiplier into a whole number, we can calculate. • We can just × 10, × 10, then ÷ 100.
	5	• Summarize the algorithms for (Decimals to hundredths) × (Decimal to tenths). • Summarize the algorithm for decimal number mutiplication in general. • Students will solve word problems involving decimal number multiplication to hundredths.	[Mathematical Way of Thinking] Students are figuring out how to calculate the product even when the multiplier becomes a decimal number to hundredths just as they did before. [Mathematical Skills] Students understand the position of the decimal point in the product and can calculate (Decimal Number) × (Decimal Number).	• We are doing the same thing as before: change the numbers to whole numbers then divide the result. • Double number lines shows quickly what calculation is needed. • The algorithm is the same as the whole number multiplication. We just need to figure out where to place the decimal point.
	6	• Students will verify that the area formula for rectangles can be used even when the lengths are given as decimal numbers.	[Knowledge and Understanding] Students understand that the area of a rectangle can be calculated using the formula even when the lengths become decimal numbers.	• Can we calculate the area of a rectangle if the lengths become decimal numbers? • If we change the unit to cm and then convert the result, it is the same. • We can express area using decimal numbers, too.
	7	• Students will know that the product will be less than the multiplicand if the multiplier is less than 1. • Students will know how to multiply two decimal numbers less than 1 using the algorithm.	[Knowledge and Understanding] Students understand the relationship between the multiplicand and the product when the multiplier is less than 1 by relating it to its double number line representation.	• Double number line representation looks different from what we have seen before. • The multiplier is on the left of 1. • What does "0.3 times as much" mean? • The answer got smaller. • There is something different when you multiply by numbers less than 1.

3 Properties of Operations	8	• Students will know that the commutative and associative properties hold even when addends are decimal numbers. • Students will know that the commutative and associative properties hold even when the factors are decimal numbers.	[Mathematical Skill] Students can verify that the commutative and associative properties hold for decimal numbers just as they did with whole numbers.	• Calculations become simple if we apply the properties. • The properties of operations are the same whether the numbers are whole numbers or decimal numbers. • Multiplication of decimal numbers is similar to multiplication of whole numbers.
	9	• Students will know that the distributive property holds even when the numbers involved are decimal numbers. • Students will apply the associative and distributive properties.	[Mathematical Skills] Students can verify that the distributive property holds for decimal numbers just as they did with whole numbers.	• Calculations become simple if we apply the properties. • The properties of operations are the same whether the numbers are whole numbers or decimal numbers. • Multiplication of decimal numbers is similar to multiplication of whole numbers.. • The associative and distributive properties are useful.
Exercises	10	• Students will find the products of decimal numbers. • Students will calculate the area of rectangles whose dimensions are given in decimal numbers. • Students will think about ways to simplify calculations using the properties. • Students will verify the relationship between the multiplicand and the product and also the associative and distributive properties.	[Mathematical Skill] Studetns can multiply decimal numbers to hundredths using the algorithm.	
	11	• Students will summarize the way to multiply decimal numbers. • Students will solve word problems involving multiplication of decimal numbers.	[Mathematical Skills] Studetns can multiply decimal numbers to hundredths using the algorithm.	•

Sub-Unit	Lesson	Main Learning Activity	Specific Evaluation Standards	Desired students' reflection
1 Calculation of (Whole Number) ÷ (Decimal Number)	1 Today's Lesson	• Studens will investigate if we can divide by decimal numbers using diagrams and tables. • Using diagrams, students will estimate the price.	[Mathematical Way of Thinking] Students can think about the meaning of (Whole Number) ÷ (Decimal Numbers), and they can explain why division is the calculation with reasons.	• We can use double number lines we used for per-unit quantities. • Since there is × (Decimal Number), so it's opposite must be ÷ (Decimal Number) • We can find the operation to use if we use double number lines. • The equation with words will be (Price) ÷ (Length) = (Price for 1m) • Since we have ÷ (Whole Number), there must be ÷ (Decimal Number) • If it's only a half, it will be about ❍ Yen. • To find per-1, we divide. • We can't use the idea for making equal groups. • Double number lines look different from how they looked with multiplication.
	2	• Students will explain their ways of calculating (Whole Number) ÷ (Decimal Number) using words, numbers, mathematical expressions, diagrams and number lines. • Students will think about ways to use the algorithm to calculate (Whole Number) ÷ (Decimal Number). • Summarize the algorithm for (Whole Number) ÷ (Decimal Number).	[Mathematical Way of Thinking] Students are thinking about the meaning of dividing by decimal numbers that can also be applied to whole numbers, and they are thinking about ways of dividing by decimal numbers.	• Pretend the length we bought is 10 times as long and calculate the quotient. Then multiply the quotient by 10. • We can divide by the number of 0.1, then multiply the quotient by 10. • We can calculate by changing the numbers to whole numbers. • Multiply the divisor by 10 so that we can divide, then multiply the quotient by 10. • If we ×10 and × 10 then divide and keep the quotient. • Ways to calculate the quotient is different from the way to calculate the product.
	3	• Using (Whole Number) ÷ (Decimal Number), students will determine the missing side of a rectangle given the area and the other side.	[Mathematical Skills] Students understand how to calculate (Whole Number) ÷ (Decimal Number) using the algorithm, and they can calculate.	• Can we use division of decimal numbers with area of rectangles? • Once we change the divisor to a whole number, the method of calculation is the same as that of whole number division. • We changed numbers to whole numbers just as we did with multiplication. • I think we can do (Decimal Number) ÷ (Decimal Number).

2 Calculation of (Decimal Number) ÷ (Decimal Number)	4	• Students will determine the appropriate mathematical expression using diagrams. • Students will think about ways to calculate (Decimal Number) ÷ (Decimal Number). • Students will explain their ways of calculating (Decimal Number) ÷ (Decimal Number) using words, numbers, mathematical expressions, diagrams and number lines. • Students will think about ways to use the algorithm to calculate (Decimal Number) ÷ (Decimal Number). • Summarize the algorithm for (Decimal Number) ÷ (Decimal Number) in general.	[Mathematical Way of Thinking] Students are thinking about ways to calculate (Decimal Number) × (Decimal Number) using what they have learned. [Mathematical Skills] Students can find the quotients for divisions of decimal numbers to hundredths.	• Double number lines are useful to determine the appropriate calculation. • If we can change one number to a whole number, we can use what we learned before. • If we can make both numbers whole numbers, then it's exactly what we have been doing. • If we do ×10 and ×10, then the quotient is what we want. • The division algorithm is basically the same.
	5	• Think about ways to divide on by annexing 0's. • Think about ways to calculate the quotient when the one's digit is a 0. • Students will grasp that two different division expressions are possible from the exactly the same context by changing which quantity will be considered as the base quantity.	[Mathematical Way of Thinking] Students are thinking about ways to divide on with decimal numbers using what they have already learned. [Mathematical Skills] Students can calculate (Decimal Number) ÷ (Decimal Number) from a variety of situations.	• There are many different patterns in the division algorithm. • The order of steps in the division algorithm is the same. • Figuring out where to put the decimal point is hard. • I must not forget to write the decimal point. • Double number lines are useful to determine the appropriate calculation.
	6	• Students will know that the quotient will be greater than the dividend when the divisor is less than 1. • Students will know hot to divide by a number less than 1.	[Interest, Eagerness, and Attitude] Students are trying to think about problems from a variety of concrete situations using diagrams.	• I wonder if the effect of dividing by decimal numbers less than 1 is the same as that of multiplying by decimal numbers less than 1. • Where the quantities are represented on double number lines are different. • The quotient is greater than the dividend. • The algorithm is the same. • Let's check the results of calculations. • It is easier to think with the quotitive division. • Double number lines make it clear.

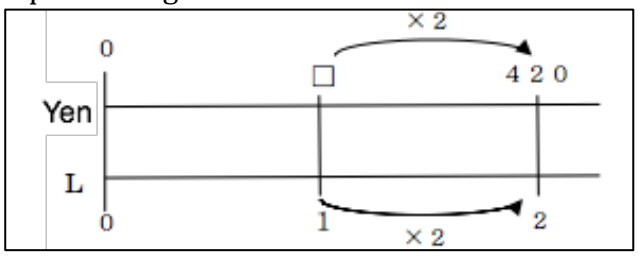
3 Division with Remainders	7	• Think about the meaning of the remainders while calculating (Decimal Number) ÷ (Decimal Number) and know how to determine the position of the decimal point.	[Interest, Eagerness, and Attitude] Students are trying to explain the size of remainders and the position of the decimal point based on a variety of situations.	• If there is a remainder, the decimal point of the quotient is the same as the decimal point in the dividend. • I can make sense if I think of repeated subtraction. • × 10, × 10, then divide. The quotient is what we need. The decimal point for the remainder is the same. • I felt satisfied after I checked the result of my calculation. • I can explain the situation.
	8	• Students will know the method and the meaning of rounding the quotient to a specified place.	[Knowledge and Understanding] Students understand the method and the meaning of rounding the quotient to a specified place.	• We can round with decimal numbers, too. • The division algorithm is tedious. • I need to remember to which place I am rounding while calculating the quotient.
4 What calculation do we need?	9	• Students will represent word problems on double number lines. • Students will grasp the structure of problems by simplifying the quantities in the problems. • Students will write the appropriate mathematical expression to find the missing quantity and solve the given problem. • Students will solve an extension problems created from the original problem. • Based on an example, write problems that will be solved by multiplication or division of decimal numbers. They will solve the problems created by each other.	[Mathematical Way of Thinking] Students are grasping the relationship of quantities to determine the necessary calculation, and they can solve problems. [Interest, Eagerness, and Attitude] Based on the original problem, students are trying to create more problems of their own.	• To decide on the appropriate calculation, double number lines are useful. • I think think of situations where we use ÷ (Decimal Number). • We are changing numbers to whole numbers when we do ÷ (Decimal Number) just as we did with multiplication. • The ways to calculate are similar with decimal numbers and with whole numbers. • We can use properties of operations.
5 Exercises	10	• Students will calculate the quotiens of various division of decimal numbers.	[Mathematical Skills] Students can calculate (Decimal Number) ÷ (Decimal Number) in a variety of situations.	
	11	• Students will find the quotient of division of decimal numbers. • Students will solve word problems. • Students will think about the relationship among the dividend, the divisor and the quotient. • Students will explain how to calculate division of decimal numbers.	[Mathematical Skills] Students can calculate (Decimal Number) ÷ (Decimal Number) in a variety of situations.	

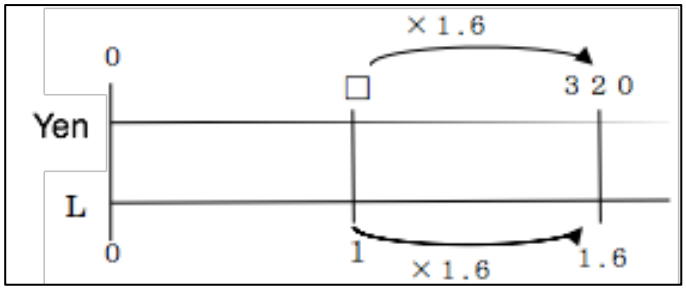
7 Today's Lesson (1/11)

(1) Goal

- Students will understand the meaning of (Whole Number) ÷ (Decimals) and be able to explain why division is appropriate calculation.

(2) Flow of the lesson

Time	Main Learning Task & Anticipated Responses	Evaluation [ ] Points of consideration (•) # Support
Understanding the Task	✪ Understand the task and have their own questions. We went to buy juice to share with everyone. We saw one package containing 1L of juice and costing 216 yen. Another package contains 2L of juice and costs 420 yen. C1: Which package is a better buy? T1: Which package would you buy? C2: 420 ÷ 2 = 210. So, the answer is 210 yen. C3: Since 1L will cost 210 yen, it is cheaper than 216 yen for 1L. T2: Why did you ÷ 2? C4: So that we can compare the price for 1 L. C5: Because one package had 2 L, I halved it to get 1L. C6: Because we are making 2 equal groups. C7: Because if we divided into 2, it will be 1 L. C7: I changed both to 1L. C9: Explain using double number line.  C10: □ × 2 = 420 □ = 420 ÷ 2 □ = 210 C11: I used the idea of per unit quantity and ÷ 2. Display a 1.6L package with the price of 320 yen. T3: What calculation do we do to find the price of 1L of juice if 1.6L is sold for 320 yen? C12: (1) 320 ÷ 1.6	• Make sure students can use words, mathematical expressions and diagrams to explain their ideas. • Have students think about the meaning of their expressions. • Write on the board terms that connect to the goals such as "half," "make 2 equal groups," "for 1L," and "split it into 2."

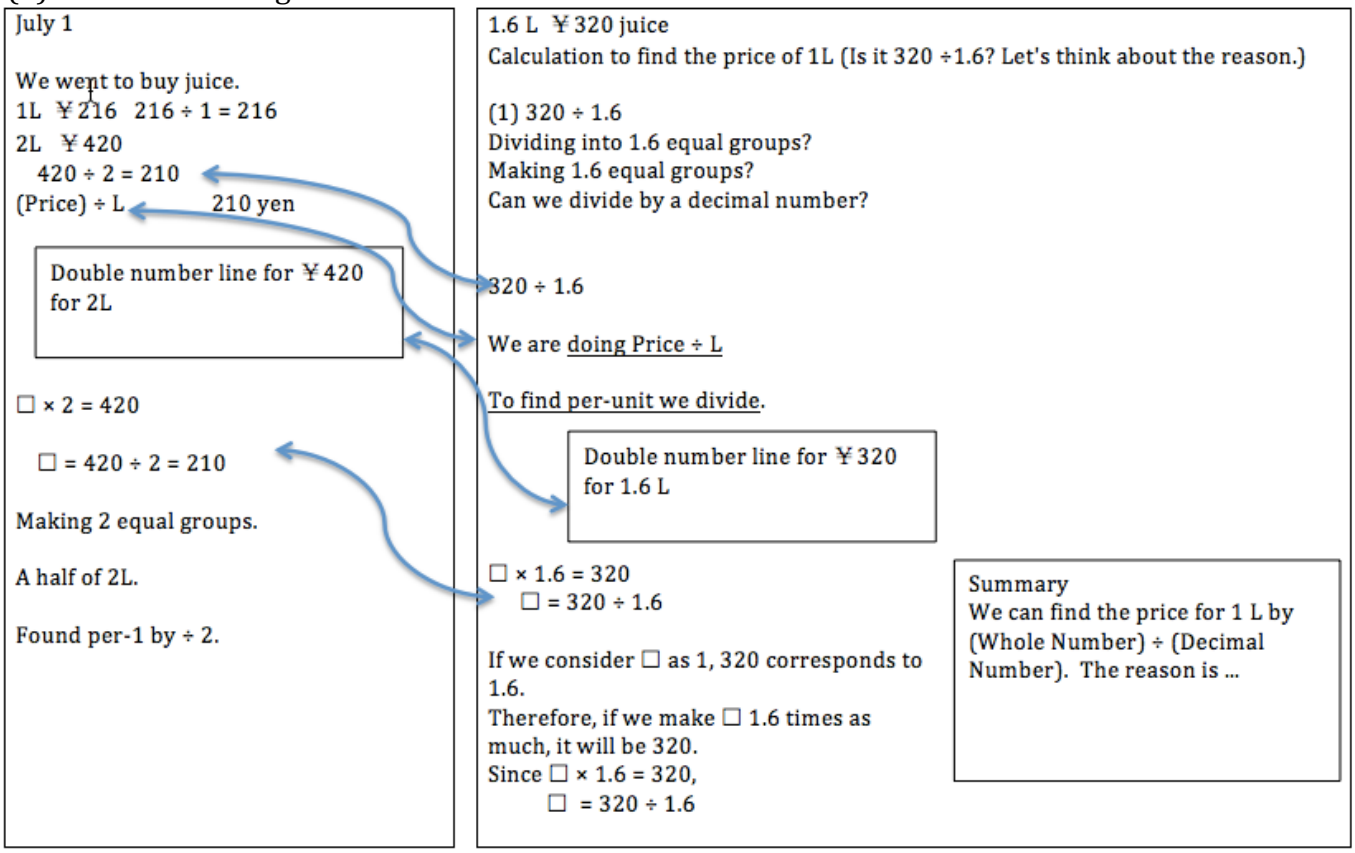
	C13: (2) Can we get the price for 1 L by doing 320 ÷ 1.6? C14: Is there such a thing as making 1.6 equal groups? C15: We can't make 1.6 equal groups. C16: Can we divide with decimal numbers? C 17: We haven't learned ÷ (Decimal Number) yet. Whole Number ÷ Decimal Number To find the price of 1L, can we use ~~320 ÷ 1.6~~? Let's think about the reason. T4: What is different from what we have learned so far? C18: The divisor is a decimal number. T5: 320 ÷ 1.6 is (Whole Number) ÷ (Decimal Number), isn't it?	• Use the language of partitive division to help students feel that we cannot use the same explanation we used with 2L. • Make sure students think about the meaning of expression.
Independent Problem Solving	✪ Write down their own ideas. C19: We are doing (price) ÷ L. C20: When we want to calculate the per-unit quantity, we use division. C21: Using double number line.  C22: □ × 1.6 = 320 □ = 320 ÷ 1.6 C23: If we consider □ as 1, 320 corresponds to 1.6. C24: 320 is 1.6 times of □.	[Mathematical Way of Thinking] Students can think about the meaning of (Whole Number) ÷ (Decimal Number) and explain why that is the correct calculation with reason. # Have students attend to words, mathematical expressions and diagrams used with whole number division. # Instruct those students who came up with their own idea to give hint to other students who may be struggling.

Whole class comparison and analysis	✪ Share own ideas. T6: Can you explain why 320 ÷ 1.6 will calculate the price for 1L? If you are not sure, let's share what you are unsure. C25: I understood when we were using whole numbers, but I got confused when we started working with decimal numbers. C26: I couldn't think about double number lines. T7: Can someone explain why the calculation should be 320 ÷ 1.6? C27: When we had whole numbers, we did (Price) ÷ L. For example, when the package had 2L, we did 420 ÷ 2. Even when the package contained 1L, we can do 216 ÷ 1. So, even though we have a dciemal number, to find the price for 1L will be 320 ÷ 1.6. C28: To find the per-unit quantity, we used division. So, even when we have a decimal number, I think we can use division to find the per-unit quantity. So, 320 ÷ 1.6. C29: I looked at the double number line diagram. When we had 2L, arrows indicated × 2. So, if we have 1.6L, they should be × 1.6. Since nothing else has changed, I thinkthis must be correct. C30: Because 320 is 1.6 times as much of the unknown quantity, ☐ × 1.6 = 320 So, ☐ = 320 ÷ 1.6.	• Have the students explain why they could not come up with their own ideas. Make sure other students understand their questions and puzzlement. • On the blackboard, draw arrows to highlight the common ideas between whole number and decimal number situations so that we can leave an explicit record of expansion of the meaning of division. • Highlight the words that could be useful in summary such as × 1.6, 1.6 times as much, and double number line.
Summary	✪ Think about the summary of the lesson. T8: Let's summarize today's lesson. The reason we can find the price for 1L even when we do (Whole Number) ÷ (Decimal Number) is ... • Because we drew double number line. • Because I wrote equations with words using both whole numbers and decimal numbers. • Because to find the per-unit quantity, we use division. • The way of reasoning was similar for both whole numbers and decimal numbers.	• Have students summarize using their own words.

(3) Evaluation of the Lesson as a Whole

- Students raise the question about the idea of making 1.6 equal groups and ÷ 1.6 based on the meaning of making 2 equal groups and ÷ 2, and they tackle the challenge of thinking about the meaning of (Whole Number) ÷ (Decimal Number).
- Students can solve problems using multiple tools such as words, expressions, diagrams, etc.
- Students can explain the reason for (Whole Number) ÷ (Decimal Number) using the analogical reasoning such as equations with words, as well as the deductive reasoning with double number line.
- By using double number line and ☐ in equation, students grasp division as the inverse operation of multiplication. They also understand the meaning of the mathematical expression.

(4) Board Writing Plan



8 Seating chart and their current state

How did they justify the choice of operation for (i), (ii), and (iii) [(i): Whole Number × Decimal Number; (ii) Decimal Number × Decimal Number; (iii) Decimal Number × Decimal Number < 1] (iv) Reflection on the unit (see bellow)

Boy1 (i) W, DNA (ii) DNB (iii) DNB, W (iv)	Girl1 (i) DNB (ii) DNB (iii) DNC (iv)	Boy2 (i) DNC (ii) O (iii) O (iv)	Girl2 (i) O (ii) DNB (iii) DNC (iv)	Girl3 (i) O (ii) DNB (iii) DNC (iv)	Boy3 (i) W, E1 (ii) DNB, W (iii) DNC (iv)
Girl4 (i) E1 (ii) DNB, E1 (iii)DNB (iv)	Girl5 (i) O (ii) W (iii) Abs. (iv)	Boy4 (i) W (ii) O (iii) DNB (iv)	Girl6 (i) O (ii) DNB (iii) DNC (iv)	Boy5 (i)DNB (ii) W (iii) E1 (iv)	Boy6 (i) O (ii) DNB (iii) DNA (iv)
Girl7 (i) O (ii) DNA (iii) DNB (iv)	Boy7 (i) E1 (ii) W (iii) Abs (iv)	Girl8 (i) DNB (ii) DBB (iii) O (iv)	Boy8 (i) E1 (ii) E1 (iii) DNB (iv)	Boy9 (i) O (ii) O (iii) DNC (iv)	Girl9 (i) E2, W (ii) DNB (iii) DNB (iv)
Girl10 (i)E2 (ii) DNB, W (iii) E1 (iv)	Girl11 (i) DNA (ii) DNB (iii) E2 (iv)	Boy10 (i) DNA (ii) DNB (iii) DNC (iv)	Girl12 (i) O (ii) DNB (iii) E2 (iv)	Boy11 (i) DNB (ii) DNB (iii) DNB (iv)	Girl13 (i) W (ii) DNB (iii) DNC (iv)
		Girl14 (i) O (ii) DNB (iii) DNA (iv)	Girl15 (i) O (ii) O (iii) DNC (iv)	Girl16 (i) E1 (ii) DNB (iii) DNC (iv)	Boy12 (i) E1 (ii) E2 (iii) DNA (iv)

E 1: Using equation to show the way to calculate.
E 2: Equation with words: (price for 1 meter) × (meter)
W: times as much
DNA: Double number line with 3 numbers and a □
DNB: Double number line with arrows, × △ or □ to set up an expression
DNC: Other double number line
O: Other - including incorrect answer or no answer

Reflections
Boy1 I understood that there are many different expressions and strategies.
Boy 2 (blank)
Boy 3 (blank)
Boy 4 I understand strategies for calculation well.
Boy 5 If I multiply by a number less than 1, the product will be smaller; we can calculate more quickly if we use strategies
Boy 6 I understood that if we multiply the multiplicand and/or multiplier by 10 then we have to ÷ 10.

Boy 7 I understood that calculation will be simpler if we use strategies.
Boy 8 I thought easy to understand is only I need to focus on, but I understood that the description in words and expressions need to match up.
Boy 9 I made mistakes of multiplying instead of adding. It was a good review.
Boy 10 I understood that we can calculate quickly if we use strategies.
Boy 11 I understood different strategies for calculation.
Boy 12 I understood that if we multiply by 10, we must ÷ 10.

Girl1 I understood we can use times as much to calculate and also strategies for calculation.
Girl2 When I heard my friends' questions, "That makes sense," but at the end I felt "This is it."
Girl3 I learned calculate with decimal numbers. I understood that we can use diagrams even for multiplication of decimal numbers.
Girl4 We can explain using words and diagrams. I thought today was both a review and extension of what we learned in Grade 4.
Girl 5 (blank)
Girl 6 I understood that we just don't bring down the decimal point when we multiply decimal numbers.
Girl7 I was glad to see different calculation strategies from mine.
Girl8 It's easier to understand when we use calculation strategies.
Girl9 It was fun because we shared different ideas and discussed about them.
Girl10 When it's difficulty to calculate, we can × 10 ÷ 10. If we change numbers to convenient numbers, it is simpler. Other people's explanation was easy to understand.
Girl11 It was difficult to understand how the decimal point was moved around.
Girl12 I understood that we can think like we did with whole numbers even when we have decimal numbers.
Girl13 It was good that the lesson moved along to deal with my friends' questions.
Girl14 It is good to use diagrams. Calculation became simpler by doing × 10 ÷ 10.
Girl15 I understood that we can elimiate the decimal point by multiplying by 10. I also learned we can make use of what we learned in Grade 4.
Girl16 (blank)