
Mathematics Department

What is Left and What is the Difference?

Place: Grade 1, No. 2 Class

Research Lesson Discussion Place: Grade 4, No.2 Class

Grade 1 Mathematics Lesson Plan

Instructor: NOMURA, Miyoko

Students: Grade 1, No.2 class (35 students)

1. About the Unit:

My view of the students

Many of my students have experienced counting numbers and finding the answers to simple addition and subtraction informally before becoming the first grade. During students' daily conversations and in mathematic class, there have been quite a few times when I heard students talking about "addition," "subtraction," or representing subtraction as " $\bigcirc - \square = \triangle$."

When they became the first grade, they learned to group and counted things as a set, compared two quantities of objects by aligning them in two lines and making one-to-one correspondence between the two sets, compared two sizes of quantities using counting and one-to-one correspondence, and stated and wrote numbers accurately. By learning to quantify and compare in this way, students became able to see numbers as sets and recognized how counting objects in a set quantifies the set.

In addition, students learned about the structure of numbers to 10 and composed and decomposed those numbers. They also learned about addition problem situations and the meaning of addition with two 1-digit numbers, then tried to become fluent with calculations. When they were learning about addition, students experienced extensive periods of time manipulating concrete materials, expressing their ideas using diagrams and pictures, and explaining their ideas to other students. That is, these students not only spent time practicing addition calculations, but they also spent time thinking about the meaning of addition.

Calculation of addition and subtraction is very important content for mathematics learning that will become the foundation of elementary and secondary mathematics. At this point in this lesson, I would like to focus the students' learning on understanding the meaning of calculations and why we use a particular operation to answer in particular problem situation. It is important to teach students how to think in this way instead of focusing only on their practicing calculations and teaching calculation procedures.

Subtraction is the operation for finding the number of elements in one of the two sub-sets of a whole when the whole is split into two subsets. In the previous unit, students learned that addition can be used in problem situations of *put together* and *add to* with the understanding of numbers as sets (smaller sets composing the larger or whole set). In the subtraction unit also, students learn that subtraction can be used in the problem situations of *take from*, *take apart*, and *compare* with the understanding of numbers as sets (whole or

composite set decomposed into smaller subsets, or two sets compared to find the difference between two sets).

In the case of *take from*, one of the subsets is taken away from the whole set and it involves change in time and a decrease in the whole quantity. On the other hand, *take apart* does not involve a change over time or a decrease in quantity. After students learn about *take from* and *take apart*, they learn about the *compare* situation of subtraction. The *compare* situation involves finding the difference between two sets. At this point, the meaning of subtraction is expanded.

In this research lesson, I will focus on the *compare* problem situation. I will help students understand how subtraction can be used in the case of the *compare* situation by utilizing their previously-learned knowledge of subtraction. I believe that providing opportunities for students to think about the meaning of operations, beginning in first grade, will allow them to develop an interest and disposition for understanding the meaning of multiplication and division operations in second grade.

At the start of the school year, the students acquired only few mathematical knowledge and experiences; however, by utilizing the learning from the previous addition unit, I would like to help the students recognize how they can represent and express the *compare* problem situation in a math sentence. To do this, I will provide opportunities for the students to manipulate concrete materials and explain their ideas to other classmates using diagrams and pictures. Through these activities, I would like to engage students in discussing questions, such as “why this situation is a subtraction situation?” and to increase students’ ability to explain their own thinking in logical manner.

2. Goals of the Unit:

- Students recognize and know the problem situations that use subtraction, such as *take from*, *take apart*, and *compare*; and students understand the meaning of subtraction. (knowledge and skill)
- Students are able to perform subtraction with minuends up to and including 10. (knowledge and skill)
- Students recognize that the ideas underlying *take from*, *take apart*, and *compare* situations are all connected as kinds of subtraction. (thinking, justification, expression, etc.)
- Students are able to think about how to calculate subtraction with minuends up to and including 10 by paying attention to the structure (composition & decomposition) of the 1-digit number and are able to manipulate concrete materials to represent and express the calculation process. (thinking, justification, expression, etc.)
- Students are able to identify the *take from*, *take apart*, and *compare* problem situations in daily life phenomena. They understand the merit of expressing these situations using math sentences and are eager to recognize and use subtraction in daily life. (attitude for learning, human nature, etc.)

3. Connection between this Subtraction Unit and the School’s General Research Goals

(1) Image of Students Learning that “Approach the Essence of Mathematics” in this Unit

The mathematical views and thinking that I would like students to perform are: Students use concrete materials, diagrams, and subtraction math sentences to think about how to manipulate concrete representations and how to represent a problem situation with subtraction math sentences to find the answer. In the process of thinking through this

lesson, I also hope to see the students reflecting back on their process of finding the answer and attempting to explain and discuss why they thought about the problem in this way.

In this unit, at first, students learn about *take from* and *take apart* problem situations. I will help students understand that subtraction can be used in the *compare* situation by providing instruction that leads students to consciously consider how *compare* situations expand the meaning of subtraction. In each subtraction situation, I will help students understand the meaning of subtraction and its mathematical terms and signs; and, finally, help students to be able to perform subtraction with minuends up to and including 10. Through their learning in the previous addition unit and this subtraction unit, my goal is to help students develop a solid understanding of the concept of numbers and make connections with the meanings of subtraction and subtraction math sentences.

In our mathematics department, we believe that we can approach the essence of the mathematics department's goals by deepening students' learning as they actively apply various views of and thinking about mathematics. The image of student learning in this unit includes the following: a.) Students will think about why they can use subtraction in a particular problem situation and the meaning of an operation while manipulating concrete materials; b.) Students will be able to find the similarities and differences across their learning about addition situations and the subtraction situations of *take from* and *take apart*; and c.) finally, students wonder and think further about whether there is another situation that can be represented by subtraction, etc. In this unit, when students' experience these kinds of learning, I believe they will perform well when they extend their thinking and learning in the future, such as how to add and subtract with regrouping, and grasping the meaning of other operations, such as multiplication and division.

(2) About Necessary Measures for a Lesson to Approach the Essence of Math Department Goals

① Create each lesson with questions that students should be asking.

Our mathematics department cares about lessons that focus on students' questions. In this unit, the following questions should surface during the lesson: "What is similar and what is different about these calculations, compared to calculations we learned previously?" "How should we manipulate the concrete materials?" "What kind of math sentence should we write?" "What are the similarities among our ideas or thinking?" "If we change the number will the subtraction calculations still work?" By placing these questions at the center of the lesson, as a driving focus, we constructed the lesson.

② Constructing Lessons that Consider Mathematical Views and Thinking

When student think about how to calculate something, it is important for them to understand the meaning of the calculation they are considering. In order for students to be able to do this reflection, we need to provide more opportunities for them to think about how to calculate by utilizing what they have learned previously. Then we need to help them to be able to explain their thinking and calculations using concrete materials, diagrams, and math sentences. Moreover, we need to help students increase their expression and use of important mathematical words to support their ability to conduct and learn mathematically.

③ Explore Students' Activities that Reflect What Their Own Learning Should Be

It is important for students and teachers to develop and demonstrate the disposition to reflect on one's own learning. Because these students are in the first grade, I will

create opportunities for them to reflect intentionally and purposefully during lessons. By doing so, I believe that we can foster students who are able to identify situations when they need to reflect consciously on their own learning in future lessons. One measure we can monitor is whether students are able to reflect on their own learning by making sure they write learning reflections in their math notebooks. Therefore, I will plan the lesson, so it shows students' reflecting on their own learning by looking back to consider what they learned during the lesson and writing a reflection.

4. Instruction and Assessment Plan (Total of 8 Lessons)

Assessment: knowledge and skills; Thinking, justification, and expression; Disposition for active learning

Lessons	Goals	Learning Activities	Views and Thinking	Assessment
What is left?				
1	Understand the meaning of a <i>take from</i> situation and how to express the situation in a math sentence.	<ul style="list-style-type: none"> Grasp the problem situation as finding what it left (<i>take from</i>) and represent the situation by manipulating concrete materials and diagrams. Express the <i>take from</i> situation in a math sentence. Know the meaning of the term "subtraction." 	Think about the meaning of subtraction by attending to numbers as sets.	<input type="checkbox"/> Identify a problem situation from daily life as a <i>take from</i> situation; and try to express it in a math sentence from the manipulating of concrete materials. <input type="checkbox"/> Understand the meaning of subtraction in <i>take from</i> situations and know how to express <i>take from</i> subtraction in a math sentence.
2	Are able to do subtraction calculations with minuends up to and including 10.	<ul style="list-style-type: none"> Practice subtraction calculations with minuends up to and including 10. 		<input type="checkbox"/> Be able to do subtraction calculations with minuends up to and including 10.
How many are there when you subtract?				
3	Understand the meaning of a <i>take apart</i> situation and how to express the situation in a math sentence.	<ul style="list-style-type: none"> Grasp the problem situation as <i>take apart</i> and represent the situation by manipulating concrete materials and diagrams. Express the <i>take apart</i> situation in a math sentence 	Think about the <i>take apart</i> situation as similar to the <i>take from</i> situations and unify them as subtraction situations.	<input type="checkbox"/> Identify a problem situation from daily life as a <i>take apart</i> situation, and try to express it in a math sentence from the manipulating of concrete materials. <input type="checkbox"/> Understand the meaning of the <i>take apart</i> situation and know how to express <i>take apart</i> subtraction with a math sentence.

4	Improve subtraction calculation skills	<ul style="list-style-type: none"> Practice subtraction calculations with minuends up to and including 10 using calculation cards. Look at how minuends and subtrahends appear on subtraction cards organized on a chart, through activities such as identifying missing cards and finding the cards that have the same answers. Understand a number can be represented as a difference of two numbers. 		<input checked="" type="checkbox"/> Be able to do subtraction calculations with minuends up to and including 10.
Subtraction with 0 (zero)				
5	Understand the meaning of subtraction calculations involving zero (0).	<ul style="list-style-type: none"> Express problem situations that involve 0 in math sentences and understand the meaning of the problem situations and sentences. 		<input checked="" type="checkbox"/> Understand that even when problem situations involve 0, the situations can be expressed in subtraction math sentences.
What is the difference?				
6 (This Lesson)	Understand the meaning of subtraction in a <i>compare</i> situation.	<ul style="list-style-type: none"> Identify the situation as a <i>compare</i> problem situation and express the situation by manipulating concrete materials and using diagrams Express the <i>compare</i> situation in a math sentence. 	Knowing two numbers as two sets, think about how to connect and unify the <i>compare</i> situation (finding the difference between two sets) with the subtraction situations that were learned previously.	<input checked="" type="checkbox"/> Identify a problem situation from daily life as a <i>compare</i> situation and try to express the situation in a math sentence by manipulating concrete materials. <input checked="" type="checkbox"/> Understand the meaning of subtraction in a <i>compare</i> situation and know how to express it in a math sentence.
7	Deepen understanding of the meaning of the <i>compare</i> subtraction situation by solving word problems.	<ul style="list-style-type: none"> From word problems and pictures of situations, think about how to answer questions, such as “Which one has more, how many more?” and “What is the difference between ○ and △?” Confirm the problem situation is a <i>compare</i> situation using manipulation of concrete materials or diagrams and expressing the situation in a math sentence. 		<input checked="" type="checkbox"/> Identify the problem situation from the written problem as <i>compare</i> situation, represent the situation with math sentence, and be able to solve the problem.

Creating story problems				
8	Deepen understanding of subtraction by creating problem situations by and interpreting a math sentence.	<ul style="list-style-type: none"> • Create problems by looking at a picture from various view points; and identify <i>take from</i>, <i>take apart</i>, and <i>compare</i> situations by interpreting a math sentence. • Express subtraction situations using pictures and math sentences; present one's work to the class. 		<input type="checkbox"/> Try to create subtraction problems and stories based on their daily life experiences. <input type="checkbox"/> Able to identify subtraction situations from a picture. Think about subtraction problems and be able to express them using pictures, diagrams, and words.

5. About this Lesson

(1) Date and Time: 9:00 a.m. to 9:45 a.m. on Saturday, June 22, 2019

(2) Place: Grade 1, Number 2 classroom, University of Yamanashi Faculty of Education Attached Elementary School

(3) Goal of this Lesson:

○ Students understand the meaning of subtraction in the *compare* problem situation.

(4) Intention and Purpose of Instruction:

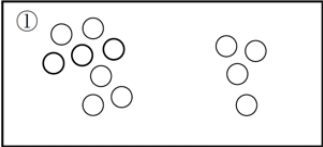
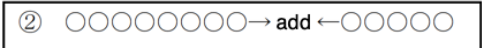

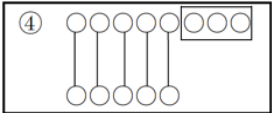
The image of students who “approach to the essence of mathematics” that I would like to see in this lesson includes: students who can identify the difference between addition and subtraction; students who understand the difference between *take from* and *take apart* problem situations; and students who are able to use subtraction appropriately to represent and solve *compare* problem situations.

In this lesson I will start with students recognizing the difference between *take from/take apart* situations, which students learned previously, and *compare* problem situations. The subtraction the students learned prior to this lesson pertained to the problem situation that deals with one (whole) set that is split into two subsets (parts), and subtraction is used to solve and find one of the subsets. However, in this *compare* lesson, the problem involves two distinct sets and students need to find the difference between the two sets. In the previous lessons, it was easier for students to identify “what is left” when they manipulated concrete materials. The big difference for students in today’s subtraction situation is that they will need to think about the number of objects that cannot be aligned to make a one-to-one correspondence when objects in the two sets are compared. In a *compare* situation, manipulation of concrete objects is different from the *take from* and *take apart* situations; however, I would like to help students understand that - even though the problem is a *compare* situation - they can use the subtraction operation just as they used subtraction for *take from* and *take apart* situations.

The questions students should be asking in this *compare* problem situation lesson are, “How should we manipulate concrete materials?” and “What math sentence should we use?” The anticipated responses for the question, “How should we manipulate concrete materials?” include: “This situation is different from *take from* and *take away* situations;” and “It looks like manipulation of concrete materials will be different from what we did with *take from* and *take apart* situations.” The anticipated response for the question “What math

sentence should we use?” include: “When we take away the parts that are connected with lines (one-to-one correspondence between two sets), we can find the answer;” and “We are taking away the part that is connected with lines, so we can use subtraction.” By connecting the questions that should be asked to bring out student responses (as mentioned above), I will help students recognize the commonality and the difference between how blocks are manipulated in *take from*/*take apart* situations versus a *compare* situation. Moreover, I will help students understand that subtraction can be used to represent and solve a *compare* situation just as they used subtraction in *take from* and *take apart* situations.

(5) Progress of Learning:

Minutes	Main Content and Learning Activities ○ Student Anticipated Responses	Instructional Points to Remember ○ Ideas for making the “lesson approach the essence”
5	<p>1. Grasp the task of this lesson</p> <ul style="list-style-type: none"> Grasp today’s problem by looking at the picture and reading the problem <div data-bbox="296 846 1058 1008" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Teachers played <i>Karuta</i> Japanese card games. Mr. Nobuyuki took 8 cards. Mr. Yukinao took 5 cards. How many more cards did Mr. Nobuyuki get than Mr. Yukinao?</p> </div> <p>○ 3 more. ○ I don’t know.</p> <div data-bbox="387 1126 1149 1182" style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>How did you think about solving the problem?</p> </div>	<p>○ By posing the problem with a picture, help students grasp what they know and what they need to find out.</p>
5	<p>2. Solve the problem on their own</p> <ul style="list-style-type: none"> Think about how to solve the problem. <p>① Student thinks about the problem using a picture, diagram, or blocks, but does not successfully find the answer.</p> <p>② Student added the minuend 8 and the subtrahend 5 and found the answer 13.</p> <p>③ Student subtracted the answer 3 from the minuend 8.</p> <p>③ Student made one-to-one correspondence using a picture, diagram, or blocks and found the answer 3.</p>	<ul style="list-style-type: none"> Help students to be able to explain a student’s idea to classmates, i.e., how the student solved the problem. Help students have foresight about how they could solve the problem (e.g., blocks, picture, diagram, draw lines for one-to-one correspondence, etc.) <div data-bbox="892 1476 1217 1626" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>① </p> </div> <div data-bbox="892 1675 1377 1727" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>② </p> </div> <div data-bbox="892 1771 1334 1825" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>③ </p> </div> <div data-bbox="892 1872 1166 1989" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>④ </p> </div>

<p>25</p>	<p>3. Compare and discuss</p> <ul style="list-style-type: none"> • Think about how to arrange the blocks to represent the problem situation. <p>How should we arrange the blocks?</p> <ul style="list-style-type: none"> ○ I think we can line them up. ○ I think we can put blocks on top of each other. <ul style="list-style-type: none"> • Discuss the method that added the minuend 8 and the subtrahend 5 to get an answer of 13. <p>I wonder if the answer 13 is correct.</p> <ul style="list-style-type: none"> ○ If the answer is 13, it is different from the answer 3 that I said. ○ It is strange because the number of cards, 13, is more than the cards each teacher got. ○ I wonder if the answer is 3. <ul style="list-style-type: none"> • Rethink about how to arrange the blocks. <p>I wonder why the number of cards became more.</p> <ul style="list-style-type: none"> ○ I think the arrangement of blocks is different. ○ If you put 8 cards and 5 cards together that means you are adding the numbers of two teachers' cards, so it is not correct. <ul style="list-style-type: none"> • Think about the problem by showing one-to-one correspondence. <p>I wonder how we can rearrange the blocks.</p> <ul style="list-style-type: none"> ○ Let's rearrange the blocks. ○ Let's put the blocks on top of each other. ○ Connect corresponding blocks with lines. ○ When we connect corresponding blocks with lines, it is easy to tell who has more cards. <ul style="list-style-type: none"> • Think about the difference of numbers when one-to-one correspondence is made. <p>What are the numbers 3, 5, and 8 referring to?</p> <ul style="list-style-type: none"> ○ 8 is the number of cards Mr. Nobuyuki got. 5 is the number of cards Mr. Yukinao got. ○ There are 3 blocks more where one of the lines of blocks sticks out. ○ The part where we can't connect lines has 3 blocks. ○ Mr. Nobuyuki has 3 more cards. ○ When we subtract 5 blocks that are connected to Mr. Nobuyuki's 8 blocks, 3 blocks are left. <ul style="list-style-type: none"> • Think about whether subtraction can be used to represent the problem situation. <p>What math sentence should we write?</p> <ul style="list-style-type: none"> ○ We are taking away the part that is connected by lines, so the math sentence should be subtraction. ○ It looks like we can use subtraction. 	<ul style="list-style-type: none"> ○ Students manipulate blocks and they explain what they did, which helps them to notice a way to solve the problem. <ul style="list-style-type: none"> • Start the presentation from idea ①, and help rearrange the blocks in order to see the difference easily through class discussion. <ul style="list-style-type: none"> • Bring up idea ② and help students to think about what is wrong about this idea by helping them notice that the number of cards (13) is actually more than each teacher's number of cards to start, which does not match the problem situation. <ul style="list-style-type: none"> ○ Think about the difference from addition. <ul style="list-style-type: none"> • Confirm with students that the problem is asking to find "how many more cards," so if they add 8 and 5, it does not match what the problem is asking them to find out. So, they cannot solve the problem and find the answer by using addition. <ul style="list-style-type: none"> • Bring out idea ④ and facilitate an opportunity to talk about why this idea involves making one-to-one correspondence. <ul style="list-style-type: none"> ○ Recall what students did when they were comparing the length of two things and, likewise, to think about how they need to manipulate these blocks so they can compare the number of blocks using one-to-one correspondence. <ul style="list-style-type: none"> ○ Ask students what the numbers 3, 5, 8 represents and help them to see that it is the "difference" of 3 that the problem is asking for. <ul style="list-style-type: none"> ○ Listen for students who voice "taking away" and connect the words with the subtraction operation.
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	<ul style="list-style-type: none"> ○ $8 - 3$ ○ It is strange to subtract the answer 3. ○ $8 - 10$ ○ It is strange because you can't subtract 10 from 8. ○ $8 - 5$ ○ 8 is Mr. Nobuyuki's cards, 5 is Mr. Yukinao's cards, so we can take away the 5 that are connected with lines. ○ The answer is 3. ○ "Take away" means subtraction. ○ The difference can be found with subtraction just like "take away." <p>What is similar about everyone's ideas?</p> ○ "Take away" is the same idea. ○ We learned about "Take away (take from)" in the past. 	<ul style="list-style-type: none"> • Confirm with students that we can use subtraction for this problem, then ask them to think about the math sentence. • Discuss if we need to subtract 5 that are in pairs in one-to-one correspondence or if we need to subtract the 10 that are included in the pairs of one-to-one correspondence. Then, help students recognize that $8 - 5$ can be used to find the part that represents the "how many more." ○ Help students pay attention to which part should be taken away when one-to-one correspondence is made. ○ Help students to recall how "put together" and "add to" situations were expressed in addition and "take from" and "take apart" situations were expressed in subtraction. Then, think about how and why subtraction can be used to express "compare" situations. • The wording of the problem is different from "take from" and "take apart," but the manipulation of the blocks is the same (because we take away the paired "part" that is connected with lines of one-to-one correspondence)
10	<p>4. Look back</p> <ul style="list-style-type: none"> • Summarize today's lesson What situations can we solve using subtraction? ○ When we find what is left. ○ When we take away blocks. ○ When we split something into "how many and how many" ○ When we take away the part connected with lines (the part where we showed one-to-one correspondence). ○ When we think about how many more. • Students write their learning reflection. ○ "How many more" is in the kinds of subtraction (similar to <i>take from</i> and <i>take apart</i>) ○ When we take away the part connected with lines, we can find the answer. ○ We can use subtraction other than "take away" and "take apart" situations. 	<ul style="list-style-type: none"> ○ List up the problem situations that subtraction is used and help students to notice how many different situations that subtraction is used. ○ Students write learning reflection by looking back learning from today's lesson.

6. Observation Points of the Lesson

- Are the students understanding correctly that subtraction should be used to solve *compare* problem situations? Are the students able to express the *compare* problem situation with manipulation of concrete materials and in their own words.
 - Utilizing questions that students should be thinking of or asking, how is the teacher helping to bring out the mathematical views and thinking that are necessary to deepen students' understanding?
 - Did we hear words or questions related to mathematical views and thinking from the students?
 - Were the reflections that students demonstrated during the lesson functioning well enough to deepen their learning?

7. Reference

- 小学校学習指導要領(平成29年告示)解説 [Teaching Guide of the Course of Study for Japan (2018)]
- 『新編あたらしいさんすう』研究編 [Research Volume, Teacher Edition of the Revised Elementary Mathematics textbook] 東京書籍