# Research Theme <br> Nurturing students who can identify and solve mathematical questions and express their ideas on their own <br> $\sim$ Through problem solving in mathematics ~ 

1 Name of the Unit: Let's think about division of decimal numbers (14 lessons)
2 Goals of the Unit
Students will understand the meaning and ways of calculating quotients when the divisor is decimal numbers.

3 Goals and standards for each assessment domain

| Domain | - Goals • Assessment standard |
| :---: | :---: |
| Interest, Eagerness, and Attitude | Students try to think about the meaning of division by decimal numbers by generalizing their understanding of division of whole numbers. The think about ways to calculate the quotients based on the properties of the base10 numeration system. |
|  | - Students try to connect ways to calculate division with decimal numbers with the ways of division with whole numbers. <br> - Students recognize the merit that Decimals $\div$ Decimals can be calculated in a similar way with division of whole numbers. |
| Mathematical Way of Thinking | Students can think about ways to calculate the quotients when the divisors are decimal numbers by using number lines and properties of operations, and they can explain their ideas concisely. |
|  | - Students think about meaning of calculating division of decimal numbers by making use of their prior knowledge of calculations and number lines. <br> - Students are thinking about ways to calculate division of decimal numbers. <br> - Students think about and represent Decimal $\div$ Decimal by making use of their knowledge of division properties and calculations with whole numbers. <br> - Students think about the size of the remainders. <br> - Students think about the ways to calculate the quotients when the quotients must be rounded. <br> - Students think about represent the way to set up the appropriate division calculation by making use of number lines. <br> - Students think about ways to determine how many times as much or the base number using decimal number division just as they did with whole numbers. |

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| Skills with Numbers, | - Students can calculate the quotients even when the divisors are decimal numbers. |
| :---: | :---: |
| Quantities, and Figures | - Students can calculate Decimal $\div$ Decimal using the standard division algorithm. <br> - Students can find the quotients to the specified number of decimal places. <br> - Students can divide by decimal numbers accurately. |
| Knowledge and | Students understand the meaning and ways to calculate the quotients when the divisors are decimal numbers. |
| Understanding about Numbers, Quantities, and Figures | - Students understand the size relationship between the dividend and the quotient based on the size of the divisor (greater or less than 1). <br> - Students understand the size of the remainder (where the decimal point must be placed). <br> - Students understand that we can use the idea of times as much to compare quantities. <br> - Students understand the meaning of division of decimal numbers. |

## 4 About the Unit <br> (1) With respect to the National Course of Study

The content of this unit is described in the National Course of Study as follows.
(3) Students will deepen their understanding of multiplication and division of decimal numbers, and be able to use them appropriately.
a. To understand the meaning of multiplying or dividing by decimal numbers based on the calculation ideas for multiplying and dividing decimal numbers by whole numbers.
b. To explore way to multiply and divide by decimal numbers, and be able to calculate accurately; to understand the size of numbers.
c. To understand that the same properties of multiplication and division for whole numbers will apply to decimal multiplication and division.

## [Mathematical Activities]

Investigate and explain the meaning and the ways of calculating with decimal numbers, using words, numbers, mathematical expressions, diagrams, and number lines.

In this unit, students will expand the meaning of division by studying the meaning of division when the divisors are decimal numbers. Students cannot explain division by decimal numbers using the "fair sharing" idea that they have been using. Therefore, by using number lines and equations with words, students will understand the meaning of division as "the calculation to find the amount per unit." They will also understand ways to carry out the calculation and develop their ability to apply their learning. They will also investigate times as much relationships with decimal numbers as the foundations for their future study of ratios and rates.
(2) Scope and Sequence


5 About the students
(1) Current state of students in mathematics

Students generally approach everything with eagerness. Many are mildmannered but willing to share their ideas during lessons. However, even though they can share their ideas orally, some students find it difficult to express their ideas in writing. Some students often do not know what to write in their own notebooks. During independent problem solving time, they often require hints from the classroom teacher.

In addition, there is a wide range of mathematical achievement levels within the class. Therefore, instead of teaching mathematics with the whole-class instruction, we have been splitting the class into 2 smaller groups. To create the 2 groups, we administer a readiness test before a unit. The homeroom teacher and the partner teacher who has been assigned specifically to support small group instruction analyze the results of the test and create the 2 groups. In the area of numbers and calculations, the achievement levels vary significantly, and some students still struggle with whole number division. In this unit, we have created 2 groups, "gungun course" and "jikkuri course" considering the differences in their prior achievements.

## <About the 2 Courses>

Gungun course --- According to the readiness test results, these students have mastered the prior contents fairly well. The instruction will proceed with the suggested pacing by the textbook.
Jikkuri course --- According to the readiness test results, these students have yet to mater the prior contents at the satisfactory level. More concrete and semi-concrete materials will be incorporated in instruction so that students can engage in more hands-on activities.
(2) Results and analysis of survey on students' attitudes toward mathematics Administered to all 27 students on May 12

| (a) Students placed in gungun course (16 students) |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Definitely <br> agree | Somewhat <br> agree | Somewhat <br> disagree | Definitely <br> disagree |
| I like studying mathematics. | $12(75 \%)$ | $3(19 \%)$ | $0(0 \%)$ | $1(6 \%)$ |
| Studying mathematics is important. | $15(94 \%)$ | $0(0 \%)$ | $1(6 \%)$ | $0(0 \%)$ |
| I understand mathematics lessons well. | $12(75 \%)$ | $3(19 \%)$ | $1(6 \%)$ | $0(0 \%)$ |
| When I see a novel problem in a math lesson, it <br> makes me want to solve it. | $11(69 \%)$ | $4(25 \%)$ | $0(0 \%)$ | $1(6 \%)$ |
| When I can't figure out a problem in a math <br> lesson, I keep trying instead of giving up. | $9(56 \%)$ | $4(25 \%)$ | $3(19 \%)$ | $0(0 \%)$ |
| I think about ways to use what I learn in math <br> lessons in my daily life. | $4(25 \%)$ | $10(63 \%)$ | $1(6 \%)$ | $1(6 \%)$ |
| I think what I have learned in math lessons has <br> been useful in my daily life. | $10(63 \%)$ | $5(31 \%)$ | $1(6 \%)$ | $0(0 \%)$ |
| When I am solving a math problem, I think <br> about easier and simpler ways of solving it. | $10(63 \%)$ | $4(25 \%)$ | $2(12 \%)$ | $0(0 \%)$ |
| When we learn a formula or a rule in math. <br> lessons, I try to understand why it works. | $7(43 \%)$ | $6(38 \%)$ | $3(19 \%)$ | $0(0 \%)$ |
| I try to write ways to solve or think about a <br> problem in my notebook. | $9(56 \%)$ | $6(38 \%)$ | $1(6 \%)$ | $0(0 \%)$ |


| (b) Students placed in jikkuri course (11 students) |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Definitely <br> agree | Somewhat <br> agree | Somewhat <br> disagree | Definitely <br> disagree |
| I like studying mathematics. | $3(27 \%)$ | $5(46 \%)$ | $1(9 \%)$ | $2(18 \%)$ |
| Studying mathematics is important. | $8(73 \%)$ | $3(27 \%)$ | $0(0 \%)$ | $0(0 \%)$ |
| I understand mathematics lessons well. | $3(27 \%)$ | $7(64 \%)$ | $1(9 \%)$ | $0(0 \%)$ |
| When I see a novel problem in a math lesson, it <br> makes me want to solve it. | $6(55 \%)$ | $3(27 \%)$ | $2(18 \%)$ | $0(0 \%)$ |
| When I can't figure out a problem in a math <br> lesson, I keep trying instead of giving up. | $6(55 \%)$ | $3(27 \%)$ | $2(18 \%)$ | $0(0 \%)$ |
| I think about ways to use what I learn in math <br> lessons in my daily life. | $4(36 \%)$ | $5(46 \%)$ | $1(9 \%)$ | $1(9 \%)$ |
| I think what I have learned in math lessons has <br> been useful in my daily life. | $7(64 \%)$ | $4(36 \%)$ | $0(0 \%)$ | $0(0 \%)$ |
| When I am solving a math problem, I think <br> about easier and simpler ways of solving it. | $5(45.5 \%)$ | $5(45.5 \%)$ | $1(9 \%)$ | $0(0 \%)$ |
| When we learn a formula or a rule in math <br> lessons, I try to understand why it works. | $6(55 \%)$ | $3(27 \%)$ | $1(9 \%)$ | $1(9 \%)$ |
| I try to write ways to solve or think about a <br> problem in my notebook. | $6(55 \%)$ | $3(27 \%)$ | $2(18 \%)$ | $0(0 \%)$ |

Students in both groups generally agree that "studying mathematics is important" regardless of whether or not they like mathematics. However, there are obvious differences in the response patterns in the 2 groups. In gungun course, about 70 $\%$ of students strongly agree that "when I see a novel problem in a math lesson, it makes me want to solve it," but the students in jikkuri course are not so enthusiastic about such problems. This seems to suggest that they do not like novel problems. Relatively small portion of students in both courses definitely agreed that "I think about ways to use what I learn in math lessons in my daily life." This makes us wonder if students do not know when they can apply what they have learned in their daily lives. We need to intentionally discuss situations where what students are learning in mathematics lessons are applicable in the future.
(3) Results and analysis of the readiness test

Administered to all 27 students on May 24

| (a) Students placed in gungun course (16 students) |  |  |  |
| :---: | :---: | :---: | :---: |
| Problem | Purpose | Correct answer (\# of Ss) Percent | Incorrect answer (\# of Ss) |
| $\begin{aligned} & \text { (1) } 360 \div 30=36 \div \square \\ & \text { (2) } 680 \div 40=\square \div 4 \end{aligned}$ | Can students apply the property of division to calculate when both the dividend and the divisor end with a 0 ? | $\left.\begin{array}{ll} (1) & 3 \\ (2) & (14) \\ (2) & 88 \% \\ \hline \end{array} 15\right) 94 \%$ | $\begin{array}{\|ll} \hline \text { (1) } & 300(1) \\ & 10(1) \\ \text { (2) } & 6800(1) \end{array}$ |
| Calculate using the algorithm: <br> (3) $9.6 \div 4$ <br> (4) $47.2 \div 8$ <br> (5) $9.72 \div 27$ <br> (6) $3.36 \div 48$ <br> (7) $55.8 \div 124$ | Can students calculate Decimal $\div$ Whole? | (3) $2.4(16) 100 \%$ <br> $(4)$ $5.9(16)$ <br> (5) $0.36(15)$ <br> (6) $0.07(15)$ <br> (7) $0.45(11)$ <br>  $69 \%$ | $\begin{array}{ll} \hline(5) & 36(1) \\ (6) & 7(1) \\ (7) & 00.45(2) \\ & 0.4(1) \\ & 4 \text { rem. 6.2 (1) } \\ & 0.441(1) \end{array}$ |
| Calculate using the algorithm: <br> (8) $87.6 \div 16$ | Can students calculate Decimal $\div$ Whole with a remainder? | (8) 5 rem. 7.6 (7) $44 \%$ | (8) 5.4 rem. $1.2(2)$ <br> 5.4 rem. 12 (1) <br> 54 rem. 12 (1) <br> 6 rem 1.6 (1) <br> 999 rem. 444 (1) <br> 5.475 (1) <br> 5 rem. 0.76 (1) |
| Word problem <br> (9) There is 7.2 L of soy sauce. If we put this soy sauce into 9 bottles equally, how many L of soy sauce will there be in each bottle? | Can students solve a word problem involving division with a decimal dividend (and whole number divisor)? | (9) Expression $7.2 \div 9$ <br> (16) $100 \%$ <br> Answer 0.8 L <br> (16) $100 \%$ |  |
| Word Problem <br> (10) The length of a red tape is 4 m and the length of a blue tape is 6 m . How many times as long is the blue tape as the red tape? | Can students solve problem involving times as much with decimal number? | (10) <br> Expression $6 \div 4$ <br> (15) $94 \%$ <br> Answer 1.5 times as long <br> (14) $88 \%$ | (10) <br> Expression <br> No answer (1) <br> Answer <br> Incorrect unit (1) <br> No unit (1) |

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| (b) Students placed in jikkuri course (11 students) |  |  |  |
| :---: | :---: | :---: | :---: |
| Problem | Purpose | Correct answer (\# of Ss) Percent | Incorrect answer (\# of Ss) |
| (1) $360 \div 30=36 \div \square$ <br> (2) $680 \div 40=$ $\square$ | Can students apply the property of division to calculate when both the dividend and the divisor end with a 0 ? | (1) 3 (6) $55 \%$ <br> (2) 68 (4) $36 \%$ | (1) No answer (2) 64 (1) 6 (1) 300 (1) <br> (2) No answer (2) 88 <br> (1) 5 (1) 17 (1) 8 (1) <br> 6800 (1) |
| Calculate using the algorithm: <br> (3) $9.6 \div 4$ <br> (4) $47.2 \div 8$ <br> (5) $9.72 \div 27$ <br> (6) $3.36 \div 48$ <br> (7) $55.8 \div 124$ | Can students calculate Decimal : Whole? | (3) 2.4 (7) $64 \%$ <br> (4) 5.9 (6) $55 \%$ <br> (5) 0.36 (3) $27 \%$ <br> (6) 0.07 (4) $36 \%$ <br> (7) 0.45 (1) $9 \%$ | (3) 24 (3) 0.2 (1) <br> (4) No answer (2) 21.4 (1) 59 (1) <br> (5) No answer (2) 36 <br> (2) 37 (1) 4.7 <br> (1) <br> 3.6 (1) 1.1 (1) <br> (6) No answer (3) <br> 7 (3) 0.01 (1) <br> (7) No answer (4) <br> 45 (2) 2.25 (1) <br> 135 rem. 30 (1) <br> 11.2 (1) <br> 4 rem. 62 (1) |
| Calculate using the algorithm: <br> (8) $87.6 \div 16$ | Can students calculate Decimal $\div$ Whole with a remainder? | (8) 5 rem. 7.6 <br> (2) $18 \%$ | (8) No answer (5) 54 rem. 12 (2) 11.1 (1) <br> 0.5 rem .7 .6 |
| Word problem <br> (9) There is 7.2 L of soy sauce. If we put this soy sauce into 9 bottles equally, how many L of soy sauce will there be in each bottle? | Can students solve a word problem involving division with a decimal dividend (and whole number divisor)? | (9) <br> Expression $7.2 \div 9$ <br> (7) $64 \%$ <br> Answer 0.8 L <br> (5) $31 \%$ | (9) Expression $\quad$ No answer (2) $9 \div 7.2(1)$ $7.2 \times 9(1)$ Answer $\quad$ No answer (3) $\quad 8 \mathrm{~L}(1)$ $1.2 \mathrm{~L}(1)$ $9.5(1)$ $64.8(1)$ |
| Word Problem <br> (10) The length of a red tape is 4 m and the length of a blue tape is 6 m . How many times as long is the blue tape as the red tape? | Can students solve problem involving times as much with decimal number? | (10) <br> Expression $6 \div 4$ <br> (6) $55 \%$ <br> Answer 1.5 times as long <br> (3) $27 \%$ | (10) <br> Expression <br> No answer (3) $4 \div 6(2)$ <br> Answer <br> No answer (3) <br> 6 times (1) <br> 0.96 (1) <br> 1 rem. 2 times (1) <br> 15 times (1) <br> 1.2 times (1) |

Students in gungun course appeared to have mastered the basic calculations. However, with the division with remainder (Problem 8) some students missed the problem because they did not see that the question was asking for a "whole number quotient." There are other students who are still having trouble determining the location of the decimal point in the remainder.

Students in jikkuri course are still in the process of mastering the basic calculation. Their success rate is better if there is no empty space in the quotient, but some students even quit trying when there is an empty space in the quotient or there is a remainder. These results suggest the need for different types of support are needed by the students in these two groups.

## 6 <br> Strategies to achieve the research theme

Characteristics of ideal students (Grade 5)

- Students can notice the difference between the given problem and what they have previously learned and identify the mathematical question for the lesson.
- Students can devise a plan to solve the given problem using their prior learning.
- Students can explain their own ideas using tools such as diagrams and equations with words.
<Strategies in this lesson>
[Strategies to help students identify the mathematical question]
○ Help students develop the habit of recording "?" that arises naturally when they encounter a novel problem in everyday lessons

When students encounter a novel problem, they naturally mumble to themselves in their minds, "What calculation do I need to do?" "It's going to be difficult with decimal numbers," etc. Those silent mumbles are the natural questions arising from the problem and they lead to the mathematical question for the lesson. Thus, we want students to develop the habit of recording those quiet mumbling in their own notebook. By doing so, students will become more aware of their own questions.
[Strategies to help students make use of their prior learning to solve a problem]
© Have hint cards that are available at any time in the classroom.
Place hint cards that can be used by students in any grade level in the classroom. We will instruct students to think about "What's the hint card we need today?" and select the appropriate one (or ones) on their own. Of course, in such a set of hint cards, some might involve the ideas students have yet to learn. However, we also want students to develop the habit and the ability to reflect on what they have previously learned. Because we started making these hint cards this year, the set is not yet complete, and we plan to keep adding to the set.

|  | Goals | Learning Activities |  | Main Assessment Standards |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Gungun course | Jikkuri course |  |
| (1) Division of decimal numbers |  |  |  |  |
| $\begin{gathered} \underset{\pi}{0} \\ \underset{\sim}{0} \end{gathered}$ | - Students can set up the appropriate division expression with understanding. <br> - Students understand the meaning of dividing by decimal numbers. | - Think about how much a 1 m of ribbon costs if a 2.5 m of the same ribbon costs 300 yen. <br> - Set up an expression and think about why the expression is appropriate by using tools such as number lines and equations with words. | - Think about how much a 1 m of ribbon costs if a 3 m of the same ribbon costs 300 yen. (Re-visiting prior learning) <br> - Think about how much a 1 m of ribbon costs if a 2.5 m of the same ribbon costs 300 yen. <br> - Set up an expression using tools such as number lines and equations with words. | [Mathematical Way of Thinking] Students are thinking about the meaning of division by decimal numbers by connecting to their prior learning of calculation and number line representations. |
| L2 | - Students can think about ways to calculate Whole $\div$ Decimals using their prior learning of properties of division and number line representations. | - Think about ways to calculate $300 \div 2.5$. <br> - Summarize the method to divide by decimal numbers. | - Think about ways to calculate $300 \div 2.5$ by making connections to students' prior learning. <br> - Students learn the method to divide by decimal numbers. | [Interest, Eagerness, and Attitude] <br> Students are trying to think about ways to divide by decimal numbers by making connections to their prior learning such as calculation with whole numbers. <br> [Mathematical Way of Thinking] Students are thinking about ways to divide by decimal numbers. |



| L6 |  | Students understand the meaning of remainders in division of decimal numbers, and they can determine remainders. | - Students will set up the calculation expression to solve the problem: How many students will get a 0.7 m piece of ribbon from a 2.5 $m$ ribbon? They can explain their methods for solving this problem. <br> - Summarize how to determine the location of decimal point in the remainder. | - Students will think about how many students will get a 0.7 m piece of ribbon from a 2.5 m ribbon using concrete materials. <br> - Summarize how to determine the location of decimal point in the remainder. | [Mathematical Way of Thinking] <br> Students are thinking about the size of remainders. <br> [Knowledge and Understanding] Students understand where to place the decimal point in remainders using the standard algorithm. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L7 |  | Students understand ways to find the quotients using approximate numbers. | - Think about the weight of 1 L of sands when 1.5 L of the same sands weight 2.5 kg. <br> - Students will realize the usefulness of using approximate numbers as quotients instead of responding with quotients and remainders. <br> - Summarize that when performing division, we sometimes use approximate numbers as quotients, for example, when there are remainders or the quotients will have many places. | - Think about the weight of 1 L of sands when 1.5 L of the same sands weight 2.5 kg. <br> - Students will realize the usefulness of approximating the quotients to the $2^{\text {nd }}$ highest place instead of responding with quotients and remainders. <br> - Summarize that when performing division, we sometimes use approximate numbers as quotients, for example, when there are remainders or the quotients will have many places. | [Mathematical Way of Thinking] <br> Students are thinking about the way to use rounding to approximate the quotients. <br> [Mathematical Skills] Students can respond with the quotients rounded to the specified place. |


| L8 | - Students will deepen their understanding of how number lines can help them determine appropriate division calculations. | - Students will think about ways to set up appropriate calculation to determine the weight of 1 m of hose and the length of 1 kg of hose when 4.5 m of the same hose weigh 0.9 kg by using number lines. | - Students learn that number lines can summarize the relationship of numbers in problem situations concisely so that setting up appropriate calculation becomes easy by determining the weight of 1 m of hose and the length of 1 kg of hose when 4.5 m of the same hose weigh 0.9 kg by using number lines. | [Mathematical Way of Thinking] <br> Students use number lines to think about and represent appropriate division expressions for problem situations. |
| :---: | :---: | :---: | :---: | :---: |
| (2) Times as much with decimal numbers and division |  |  |  |  |
| L9 | - Students understand that even when both the compared quantity and the base quantity are decimal numbers, division is still the appropriate calculation to find how many times as much. | - Think about ways to determine how many times as long are 3.6 km and 1.8 km as $2.4 \mathrm{~km}(2.4 \times \square)$. <br> - Summarize that even when both the compared quantity and the base quantity are decimal numbers, division is still the appropriate calculation to find how many times as much. | - Think about ways to determine how many times as long are 3.6 km and 1.8 km as $2.4 \mathrm{~km}(2.4 \times \square)$. <br> - Summarize that even when both the compared quantity and the base quantity are decimal numbers, division is still the appropriate calculation to find how many times as much. | [Mathematical Way of Thinking] Students are thinking that the way to determine how many times as much even when both the compared and base quantities are decimal numbers is by division as was the case when those quantities were whole numbers. |
| L10 | - Students understand that even when the compared quantities are decimal numbers times as much, they can use $\square$ for the base quantity and set up multiplication equations, and division is the operation to find the value of $\square$. | - Think about ways to find the base quantity when 630 g is 1.8 times as much as the base quantity. <br> - Summarize that expressing the relationship in a multiplication equation using $\square$ for the unknown base quantity is helpful. | - Think about ways to find the base quantity when 630 g is 1.8 times as much as the base quantity. <br> - Summarize that expressing the relationship in a multiplication equation using $\square$ for the unknown base quantity is helpful. | [Mathematical Way of Thinking] <br> Students are thinking that the way to determine the base quantity even when both the compared quantity and how many times as much are decimal numbers is by division as was the case when those numbers were whole numbers. |


| L11 | - Students understand that two quantities can be compared using the difference or times as much. | - Compare how much prices increased by using times as much. | - Compare how much prices increased by using times as much. | [Knowledge and Understanding] Students understand that times as much is an appropriate way to compare quantities based on the purpose of comparison. |
| :---: | :---: | :---: | :---: | :---: |
| (3) Summary |  |  |  |  |
| L12 | - Master the content of the unit. | - Engage in practice problems. (Basic Problems) | - Engage in practice problems. (Basic Problems) | [Mathematical Skills] <br> Students can calculate division of decimal numbers accurately. |
| $\begin{array}{\|l\|l\|} \text { L13 } \\ \text { L14 } \end{array}$ | - Solidify the understanding of the content of the unit. | - Engage in practice problems. <br> (Advanced Problems) | - Engage in practice problems. <br> (Advanced Problems) | [Knowledge and Understanding] Students understand the meaning of division of decimal numbers. |

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## 8 Today's Lesson (Lesson 1 of 14) Gungun course <br> (1) Goal

- Students can think about the meaning of dividing by decimal numbers by making connections to their prior learning of calculation and number lines, and they can explain their ideas logically.
(2) Flow of the Lesson

|  | Main questions and anticipated responses | $\square$ Assessment Support <br> - Point of consideration <br> $\bigcirc$ Teaching strategy |
| :---: | :---: | :---: |
| G R A S P | 1 Encountering the problem <br> A 2.5 m of ribbon costs 300 yen. <br> How much will 1 m of the same ribbon cost? <br> T: After you read the problem, please write today's "?" <br> T: What were your "?"? <br> C1: What calculation do I need? <br> C2: Will it be easier if we change the decimal number to a whole number? <br> C3: Will it be division? <br> C4: Should it be less than 150 yen? <br> C5: Is it possible to represent with a diagram? <br> T: Do you think you can solve this problem if we change the decimal number into a whole number? <br> C6: For example, if we change 2.5 m to 3 m , it will be simple. <br> T: Indeed, if you change the decimal number into a whole number, we can solve the problem. So, what will be the calculation expression to find the answer for today's problem? <br> C7: I think it is $300 \div 2.5$. <br> T : Is " $\div 2.5$ " the appropriate calculation to find the price for 1 m ? | $\bigcirc$ When students encounter a problem, have them write down their "?" in their notebooks. <br> - By changing the decimal number into a whole number, students can use their prior learning to set up the appropriate calculation expression. <br> - Pose a question to help students develop a new question of their own. |


|  | 2 Grasp the mathematical question <br> T：I think we are beginning to see the mathematical question for today＇s lesson． <br> C8：I want to think about the reason why $300 \div 2.5$ is the appropriate calculation． <br> C9：I want to explain why dividing by 2.5 will determine the price for 1 m ． <br> T：（Listen to students＇ideas，and write down the mathematical question for the lesson on the blackboard．） <br> Is $300 \div 2.5$ the appropriate calculation to find the price for 1 m ？ | －Treat this discussion carefully so that today＇s mathematical question will arise from students＇ questions． |
| :---: | :---: | :---: |
| D E V I I S E | 3 Devise a plan <br> T：Are there things you have learned so far that you might be able to use to explain the reason for the appropriate calculation？ <br> C10：I think we can use an equation with words． （Making connections to calculations with whole numbers） <br> C11：I think we can use the idea of multiplication． <br> （Thinking about the inverse of multiplication） <br> C12：I think we can use 0.1 as a unit． <br> （Making use of per－unit quantity） <br> C13：I think we can use diagrams． <br> （Thinking about ways to represent the situation） <br> C14：I tink we can use number lines． <br> （Thinking about ways to represent the situation） | －Help students devise a plan that they might be able to follow through． |
| S O L V E \＆ D I S C U S S | 4 Solve the question and represent <br> T：Please write down your idea in your notebook． （Independent problem solving time） <br> ＜Anticipated students＇responses＞ <br> （A）Students who can explain the reason for the calculation <br> i．Making use of equations with words With whole numbers，we get $300 \div 3=100$ <br> Because <br> ［Price $] \div[$ Length $]=[$ Price for 1 m$]$ ， we can determine the price for 1 m by using division even when the length is a decimal number． |  |

ii. Thinking in terms of the reverse operation of multiplication
Since we don't know the price of 1 m , we will say it is $\square y e n$. Since the price of $2.5 \mathrm{~m}, 300$ yen, will be 2.5 times as much as the price for 1 m , we can represent this relationship in the equation, $\square \times 2.5=$ 300.

Therefore, the calculation to determine the value of $\square$ must be $300 \div 2.5$, and we know that $" \div 2.5$ " is the correct calculation to find the price for 1 m .
iii. From number lines, thinking about the unit quantity.

(B) Students who are thinking more about finding the answer
iv. Think in terms of 0.1 as a unit
2.5 m is made up of 250.1 m .

If you divide 300 yen by 25 , we get 12 . That tells us that the price of 0.1 m of this ribbon is 12 yen. Therefore, the price of 1 m of this ribbon is 120 yen.
v. Think in terms of 0.5 as a unit
2.5 m is made up of 50.5 m .

If you divide 300 yen by 5 , we get 60 . That tells us that the price of 0.5 m of this ribbon is 60 yen. Therefore, the price of 1 m of this ribbon is 120 yen.
(Whole class discussion)
C15: (iv)'s idea is how to do the calculation, and I don't think it explains why the correct calculation is division.
C16: I can see why the calculation must be division clearly from the explanations if (i), (ii), and (iii).

T: So, do you think " $300 \div 2.5$ " is the appropriate calculation to solve today's problem?

C17: I understood why division is the calculation we need for today's problem by listing to $\qquad$ 's explanation.
[Mathematical Way of Thinking]
Students think about and explain the meaning of division by decimal numbers by making use of their prior knowledge of calculations and number lines.
(Presentations/Notebooks)
© Have hint cards ready in the room and allow students to use them freely.

- Teacher will suggest an approach if students were having difficulty selecting a strategy on their own.
- Let students who finished writing their ideas to discuss their ideas with each other.
- For (iv) and (v), if any student uses these approaches, select one and have it shared with the whole class.
- Reflect on today's mathematical question and make sure students understood the main idea.
- As a class, affirm that division is the appropriate calculation to solve today's problem.



## Board Writing Plan



## Seating Chart

Blackboard

|  |  |  |  | 3 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |
| 10 | 9 | 8 | 7 | 6 | 5 |
| 16 | 15 | 14 | 13 | 12 | 11 |

