

# Grade 9 Mathematics Lesson Plan

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Period # 6

Kiyose Lower Secondary School No. 4

Grade 9 Classrooms A & B

(Standard course, 24 students)

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## I. About the Unit

1. Name of the Unit: Square Roots [Grade 9 textbook by Tokyo Shoseki, Ch. 2]

### 2. Unit Plan

Sub-Unit	Section	Goals	#
1. Square Roots	① Square Roots	<ul style="list-style-type: none"> <li>Students understand the meaning of square roots, and they can find the square roots of given numbers.</li> <li>Students can represent the size relationships of square roots using inequality symbols.</li> <li>Students understand the meaning of rational numbers and irrational numbers, and they can sort the numbers they have learned into the appropriate types.</li> </ul>	4
	② Prime Factorization	<ul style="list-style-type: none"> <li>Students understand the meaning of prime factorization, and they can factorize the given natural numbers. In addition, students can determine the square roots of natural numbers by using prime factorization.</li> </ul>	1
	• Basic Problems		1
2. Calculations of expressions with square roots	① Mult. & division of expressions with square roots	<ul style="list-style-type: none"> <li>Students can calculate expressions involving multiplication and division of square roots.</li> <li>Students can manipulate expressions with square roots. They can also transform the given expression to an equivalent expression to determine the approximate value of the expression.</li> <li>Students can rationalize the denominator.</li> <li>Students can devise ways to calculate expressions with square roots.</li> </ul>	4
	② Addition and subtraction of expressions with square roots	<ul style="list-style-type: none"> <li>Students can simplify expressions with square roots by combining like square roots.</li> <li>Students can calculate expressions with unlike square roots by transforming square roots.</li> </ul>	2
	③ Calculations of various expressions with square roots	<ul style="list-style-type: none"> <li>Students can calculate and determine the value of expressions with square roots by making use of the distributive property and the multiplication formulae.</li> </ul>	1
	④ Application of square roots	<ul style="list-style-type: none"> <li>Students can identify situations involving square roots in their surroundings.</li> </ul>	1
	• Basic Problems		1
Chapter Problems			2

### 3. Goals of instruction

The main goal of this unit is to expand the numbers to irrational numbers as we expanded the numbers to negative numbers in Grade 7. Students will learn that we can perform the arithmetic operations with irrational numbers as well as properties of operations such as commutativity, associativity and the distributive property remain true with irrational numbers. We want students to truly understand how the world of numbers are expanding; therefore, instead of simply telling students how to perform the arithmetic operations, we want students to investigate and discover ways to calculate with irrational numbers while imagining the actual quantities irrational numbers represent.

In the introduction of the unit, students tackled the problem of finding the dimensions of the square with the area of  $50 \text{ cm}^2$ . In that lesson, students actually folded a  $100 \text{ cm}^2$  square paper to create a square with the area of  $50 \text{ cm}^2$  so that they can have a concrete image of that length. Students can also experience that the number we are considering is not a part of the numbers they have learned previously by actually trying to find the number that equals 50 when it is squared using a calculator. In addition, when comparing the sizes of square roots, they tried to estimate their sizes by making use of special cases such as square roots of 4 and 9. They can also compare the lengths of sides of squares of area 2 and 5 to compare  $\sqrt{2}$  and  $\sqrt{5}$ . We believe these experiences help students as they think about ways to calculate.

As they continue their study of square roots, some students may wonder why irrational numbers can be represented by fractions. In this unit, we will touch upon the fact that an irrational number cannot be expressed as a fraction,  $\frac{a}{b}$ , as well as how repeating decimals can be converted to fractions. However, the purpose of the discussion is not to develop the expertise in proof by contradiction or converting repeating decimals into fractions. A more in-depth treatment of those topics will be done in the 10<sup>th</sup> grade mathematics, and the discussion in this unit is simply to serve as a bridge to the Upper Secondary School mathematics.

### 4. Assessment Standards

<b>Interest, Eagerness, and Attitude Toward Mathematics</b>
<ul style="list-style-type: none"> <li>Students will be interested in thinking mathematically by using square roots to make sense of various phenomena and examine relationships among them. Students are eagerly seeking ways to make use of square roots to solve mathematical problems.</li> </ul>
<b>Mathematical Way of Thinking</b>
<ul style="list-style-type: none"> <li>Students will master mathematical ways of observing and thinking such as applying the basic knowledge and skills about square roots of numbers to think logically and represent mathematically.</li> </ul>
<b>Mathematical Skills</b>
<ul style="list-style-type: none"> <li>Students calculate expressions including square roots of numbers. They also master the skills to represent and process situations using expressions involving square roots of numbers.</li> </ul>
<b>Knowledge and Understanding of Numbers, Quantities, and Geometric Figures</b>
<ul style="list-style-type: none"> <li>Students understand the meaning and the need for square roots of numbers.</li> </ul>

## II. Instruction of the Unit

### 1. Topic

Chapter 2, Square Roots, Sub-Unit, 2. Calculations of expressions with square roots, 4, Applications of Square Roots

### 2. Goal

Students can identify square roots of numbers in their surroundings.

### 3. About the students

For the mathematics instruction at our school, we have been splitting students into 2 groups (Basic course and Standard course) based on their mastery of prior materials. The research lesson is with the students in the Standard course, and many students are willing to participate in mathematics lessons actively and they have, in general, mastered the basic skills and procedures. They do not hesitate to share their ideas during the class, and they eagerly tackle problems that require students to think differently or explain their reasoning.

### 4 Key points of the lesson

In this lesson, students are introduced to the fact that irrational numbers are being used in many situations in our everyday life. In the recent lessons, we have focused on calculations with irrational numbers, and students may have developed the image that irrational numbers were created to keep the calculations consistent. Thus, we want students to realize that irrational numbers are being used in various places in our surroundings.

Students will be shown several books, and we will try to sort them based on their shapes. By doing so, we want students to pay attention to the ratios of the short and the long sides of the books. Then, each student will receive a piece of B-5 paper which is used to make the mathematics textbook. When they are asked to compare the lengths of the short and the long sides, many will fold the paper and notice that the long side is about 1.5 times as long as the short side. After students unfold the paper, they are asked to pay attention to the crease line, and some may notice that the length of the crease line is equal to the length of the long side of the paper. Students will then be asked to determine the length of the long side when the length of the short side is considered as 1, but this might be too difficult for some. Thus, we will have students work in small groups, and through the group investigation, we hope that students will realize that the length of the long side is the side of a square with the area of 2 square units (using the short side of the paper as the unit length), or  $\sqrt{2}$  units.

Moreover, when students compare the ratio of sides of the rectangle obtained by folding the B5 paper in half, they notice that the ratio is  $1:\sqrt{2}$ . Thus, the shape of the paper remains unchanged. Students may be impressed by the wisdom of people who devised these size of papers.

## 5. Assessment Standards

[Interest] Students will be interested in comparing the ratio of the lengths of the short and the long sides of the standard paper, and they are trying to apply what they have learned about square roots.

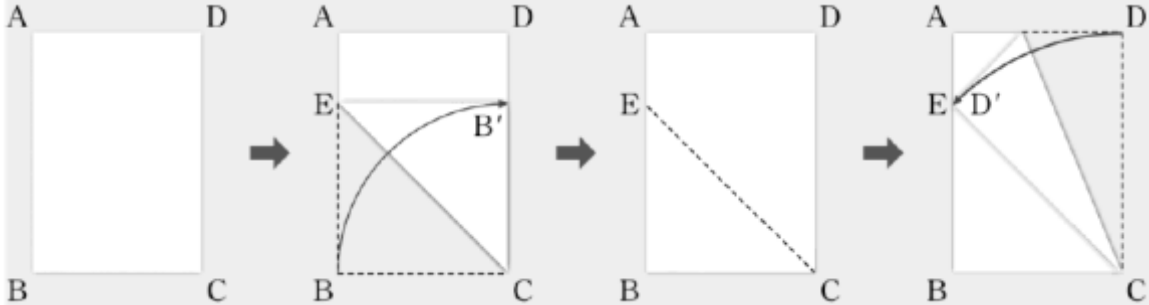
[Way of Thinking] Students can explain what the length of the long side is  $\sqrt{2}$  if the short side is considered as 1 by using diagrams.

[Skills] Students can determine the ratio of the lengths of the short and the long sides of the rectangle obtained by folding a B5 paper in half is  $1:\sqrt{2}$ .

[Knowledge] Students understand the characteristics of the standard sizes of papers.

## 6. Flow of the Lesson

Teacher Moves	Learning Activity & Anticipated Responses	Instructional Points of Consideration
<p>(Show several books of different sizes and shapes to the students.)</p> <ul style="list-style-type: none"> <li>Let's sort these books into groups.</li> <li>What do you mean by the shape of rectangles being the same?</li> <li>Let's compare the ratio of the lengths of the short and the long side of the paper that is the same size as our mathematics textbook.</li> </ul>	<ul style="list-style-type: none"> <li>Although the books were all rectangular in shapes, there are many different shapes. Students will realize, however, that some books are the same shape as the shape of their mathematics textbook.</li> </ul> <p>S: What do we need to focus on to sort them?</p> <p>S1: Their angles are congruent. S2: Their short and long sides are equal. S3: The lengths of their short and long sides are in the same proportion. S4: The ratios of the lengths of the short and the long side are equal.</p>	<ul style="list-style-type: none"> <li>Call on students and have them sort the books.</li> <li>If students focus on the contents or topics of the books, remind them that we are in a mathematics lesson, and they should focus on the shapes of the books.</li> <li>If necessary, ask additional questions to help students realize that the ratios are equal.</li> </ul>

<p>(Distribute B5 sheets of paper to the students.)</p> <ul style="list-style-type: none"> <li>Let's compare the lengths of sides using this sheet of paper.</li> <li>Can someone share what you noticed about the crease line?</li> </ul>	<ul style="list-style-type: none"> <li>Determine the ratio of the lengths of the short and the long sides of a B5 sheet of paper.</li> <li>Many students will fold the paper to match the short side and the long side.</li> </ul> <p>S1: The long side is about 1.5 times as long as the short side. S2: I think it is a little shorter than 1.5 times.</p> <ul style="list-style-type: none"> <li>Color the crease line and look at the students.</li> </ul> <p>S: I think it is the same length as the long side.</p>	<ul style="list-style-type: none"> <li>If no student colors the crease line, suggest them to do so.</li> <li>Have students fold the paper to verify their observation.</li> </ul>
		
<ul style="list-style-type: none"> <li>Please find the lengths of the long side if we consider the length of the short side as 1.</li> <li>Please share the ratios you determined.</li> </ul>	<ul style="list-style-type: none"> <li>In groups of 4, students will figure out how to determine the length of the long side when the length of the short side is considered as 1.</li> <li>Groups will present that the ratio of the length of the short side to the length of the long side is <math>1:\sqrt{2}</math>.</li> </ul> <p>S: As you can see in the diagram, if we put together 4 copies of the right isosceles triangles together, we can make a square with the area of 2. Therefore, the length of the side must be <math>\sqrt{2}</math>.</p>	<ul style="list-style-type: none"> <li>Instruct students to use the papers each has folded.</li> <li>Observe the groups, and have those groups that determined the length of the long side to be ready to present their answers to the whole class.</li> <li>If others used different approaches, have them share their ideas, too.</li> </ul>

<ul style="list-style-type: none"> <li>I wonder why they use the ratio of <math>1:\sqrt{2}</math>. Fold the paper in half and share what you notice about the new rectangle.</li> </ul>	<div data-bbox="564 203 943 584" data-label="Image"> </div> <ul style="list-style-type: none"> <li>Students discover that the ratio of the lengths of the short and the long sides of the rectangle obtained by folding a B5 sheet in half is also <math>1:\sqrt{2}</math>.</li> <li>Students will determine the ration individually.</li> </ul> <p>S1: The ratio of the length of the short side to the length of the long side is <math>\frac{\sqrt{2}}{2} : 1</math>.</p> <p>S2: The ratio is <math>\sqrt{2} : 1</math>.</p> <p>S3: The ratio is <math>1:\sqrt{2}</math>.</p>	<ul style="list-style-type: none"> <li>Have students actually fold the sheet and help them observe that the shape of the smaller rectangle is the same as the shape of the original rectangle. Have them verify that the ratio of the length of the short side to the length of the long side is <math>1:\sqrt{2}</math>.</li> <li>Have someone explain the ratios <math>\frac{\sqrt{2}}{2} : 1</math> and <math>1:\sqrt{2}</math> are equivalent by manipulating expressions.</li> </ul>
<ul style="list-style-type: none"> <li>Let's look back on what we did in today's lesson.</li> </ul>	<ul style="list-style-type: none"> <li>Students will know that the paper size standards such as B5 make use of irrational numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Have students verify that the size of a common pocket-size books are obtained by cutting an A4 sheet of paper into 4 equal parts. However, the shape of another popular series of pocket-size books is different from those, and help students see that the ratio of the lengths of sides of those books is not in <math>1:\sqrt{2}</math>. This should pique students' interest to want to find the ratio of those sides in the next lesson. (*)</li> </ul>

(\*) Extension in the next lesson

- In the 2<sup>nd</sup> type of pocket-size book, the paper is in the shape of rectangle such that the rectangle obtained from the original rectangle by removing the square portion of it is in the same shape as the original rectangle. After students learn this relationship, have them think about ways to determine the ratio of the lengths of the short and the long sides of this rectangle.

From the proportion,  $1 : x = (x - 1) : 1$ , students will obtain the equation,  $x^2 - x = 1$ , as they try to determine the value of  $x$ .

This will serve as the motivation for the topic of the next chapter, quadratic equations.

