June 21

Grade 3, Mathematics Lesson Plan

Place: Showa City Oshihara Elementary School
Teacher's Name: Midori SHIBAKITA

1. **Name of the Unit:** Let's Think about Division (Division with Remainders)

2. **Goals of the Unit:**
Students understand division when there is a remainder in the quotient. The lesson is designed to further students' understanding of the meaning of division, including applying their knowledge and understanding about remainders in division problem situations.

- Using cases of division when there is no remainder (divisible division), students try to understand the meaning of and how to calculate division when there *is* a remainder (indivisible division), by connecting division to their understanding of multiplication and representing and modeling indivisible division with concrete objects. **[Interest, Motivation, and Disposition]**
- Students see that division with divisible cases and indivisible cases both represent unified division cases. They express the meaning of division and division calculations using concrete objects, diagrams, and math sentences. **[Mathematical Reasoning]**
- Students perform indivisible division calculations and find solutions that are quotients with remainders. **[Skills and Procedures]**
- Students know what remainders represent and understand the size relationships between remainders and divisors, as their understanding of division increases. **[Knowledge & Understanding]**

3. **About the Unit:**

In Grades 2 and 3, my students thought about the meaning of multiplication and performed multiplication calculations to explore the properties of multiplication. In the everyday life problem situations they solve in my classroom, students have shown a solid understanding of how to use multiplication when a whole is divided equally into several equal parts.

In addition to previous grade levels, in this grade's fourth unit, entitled "Division," students learned that division is a reversed calculation of multiplication (i.e., division as inverse of multiplication). In that unit, the division students learned was all problems with dividends that "matched" the products of multiplication facts they were familiar with. That is, when my students thought about the divisor and the multiplication facts (multiples) of the divisor, they saw how the dividend reflects multiplication of the divisor. They used this relationship between multiplication and division to find the quotient of division problems.

However, in the present unit, the dividends do not match whole-number products of multiplication facts. Now, students need to think about how to carry out division calculation in these situations. In the process of thinking about how to solve these division problems, some students do not question when a remainder is greater than a divisor. Some do not think about how different cases of problem solving situations mean they have to think about the remainder differently, too. Therefore, it is important for students to use diagrams to represent and explain their thinking. Then they can see how they may be misunderstanding the situation, because they are not representing the situation correctly.

In this unit, students learn division with remainders, the division case that is indivisible. To do a division-with-remainder calculation, students must find the largest product that does not exceed the dividend; then, they can use that product to find the quotient and the remainder. For example, in the case of calculating 13 ÷ 4, when the missing product of 4 x [ ] or [ ] x 4 is less than 13 and closest to 13, we can find the whole number quotient and the remainder. The

quotient shows the greatest quantity of each of the equally divided parts or the greatest number of equally divided quantities. Moreover, the remainder is less than the divisor. These are important ideas students need to understand. In addition, when students understand the relationship, (divisor) x (quotient) + (remainder) = (dividend), they see how they can use this relationship to check their calculation answers. I am hoping that students will recognize and come to understand the difference between the division they learned previously (divisible division) and the division they encounter in this unit (indivisible division).

In problem situations, there are cases when it is important to find both the quotient and the remainder, consider the remainder as part of the quotient, and add 1 to the quotient. Then there are those situations when only the quotient – without the remainder – represents the problem solutions. Students need to understand and recognize the difference between these two division situations, so they identify correctly how to process remainders.

The division situations in the previous Unit 4 "Division" introduced division via the partitive division problem situation; however, in this unit, "Division with remainders," division was introduced in quotative division problem situations, because these situations help students understand the meaning of remainders. Through these situations, students learn they can write division math sentences just as before, even in the case of indivisible division situations. I would like to incorporate activities where students manipulate concrete materials and draw diagrams, using these visual representations to explain their ideas and better understand the indivisible problem situations and solution process. Finally, there are additional cases when the answer to an indivisible problem situation cannot be answered by using only the quotient and remainder, so I would like to provide opportunities for students to think, explain, and reason to help them develop their deep understanding of remainders and how to process remainders.

4. Knowledge and Skills that I Would Like to Foster Among the Students:

In students' daily life, there are situations or phenomena that students should connect to the mathematics that underlies the situations/phenomena, as they improve how to think, justify, and problem solve. In this way, students need to be equipped with skills for grasping problem situations and process these situations appropriately to find solutions. Therefore, I would like to help students develop skills to connect everyday-life problem situations with mathematics and use mathematics to think and process. In addition, I would like to help students see and feel how very useful mathematics is.

Included in not only this unit, but across units, I would like to draw from problems that are connected to students' daily lives and engage them in activities that provide ample opportunities to think and explain using concrete materials, diagrams, and math sentences to problem solve. Moreover, I would like to provide opportunities for students to reflect on the solution process and think about the meaning of the solution as part of my instruction.

5. Unit Plan of the Instruction and Assessment:

Lessons	Goals	Instructional Activities	Main Assessment Points
(1) Division with Remainders Total of 6 Lessons			
1	[Prologue] • Through discussing a review of previously learned division (divisible division) and the points students know about division, help students notice there are cases of division that are indivisible (division with remainders) and become interested in learning more about division.		
2	• Students understand how to perform calculations in which the divisor and quotient are both 1-digit and the dividend is indivisible.	• Students think about how to find the answer of 14 ÷ 3. • Students present their ideas and confirm the answer. • Students discuss how to process the remainder. • Students understand the meaning of remainders.	[IMD]* Students are thinking about how to perform division calculations in indivisible cases using their previously acquired knowledge of divisible cases as a basis. [MR] Students are thinking about how to perform division calculations in indivisible cases using their previously acquired knowledge of divisible cases as a basis. They explain their methods using aids such as concrete objects, diagrams, and math sentences.
3	Students understand the relationship between the remainder and the divisor.	• Students investigate the relationship between remainder and divisor in the case of division, 14 ÷ 3.	[K&U] Students understand that the remainder is to be smaller than the divisor.
4	Students understand that indivisible cases of division can be applied in partitive division, as well as quotative division.	• Students grasp the problem situation, establish a math sentence, 16 ÷ 3, and think about how to find the answer. • Students present their ideas and confirm the answer. • Students solve word problems.	[MR] Students think about how to calculate indivisible partitive division problems using divisible cases of partitive division as a basis; and they explain using tools or models such as concrete objects, diagrams, and math sentences.
5	Students understand how to check the answers of indivisible division calculations.	• Students think about how to check the answers of indivisible division calculations.	[K&U] Students understand how to check answers to indivisible division problems.
6	Students practice division calculations, including those that are indivisible.	• Students practice division calculations and check the answers.	[S&P] Students can perform indivisible division calculations and solve for the quotient and remainder.

(2) Problems Dealing with Remainders Total of 2 Lessons			
7 This Lesson	Students think about how to process remainders based on problem situations and be able to explain the process. (Problem situation: quotient plus 1).	• Students grasp the problem situation, establish a math sentence, 23 ÷ 4. • The calculation shows the quotient is 5 and remainder is 3. Students discuss if the answer should be 5 or not. • Students summarize why the answer will be the quotient plus 1.	[MR] Students understand how to process quotients and remainders based on the problem situation and are able to explain why they used this process.
8	Students think about how to process remainders based on problem situations and are able to explain the process. (Problem situation: No need to add 1 to quotient)	• Students grasp the problem situation, establish a math sentence, 30 ÷ 4. • The calculation shows the quotient is 7 and remainder is 2. Students discuss if the answer should be the quotient plus 1 or only use the quotient.	[MR] Students understand how to process quotients and remainders based on the problem situation; and they are able to explain why they used this process.
Summary Total of 2 Lessons			
9	Student apply what they have learned about division in this unit to solve problems.	• Students solve “Power Builder” problems.	[S&P] Students solve problems using previously-learned content appropriately.
10	Students check and reinforce their understanding of the math content in this unit	• Students solve “Mastery Problems.”	[K&U] Students have acquired a basic understanding of the content in this unit.

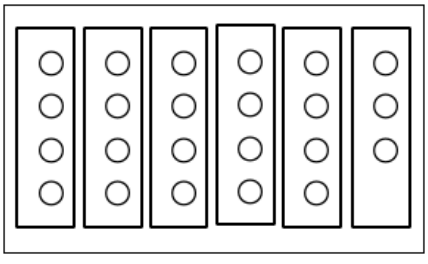
*[IMD] Interest, Motivation, and Disposition, [MR] Mathematical Reasoning, [S&P] Skills and Procedures, [K&U] Knowledge & Understanding

6. About this lesson

(1) Date and time: 5th period, Friday, June 21, 2019
(2) Place: Grade 3, Class 2 classroom
(3) Goal of the lesson:
Students think about how to process remainders based on problem situations and are able to reason about and explain the process.

(4) About Instruction
In the past, students have learned about division problems by answering questions, such as: “How many people receive [*n*] sheets of papers and how many sheets will be left?” “How many sheet of paper will each person get and how many sheets will be left?” These problems ask, “How many are left?” so students become accustomed to finding the remainders and using the quotient and remainders to answer these questions. However, the problem the students face in this lesson do not have a question that directly asks about the remainder. Therefore, students will learn to consider how the remainder should be processed to find an appropriate answer to the question the problem is asking. This is the primary task and object of this lesson. In this lesson, the problem the students solve is: “To carry all the tennis balls, how many containers do we need?” Since the answer is the quotient plus 1, the students need to think about how to deal with the remainder. In other words, students must think about whether the calculation and answer (quotient and remainder) is appropriate for answering what the problem is asking. I would like to provide opportunities for students to explain their own solution ideas and use diagrams and math sentences to try to understand their friends’ ideas. In addition, I would like to help them think about how to process the remainder appropriately, so they can be confident that they answered the question the problem is asking.

(5) Instructional process

Time	Instructional Activities ○ Anticipated Student Responses	Things to Remember	Notes
Grasp 10 min.	1. Grasp Today's task. ① Grasp the problem situation There are 23 tennis balls. A tennis ball container can hold 4 balls. How many containers do we need if we want all the balls in containers so we can move them to another location? ② Think about the math sentence ○ 23 ÷ 4 ③ Grasp Today's task Let's think about how many containers we need.	• Help students understand that containers are necessary to carry balls from place to place. • Discuss the problem, so students confirm that a container can hold only 4 balls. • Show concrete materials. • Confirm with students that the math sentence is 23 ÷ 4, from the discussion. • Write the task on the board so the students grasp the task clearly.	
Think 10 min.	2. Write your own idea in notebook. A: 23 ÷ 4 = 5 R3 Answer: We need 5 containers and 3 balls are left The answer to the calculation is 5 R3, so we need 5 containers and 3 balls will be left. B: 23 ÷ 4 = 5 R3 Answer: 5 containers The 3 balls left over cannot be in the container, so the answer is 5 containers. C: 23 ÷ 4 = 5 R3 Answer: 6 containers We need to put the 3 balls in a container also. So, we need one more container. The answer is 6 containers. (If we show it with a diagram) 	• When finding students who are having difficulty calculating, ask them to recall what they did in the past. • Walk around the classroom and monitor students' progress and ideas. • When finding students writing their own explanation in addition to their answers, provide some positive feedback. • So they make connections among diagram, explanation with words, and math sentences in later discussions, encourage students to represent their ideas in multiple different ways.	◉ Students understand how to process quotients and remainders based on the problem situation and are able to explain why they used that process. (Notebook)

Pursuit 20 min.	3. Discuss the solution (whole class). ① Explain your solution. A: ○ The answer to the calculation is "5 R3," so we need 5 containers and 3 balls are left. ○ There are only 3 balls left, so we can't put them in the container. I think we need only 5 containers. ○ We don't need to think about the remainder. B: ○ 5 containers and 3 balls left, so 5 containers. C: ○ We need to put the 3 balls in a container, so we need 6 containers. ○ We need to put all the balls in the containers, so we need to put the 3 balls in a container. ○ We need 5 containers to put all the balls away. ○ If we draw a diagram, we can see that we need another container. ○ If we write it in a math sentence, 5 + 1 = 6. ② Compare each solution and relate the solutions to each other: ○ There are answers that show the remainder and add one more container to include the remainder. ○ There is an answer that does not think about the remainder. ○ If we don't use another container, we need to carry the 3 balls by themselves. ○ The problem says, "all the balls in the containers," so we need to put the three balls in a container also. ○ 6 of the 6 containers means 5 + 1. ③ Summarize the discussion ○ I thought I needed to find the remainder at first, but I understand that I need one more container to carry all the balls. ○ I thought I didn't need to think about the remainder, but now I understand that I need to add one more container. ④ Think about if the remainder is 1 or 2 (instead of 3) ○ We need to use another container to put in the remaining balls, even when there are only 1 or 2 balls.	• Write down each student's idea on the board and help students organize their thinking. • Carry out the discussion focused on how to deal with the remainder 3. • Carry out the discussion focused on how to deal with the remainder 3. • Compare with the problem situation that requires showing remainders. Confirm that in this problem they do not need to find the remainder. • Confirm that the answer is the quotient plus 1. • Confirm that there are cases that even if the remainder is 2 or 1, they will need to add 1 to the quotient to answer the problem.	◉ Students understand how to process quotients and remainders based on the problem situation. They are able to explain why they used that process. (Notebook, presentation)

Summarize 5 min.	4. Summarize how to process quotient and remainder ○ If we have 3 or 2 left, we need another container. ○ Even if there are only 2 or 1 left, there are cases when we will need to add 1 to the quotient. There are cases when we need to add 1 to the quotient, because there is a remainder.	• Summarize the learning using what students discussed. • Confirm that there are cases when we need to find the remainders, when we don't need to find the remainders, and when 1 is added to the quotient to answer the problem.	
	5. Students write reflections ○ I understand that sometimes I have to add 1 to the quotient based on the problem situation. ○ I understand that when we have 3 or 2 left, we need to add 1 to the quotient. ○ I did not know what to do with the remainder at first; but as I listen to my friends' explanations, I understand why we need to add 1 to the quotient.	• Remind students that they need to write in their reflections what they understand, what they noticed, and their thinking after they listen to their friends' explanations. • Tell students that there are problem situation cases when they won't need to think about remainders at all. (This should pique their interest, as they foresee what is coming in the next lesson.)	◉ Students understand how to process quotients and remainders based on the problem situation; and students are able to explain why they incorporated a given process. (Notebook)

(6) Evaluation of the lesson:

Did students understand how to process quotients and remainders based on the problem situation and were they able to explain why they chose a specific process? [Mathematical Reasoning]

7. References:

Tokyo Shoseki. *Atarashii Sansu [New Elementary Mathematics] Grade 3A, Teacher Guide, Instructional Volume*. Tokyo: Tokyo Shoseki.

Tokyo Shoseki. *Atarashii Sansu [New Elementary Mathematics] Grade 3A, Teacher Guide, Research Volume*. Tokyo: Tokyo Shoseki.

Ministry of Education, Culture, Sports Science and Technology. 2017. *Elementary School Course of Study: Mathematics*. Tokyo: Nippon Bunkyo Shuppan.

Japan Society of Mathematical Education. *The Key Points for Developing Mathematics Lessons: Paying attention to basics.* Tokyo: Toyokan Shuppan

Tsukuba University Attached Elementary School's Mathematics Research Group. 2013. *Sansujyugyo Ronkyu [Elementary Mathematics Lesson Research].* Tokyo: Toyokan Shuppan.

Shin Sansu Kyoiku Kennkyukai. 2010. *Koza: Sansujyugyo no shintenkai [Lecture: New Development of Mathematics Lessons].* Tokyo: Toyokan Shuppan.