∞ IMPULS 国際算数数学授業研究プロジェクト
International Math-teacher Professionalization Using Lesson Study

Lower Secondary Grade 1 (Grade 7) Mathematics Lesson Plan

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1. Title of Unit: Positive and Negative Numbers

2: About the Unit:

In this unit, students' number world will expand from non-negative rational numbers they learned in elementary schools to the entire rational numbers. Up to this point, not all subtraction problems were solvable; however, in this new number world, all four arithmetic operations are possible all the time. In lower secondary school, solidifying students' understanding of rational numbers is a central focus; therefore, this unit has a particular importance as the foundation of mathematics learning in lower secondary school.

With the introduction of negative numbers, students will learn that subtraction may be changed to addition because of the existence of "a number that will make the sum of 0" (additive inverse). Students have learned that division may be changed to multiplication because of the existence of "a number that will make the product of 1" (multiplicative inverse). Although terms such as "identity" or "inverse" are not a part of the instructional content, it is hoped that students will understand that addition and multiplication have the same structure by recognizing these types of numbers exist with respect to each operation. Such an understanding may lead to students' realization that steps in solving equations -- such as transforming equations in the form, ax = b, or changing the coefficient of x to 1 -- utilize the additive and multiplicative inverses. With respect to the four arithmetic operation, representations with number lines will be used to help students make sense of the meaning of the operations and to deepen their understanding.

Positive and negative numbers are also used in our everyday life. Thus, in the unit, we will incorporate activities to identify situations in our lives where positive and negative numbers are used. In addition, by applying positive and negative numbers in problem solving, we want to develop the disposition to seek the merit of using mathematics (positive and negative numbers in this case) such as simplifying calculation and easily grasping the differences from the point of reference.

3. Goals of Unit:

1. Students will be interested in thinking and representing various phenomena mathematically by grasping them using positive and negative numbers and discovering their characteristics and properties. They will also actively use mathematical ideas in reasoning and making judgments as they solve problems.
2. Students will be able to reason clearly and logically and represent phenomena using their knowledge and skills of fundamental patterns and relationships of positive and negative numbers. They can also deepen their understanding by reflecting on their reasoning.
3. Students will master the ability to calculate with positive and negative numbers. They can also use expressions and equations with positive and negative numbers as representation tools and interpret them.
4. Students will understand the meaning and need for positive and negative numbers. They also understand the meaning of the four arithmetic operations with positive negative numbers and master the calculation skills.

4. Evaluation Standards for the Unit:

Interest, Eagerness, and Attitude Toward Mathematics	Mathematical Way of Thinking	Mathematical Skill	Knowledge and Understanding Regarding Numbers, Quantities and Geometrical Figures
Needs for and meaning of positive and negative numbers			
Students will be interested in positive and negative numbers and think about their needs and meaning. They will try to represent various phenomena in their surroundings using positive and negative numbers.	Students will be able to think about how positive and negative numbers may be used by identifying specific situations in which positive and negative numbers are used such as expressing the differences of high temperature between yesterday and today.	Students will be able to represent various phenomena in their surroundings using positive and negative numbers. Students will be able to represent characteristics and directions that are opposite of each other using positive and negative numbers. Students can represent positive and negative numbers on a number line and express their relationships using the equal and inequality signs.	Students understand the need for and the meaning of positive and negative numbers. Students understand the meaning of the size relationship of various numbers (natural numbers, whole numbers, positive and negative numbers), the meaning of positive and negative signs, and the meaning of absolute values.
Meaning of four arithmetic operations and calculations			
Students will be interested in the four arithmetic operations with positive and negative numbers. Students will think about ways to calculate with positive and negative numbers and carry out the calculations.	Students will be able to figure out ways to calculate with positive and negative numbers based on their previous knowledge of calculations. By expanding the range of numbers to include both positive and negative numbers, students will be able to consider addition and subtraction operations from a unified perspective. As a result, students can consider an expression involving both addition and subtraction as a sum of terms with positive and negative terms.	Students will be able to calculate with positive and negative numbers. Students will be able to represent an expression with both addition and subtraction operations as a sum of positive and negative terms.	Students understand ways to calculate with positive and negative numbers. Students understand that addition and subtraction operations can be considered from a unified perspective by expanding the numbers to include positive and negative numbers.
Processing and representing with positive and negative numbers.			
Students will be interested in using positive and negative numbers. They will try to represent and process various phenomena using positive and negative numbers.	Students can examine various phenomena and situations involving changes by using positive and negative numbers to express the differences from the established target value.	Using positive and negative numbers, students will be able to represent and process various phenomena in their surroundings such as determining the arithmetic mean using an estimated mean.	Students understand that some phenomena and situations involving changes may be represented clearly or processed efficiently by using positive and negative numbers.

5. Unit Plan (Total of 28 Lessons):

Sub-Units	# of lessons	
1. Positive and negative numbers	5	
2. Addition and subtraction	8	
3. Multiplication and division	11	28 total
4. Using positive and negative numbers (today's lesson)	1	
5. Projects	2	
Summary of the unit	1	

6. Today's Lesson:

1. ***Date:*** Thursday, June 27, 2012   14:10 ~ 15:00

2. ***Location:***

>   Lower Secondary School Attached to College of Education of
>   Yamanashi University, Red Brick Building

3. ***Title:*** Let's split the team into two groups with an equal average height

4. ***Objectives:***
   - Students will learn that the calculation of arithmetic mean will be simplified by using the idea of tentative average and positive/negative numbers.
   - Students will foster the disposition to use mathematics in problem solving by learning about the merits of the idea of tentative average and the use of positive/negative numbers.

5. ***Strategies for helping students develop own questions:***

   In order to create combinations that have the equal average height, the average height for each combination must be calculated. By making students experience the tediousness of the process may prompt students to ask, "Is there a way to make the calculation simpler?" Furthermore, by selecting the numbers (heights) so that there are more than one way to split the team into two groups with an equal average height, the need to repeat the process again even after students find one combinations that have the equal average will emerge. That will make it even more likely for students to ask, "Is there a way to make the calculation simpler?" In this lesson, both of these points were considered to create the main task.

   While students are working on the task, those pairs who are using the differences between the average height and each data point will be identified and noted. If no such pair is present, those who are focusing on the average value in their attempts. That is because if they are focusing on the average height, the chance is good that they are thinking about the differences between the height and players' heights. By bringing out those students' ideas, it is hoped that the class can experience the merits of using the differences between the average and individual data points as the numbers will become smaller, thus easier to process. By making the merit of using the differences between the average and individual data points a shared understanding, it is aimed to make a connection to the main question for the lesson, "Is there a way to make calculation simpler?"

6. Flow of the lesson:

Step	Instructional Activity	Anticipated Students' Responses	Notes
Introduction	1. Understand the problem situation (3 min.) **[Problem]** There are 10 new members on our school basketball team. In order to evaluate the skill levels of these new members, Mr. Sakuragi who is the faculty sponsor of the team decided to have them play intra-squad games by creating two teams, X and Y. Mr. Sakuragi figured out 3 different ways to create two teams so that the average heights are the same, and they played 3 intra-squad games. How did Mr. Sakuragi made these teams. Here are the heights of the players. <table><tr><td>Name</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td></tr><tr><td>Height (cm)</td><td>166</td><td>164</td><td>161</td><td>156</td><td>153</td><td>151</td><td>150</td><td>146</td><td>143</td><td>138</td></tr></table>	• The players are listed in order of their heights, from the tallest to the shortest.	• Distribute the sheets with the problem. • Check to see if there is anything that is unclear.
Development	2. Tackle the problem in pairs. (10 min.)	1. Distribute the players alternatively from the tallest to the shortest. Team X: A, D, E, H & I Average Height 152.8 cm Team Y: B, C, F, G, & J Average Height 152.8 cm 2. Pair up the tallest and the shortest and assign them to a team. Then, pair the tallest and the shortest of the remaining players and assign them to the other team. Repeat this process. Team X: [A, J], [G, H] & E Average Height 152.8 cm Team Y: [B, I], [D, G] & F Average Height 152.8 cm 3. Create a group of 4 players by selecting the two tallest and two shortest players. Assign them to team X. Do the same with the remaining players and assign them to team Y. Assign the tallest of the remaining to team X and the last player to team Y. Team X: [A, B, J, I] & E Average Height 152.8 cm Team Y: [C, D, H, G] & F Average Height 152.8 cm 4. Distribute the players alternatively starting with the player whose height is closest to the average height of the 10 players, 152.8cm. Team X: E, G, C, I, & A Average Height 152.8 cm Team Y: F, D, H, B, & J Average Height 152.8 cm	• Have students sitting next to each other work together. If any student is missing, form a group of 3. • Distribute graphing calculators, one to each pair. • For sharing, have each pair write their response on a magnetic white board. • Identify pairs who are using either the minimum (138) or the average (152.8) as the base and representing other data points by their differences from the base.

Development	3. Sharing (12 min.)	• Share the solutions 1 ~ 4 above. • It is tedious to calculate the average height of each team every time we try a different way to split the tplayers into two groups. • Is there an easier way to calculate the average?	• Incorrect or incomplete solutions should also be shared. • Ask students "What was challenging as you tried to solve the problems?"
Deepening	4. Look for easier ways (15 min.) [Task] To make two teams with equal average height, is there ways to simplify the calculations?		
		• Have students share the idea of using a base value and re-organize the table using the differences from the base. 　a. Using the average for the 10 players (152.8) as the base (0), express all data points using positive/negative numbers. 　b. Using 153 as the base (0), express all data points using positive/negative numbers. 　c. Using 150 as the base (0), express all data points using positive/negative numbers. 　d. Using 138 (minimum value) as the base (0), express all data points using positive/negative numbers. • The numbers become smaller and that makes calculations simpler. • By using positive and negative numbers, the sums become smaller because + and - cancel out and that makes calculation simpler.	• Ask students, "What's the merit of re-organizing the table using a base value?" **"What is the merit of re-organizing the table?" (explore merits)**
Summary	5. Think about the merits of using the idea of tentative average and positive/negative numbers. (10 min.) • Journal writing	• The idea of tentative average makes the numbers smaller and make the calculation simpler. • By using positive and negative numbers, the sum of values become smaller, and the smaller values are easier to calculate.	• Explain the idea of tentative average.