



Date: June 24, 2017
Yamanashi University Attached Elementary School
Grade 6, Classroom 2 (32 students)
Teacher: YAMAGUCHI, Kuniyuki

Mathematics

Organizing Data

1 About the Unit

The mathematics working group for the Central Education Council examined possible points of improvement in school education toward nurturing of 3 types of dispositions and abilities in our students. One of the recommendations presented in the summary report is the improvement and enrichment of statistics education. In particular, following four types of instructional activities were suggested as examples.

- After students represent a given data set in a graph, they will examine it and generate a new set of questions. Then, they will revise the graph to match the new purposes.
- Critically examine a graph which has been created for a certain purpose from multiple perspectives.
- Represent two or more sets of data on the same graph (bar graph, broken line graph and/or histogram) to compare them.
- Discuss representative measure of data set other than mean.
- Intentionally make connections to the content in other subject areas.

In addition, in the new National Course of Study released on March 31, 2017, a new domain, D. Use of Data, was added in the elementary mathematics, and terms such as mean, median, mode, and ranks have been shifted from Grade 7 (in lower secondary schools) to Grade 6 (in elementary schools).

As we planned lessons in this unit, these recommendations and changes were always kept in mind. For our unit, we decided to focus on devising rich activities related to problem solving and decision making based on statistical analysis. In addition, we also wanted to focus on critical examination of statistical methods and conclusions. Our overarching goal is set as “Helping students purposefully collect data, organize them in appropriate tables and graphs, and then identify trends in the data set by observing the representative values and the distribution.”

Students have learned about using pictures and diagrams to represent ideas in Grade 1. In Grade 2, they learned 1-dimensional tables and pictographs, and in Grade 3, they learned 2-dimensional tables and bar graphs. Broken line graphs were discussed in Grade 4. In Grade 5, they learned about percent, circle graphs and percent bar chart as well as average of measured quantities. In each of these units, students engaged in activities of organizing and interpreting

phenomena by representing phenomena in tables and/or graphs and also interpreting tables and graphs.

The goal of the unit is, "Students understand mean as the representative value of a data set, spread of a data set, and frequency distribution, and make use of those ideas purposefully to examine situations statistically and make appropriate representations." Students will collect data purposefully and organize them in a frequency distribution table and also in a histogram. Using those tools, students will interpret characteristics and trends in the data set. Although students have learned about mean previously, in this unit, we will study mean as a type of representative measure of data set. Students will also try to solve problems by making use of data based on what they have learned previously. Students will engage in various learning activities generally sequenced as Identify the Problem → Devise a Plan → Collect Data → Analyze Data → Develop a Conclusion (PPDAC cycle).

In teaching this unit, we want students to understand that, instead of looking at the data set as collected, it is helpful to grasp trends when the data are organized in a frequency distribution table. Moreover, we also want them to understand that a histogram is also helpful to observe how the data are distributed. We want to make sure students understand the merits of various ways to organize data. As students make use of data to solve problems, we also want them to critically reflect on both the methods and conclusions obtained statistically by employing tools such as frequency distribution tables, histograms, and representative values.

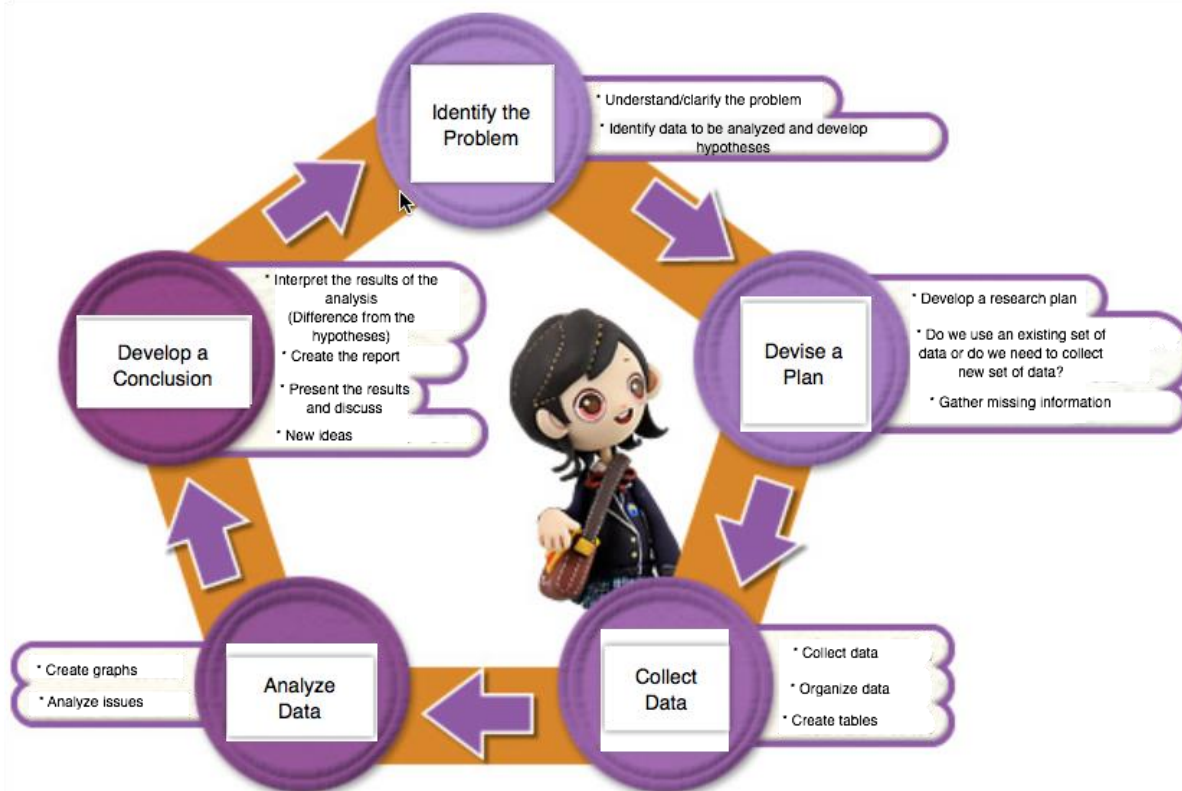


Figure 1 PPDAC cycle

2 Goals of the Unit

- ◆ Students will understand mean as a representative value, dispersion of data, and frequency distribution. They can use those ideas purposefully to examine and represent phenomena statistically.
- Students recognize the usefulness of mean as a way to represent characteristics of a data set, and they try to use it purposefully to examine and represent phenomena in their surroundings statistically. [Interest, Eagerness, and Attitude]
- Students can examine data sets to identify their characteristics using tools such as mean and the way data are scattered. [Mathematical Way of Thinking]
- Students can represent data sets in frequency distribution tables and histograms as well as interpret given tables and histograms. [Mathematical Skills]
- Students understand mean as a representative value of a data set. They also understand frequency distribution tables and histograms. [Knowledge and Understanding]

3 Relationship between the Unit and the Research Theme

(1) About the dispositions/abilities we want to nurture in this unit

In the mathematics group, in order to realize lessons in which students create their own mathematics, we utilize lessons that focus on problem solving (*mondai kaiketsu gakushu*). In the learning processes in problem solving lessons, the four dispositions/abilities and "questions" are closely related.

In the stage of grasping the learning task, students will have the question, "What have I learned so far?" and put the problem situations from their daily lives onto the mathematical playing field as the first step of problem solving. In the independent problem solving stage of the lesson, students will ask themselves, "Which of what I have learned may be useful in this problem?" and tackle the problem by comparing it to previously solved problems. However, it is not always possible for students to have their own ideas. Thus, their peers will become an important component of their learning. During the comparison and critical reflection stage, students compare and contrast their own ideas with those of their peers to generate better solutions, approaching the goals of the lesson. In this stage, students will ask about the rationale, commonality, differences, and generalizability. During the reflection stage, students will ask about the merits and extendability of ideas so that they can use what they learned in other situations. In mathematics, we believe the engine for learning is "question." In the process of learning, when one problem is solved, a new "question" arises. "Questions" are continuously generated. What supports this learning process is students' disposition to tackle problem solving autonomously.

In this unit, questions will be generated as students represent the data set in tables and graphs. Students will hold various questions as they go through the learning activity sequence. In the step of Identify the Problem, they will ask themselves, "What is the issue in this situation?" As they Devise a Plan, they will ask, "What information is needed to solve this problem?" They will ask, "What method should we use to collect the data?" in the Data Collection stage. As they engage in data analysis, the question may be "What can we tell from the data?" Finally, as they try to generate a conclusion, they will be asking "What can we conclude from our analysis?" As students engage in problem solving through these steps, there will be a question that is the question that must be asked. In other words, those are the questions, students want to answer as they identify the problem. By collecting,

organizing, and analyzing data, students will then generate the solution for the problem. We hope students will actively engage in this problem solving process.

(2) About strategies for “lessons in which students feel the values of learning”

In the mathematics group, we consider “values of learning” is realized in experiencing the merits of mathematics. The strategies necessary for “lessons in which students feel the values of learning” are as follow:

- ① Strategies to devise lessons in which students have questions and the sense of expectations
 - Set up the learning tasks that generates “questions.”
- ② Strategies to devise lessons in which students are engaged independently and with each other
 - Investigate “questions” that must be asked.
 - Anticipate how a series of “questions” may be generated.
- ③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction
 - Record “questions” on blackboard and reflect on the learning.
 - Secure time for students to write reflective journal entries.

In this unit, we will set up the learning tasks that are easy for our students to relate to by picking situations from their everyday life. Thus, it is hoped that students will feel that they are dealing with problems that are relevant to them. As they progress through their problem solving processes, they will generate their own questions and make own decisions. We will try to anticipate what questions students might generate and how they may be connected by carefully observing them during the lessons and reading their reflective journal entries.

(3) About methods for assessing the quality of individual student’s learning

As a strategy to assess the quality of individual student's learning, we will make use of their notebooks. We have been encouraging students to make their notebooks align with the process of problem solving lessons, "grasping the learning task → independent problem solving → comparison and critical reflection → reflection." Thus, in the independent problem solving stage, students will write their own ideas. In the comparison and critical reflection stage, they try to record their peers' ideas. When they do so, instead of simply writing down the answers, we have encouraged them to include the steps and process of getting the answers, using words, pictures, diagrams and mathematical expressions. By writing their learning journal entries, students can organize their ideas, reflect on them deeply and make use of the ideas in new problem situations. This way, students can reflect on the development of their ideas. By checking students' entries in independent problem solving/comparison and critical reflection/learning journal against each other, we will know what ideas students initially had, what challenges they faced, and how their ideas evolved. In this way, we want to assess our efforts to increase the quality of individual student's learning.

In this unit, students will be solving a common problem in collaboration instead of each one solving his or her own problems. As students devise a plan, figure out the method of data collection, and chose ways to analyze the collected data, we will try to grasp each student’s ideas from their notebooks and reflective journal entries. We want to make use of students’ ideas in the following lessons. Moreover, we want to carefully observe what each student thinks about to reach his or her conclusion as students discuss their ideas with each other so that we can assess individual student’s learning.

4 Unit Plan and Assessment (Total of 10 lessons)

#	Goals	Learning Activity	Assessment Standards
① How to analyze data (6 lessons)			
1	○ Students understand mean as a representative value.	<ul style="list-style-type: none"> • From a table, summarize the data by identifying features such as the greatest value, the least value, and the total. • Think about the usefulness of using means to compare two more more sets of data. 	<p>[Interest] Students recognize the usefulness of using means to compare sets of data.</p> <p>[Knowledge] Students understand that we sometimes use means to analyze characteristics of data sets.</p>
2	○ Students can observe and think about the spread of data.	<ul style="list-style-type: none"> • Students investigate how data are spread from a table. • Students note that if they investigate how data are spread then it will be easier to see characteristics of a data set. 	<p>[Thinking] Students think about the needs for investigating the spread of data and analyze a data set statistically.</p>
3	○ Students understand the methods to organize a data set into a frequency distribution table, and they can also interpret a frequency distribution table.	<ul style="list-style-type: none"> • Students will organize the spread of data in a table. • Students will identify the frequencies of data in various classes and also calculate the relative frequency of each class to the total number of data. 	<p>[Skills] Students can organize a data set in a frequency distribution table. They can interpret a frequency distribution table.</p> <p>[Knowledge] Students understand that if we investigate the spread of data, we can more easily see characteristics of a data set.</p>
4	○ Students understand how to draw and interpret bar graphs.	<ul style="list-style-type: none"> • Students identify characteristics that can be observed from a graph. 	<p>[Skills] Students can draw and interpret bar graphs.</p> <p>[Knowledge] Students understand that if we organize data in a bar graph, we can more easily see characteristics of a data set.</p>
5	○ Students will deepen their understanding of methods of analyzing data by summarizing what they observed using different statistical methods.	<ul style="list-style-type: none"> • Discuss what they noticed about various ways of comparisons and conclusions obtained. 	<p>[Thinking] Students can identify and explain trends in data using tools such as representative values and histograms.</p>

② Use of data (5 lessons)			
1	○ Students can identify a problem and clarify the purposes for collecting data.	● Look for and identify a problem and clarify the purposes for collecting data.	[Interest] Students try to identify a problem and examining the purposes.
2	○ Students can devise a plan for conducting a study.	● Devise a plan for data collection. ● Identify various conditions for data collection.	[Interest] Students try to devise a plan to collect data purposefully.
3	○ Students can collect data purposefully.	● Collect data purposefully and organize them in a frequency distribution table.	[Interest] Students try to represent data in a frequency distribution table. [Skills] Students can represent data in a frequency distribution table.
4	○ Students can analyze the collected data (measurements) and think about ways to compare data sets that match the purposes they have identified.	● Analyze the collected data (measurements). ● Based on the purposes they have identified, generate their own interpretations of data.	[Thinking] Students can identify trends in data sets by using representative values and bar graphs, and they can explain their reasoning.
5 Today's Lesson	○ Students can organize data and identify trends so that they can solve the identified problem. They can explain their reasoning.	● Identify trends in data and explain their own interpretation.	[Interest] Students become interested in making use of frequency distribution tables, histograms, relative values, etc., to solve problems, and they are trying to apply them in other problems. [Thinking] Students can identify and explain trends in data using representative values and histograms.

5 Today's Lesson

(1) Date: Saturday, June 24, 2017

(2) Location: Yamanashi University Attached Elementary School

(3) Goal of the lesson

- Students can organize data to solve problems and explain their reasoning.

(4) Reason for teaching this lesson

Problem B[5] in the 2016 National Assessment asked students to explain mathematically the reason why they made a certain judgment based on data. The report of the assessment recommends that "it is important to help students make appropriate judgments based on trends in data through activities in which they determine

representative values of data or examining the spread of data." In today's lesson, students will be presented with a problem situation that is familiar in their everyday life. It is hoped that students will be able to discuss the issues mathematically using what

In order to greet as many people as possible, from what time to what time should we be standing?

they observed in data as the rationale.

This lesson is the second of a 2-lesson sequence in the second sub-unit of the unit. Students have been trying to find the answer to the question, " " They have collected on what time students tend to come to school, and they have determined representative values and summarized the data in a frequency distribution table and a histogram. Based on the data, they have reached their own conclusions.

In today's lesson, students will share the conclusions they have reached about what time they should be coming to school. First, we will focus on various representative values for the data set, and discuss what we can observe from those.

First, we will discuss the idea of mean. Students have learned about mean in Grade 5. In addition, in the sub-unit 1, students used means while comparing data from multiple groups. The word "mean" is used in our daily life frequently, therefore, we expect it is easier for students to think of mean as the representative value than other measures. On the other hand, if we draw a conclusion on what time to be standing based only on the mean, it is not necessarily the case that the data will be around the mean. Thus, we need to question the validity of using the mean to make a conclusion. We also need to consider mode, the frequency distribution table and/or the histogram.

Next we will turn our attention to those who considered mode. This discussion may help those who only used the mean to make decisions realize that looking at the mean alone will not be sufficient. Moreover, in addition to considering the mode, if we look at the histogram, we can see the time frames that are second or third most frequent as well. Based on students' own experiences, they may hypothesize that those peak times may relate to the bus schedule. Based on this hypothesis, some students may want to create a histogram for those students who are coming to school by buses alone. This way, we will create 2 different histograms, one for those students who come to school by buses and one for those who walk. The reason for creating the 2 histograms is to give students an experience that by doing so they can actually make new interpretations. This will, in turn, develop the ability to more critically examine results obtained from the analysis instead of simply creating a graph. They will have the ability to reflect on the data and make better decisions. Because the experience of creating 2 different histograms is critical, if that idea does not come from the students, the teacher will make this suggestion.

Based on the histogram for bus riders, the class will discuss what they can observe from the graph and make suggestions about what time they should be standing to greet others. Some students might notice that the suggestions based on the histogram for bus riders only will focus on narrower range of time than the suggestions made from the histogram for all students. We want to emphasize the values of making conclusions by comparing 2 different graphs.

After discussing the histogram for bus riders, some students may wonder about the distribution of times for those students who come to school on foot. Based on that wondering, we will examine the histogram for walkers, and try to interpret the graph. Finally, based on the 3 histograms, total, bus riders and walkers, the class will come up with the conclusion when they should be standing to greet arriving students.

(5) Flow of the lesson

min.	Main learning activity/content Anticipated responses	<ul style="list-style-type: none"> • Points of consideration • Strategies for “lessons in which students feel the values of learning”
5 G R A S P	1. Look back on the previous lesson <ul style="list-style-type: none"> • Make sure students understand the time for the "meet & greet" <ul style="list-style-type: none"> ○ We were standing between 8:05 and 8:15. ○ It was for 10 minutes. • Make sure students understand the issues for the "meet & greet" <ul style="list-style-type: none"> ○ Not many students came. ○ It is hard for the members of the students council to be at school so early. 2. Understand the task and devise a plan <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> In order to greet as many people as possible, from what time to what time should we be standing? </div>	<div style="border: 1px solid black; padding: 10px; margin-bottom: 10px;"> In the process of “lessons in which students feel the values of learning” ◎ Strategies <ul style="list-style-type: none"> • Motivate students to tackle the task with the understanding of the problem situation. • Set up the learning tasks that generates “questions.” </div> <ul style="list-style-type: none"> • Make sure students understand the issues in the "meet & greet" • Instruct students to make recommendations and the reason why
35 E X P L O R E	3. Develop recommendations based on representative value and histograms <p>"I wonder what time is the best if we use the mean arrival time?"</p> <ul style="list-style-type: none"> • Thinking about the mean arrival time, 7:53. <ul style="list-style-type: none"> ○ 7:48 - 7:58 --- ① 5 minutes before and after 7:53. Because this is the mean, I think we get a lot of students coming around that time. ○ 7:50 - 8:00 --- ② I included the mean arrival time and surrounding time with a lot of students. ○ Both includes the mean arrival time of 7:53. 	<ul style="list-style-type: none"> • From the journal entries from the previous lesson, bring up the idea of the mean. <div style="border: 1px solid black; padding: 10px; margin-bottom: 10px;"> In the process of “lessons in which students feel the values of learning” ◎ Students who feel the values of learning <ul style="list-style-type: none"> • Explaining the conclusions they reached, listen to their friends' explanations, and look for similarities and differences. • Through data analysis, grasp trends in the data set. </div> <ul style="list-style-type: none"> • Ask students to identify commonalities. Make sure students see that all ideas include the mean arrival time.

<ul style="list-style-type: none"> • Thinking about other times <ul style="list-style-type: none"> ○ Many of bus riders arrive at different times. ○ There are a lot at 7:45. ○ There are a lot at 7:55, too. ○ I walk to school, and I'm usually at school earlier. • Thinking about the mode, 7:57 <ul style="list-style-type: none"> ○ 7:52 - 8:02 --- ③ 5 minutes before and after 7:57. Because many students arrive then. ○ 7:55 - 8:05 --- ④ I used the interval with the most students and the interval to the right. ○ 7:50 - 8:00 --- ⑤ I used the interval with the most students and the interval to the left. ○ ⑤ includes the mean arrival time, too. <p>"I wonder what time is the best if we use the graph?"</p> <ul style="list-style-type: none"> • Thinking about a histogram. <ul style="list-style-type: none"> ○ With ①, it includes an interval with small number of students, so we can't greet that many students. ○ I don't think we can tell which interval will have a lot of students. ○ I think either one of the intervals based on the histogram is the best, 7:50 - 8:00 or 7:55 - 8:05. • Think about the second most frequent interval. <ul style="list-style-type: none"> ○ I think 7:45 - 7:50 is because of bus riders. ○ I arrive at that time, but I walk to school. ○ I think it would be helpful to have a histogram for bus riders only. <p>4. Make recommendations using the histogram for bus riders</p> <p>"I wonder what time is the best if we use the histogram for bus riders?"</p> <ul style="list-style-type: none"> • Share what they observed from the histogram 	<div style="border: 1px solid black; padding: 10px; margin-bottom: 10px;"> <p>In the process of "lessons in which students feel the values of learning"</p> <p>◎ Students who feel the values of learning</p> <ul style="list-style-type: none"> • Explaining the conclusions they reached, listen to their friends' explanations, and look for similarities and differences. • Through data analysis, grasp trends in the data set. </div> <ul style="list-style-type: none"> • Help students realize that if they use other tools in addition to the mean, they can make a better decision. <div style="border: 1px solid black; padding: 10px; margin-top: 10px;"> <p>In the process of "lessons in which students feel the values of learning"</p> <p>◎ Strategies</p> <ul style="list-style-type: none"> • Students generate their own "questions" from the task. <p>"I wonder what time is the best if we use the mean arrival time?"</p> <p>"I wonder what time is the best if we use the graph?"</p> <p>"I wonder what time is the best if we use the histogram for bus riders?"</p> <p>"I wonder what time is the best based among all of the recommendations?"</p> </div> <ul style="list-style-type: none"> • Make sure that the mean arrival time is clear in the histogram.
---	---

<p>for bus riders.</p> <ul style="list-style-type: none"> ○ There are a lot of students arriving 7:55 - 8:00. ○ 7:45 - 7:50 is the next. ○ There are very few before 7:35. ○ I thought many arriving 7:45 - 7:50 are bus riders, but many of them are actually walkers. <ul style="list-style-type: none"> ● Make a recommendation for the best time. <ul style="list-style-type: none"> ○ 7:50 - 8:00 --- ⑥ Because this interval has the most students in the histogram for bus riders. It's the same for the whole student body. ● Share what they observe in the histogram for walkers. <ul style="list-style-type: none"> ○ 8:00 - 8:05 has the most students. ○ 7:55 - 8:00 also has many students. ○ Unlike for bus riders, the students are spread out more widely. ● Make a recommendation for the best time. <ul style="list-style-type: none"> ○ 7:55 - 8:05 --- ⑦ There are most walkers arriving in this time interval. <p>"I wonder what time is the best based among all of the recommendations?"</p> <ul style="list-style-type: none"> ● Make recommendations using the mean and the histograms. <ul style="list-style-type: none"> ○ 7:50 - 8:00 Because it came up in ②, ⑤, and ⑥. ○ 7:55 - 8:05 Because the most popular intervals are different for bus riders and walkers. ○ We split ourselves to 2 groups, one for 7:50 - 8:00 to greet bus riders and the other for 7:55 - 8:05 for walkers. 	<ul style="list-style-type: none"> ● Using a spreadsheet software, display histograms for bus riders and walkers. ● Acknowledge the value of comparing the 2 histograms, one for bus riders and one for the entire students. ● Help students realize that the data are more wide spread compared to the data for bus riders. ● Even though the total interval becomes 15 minutes, accept as a recommendation.
---	---

5. Look back on today's lesson

- First, reflect on the main task.
- Not many students came at the time we tried
→ We learned from the graph that there aren't many students arriving in 8:05 - 8:15.
- Early morning is difficult for the council members → There are many students who arrive early, too.
- Develop a class recommendation.
- Write reflective journal entries.
- I was glad that we were able to make a recommendation for the "meet and greet."
- By separately looking at bus riders and walkers, we were able to make a better recommendation.

In the process of “lessons in which students feel the values of learning”

◎ Methods for assessing the quality of individual student’s learning

- Through students' reflective journal entries, assess if lesson goal was met.
- From students' notebooks, assess how students ideas developed across the time.
- From reflective journal entries, assess the effectiveness of the lesson and students' learning.

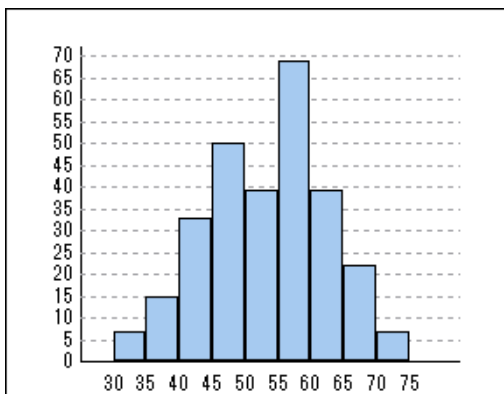


Figure 2 Histogram for the whole student body

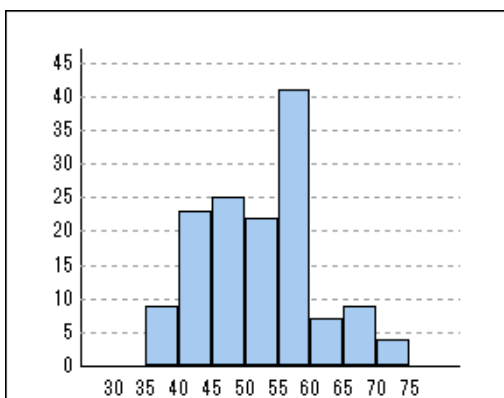


Figure 3 Histogram for bus riders

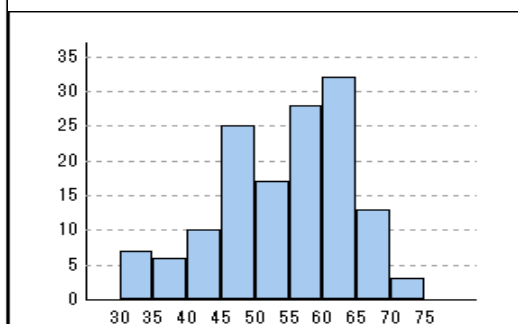


Figure 4 Histogram for walkers

(6) Points of observation

- Were the strategies to help students feel the values of learning?
 - ① Strategies to devise lessons in which students have questions and the sense of expectations
 - ② Strategies to devise lessons in which students are engaged independently and with each other
 - ③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction

(7) References

Omitted