2013 Grade 9 Mathematics Set A

The English translation is prepared by the Project IMPULS at Tokyo Gakugei University, Tokyo, Japan. (http://www.impuls-tgu.org/)

[1] Answer the following questions (1) through (4).

(1) Calculate $\frac{5}{8} \times \frac{3}{4}$.

(2) Calculate $5 \times (4 - 7)$.

(3) If a and b are whole numbers, which of the following calculations (a) through (d) may not result in a whole number? Select the correct expression. In division, assume that the divisor is not 0.

(a) $a + b$

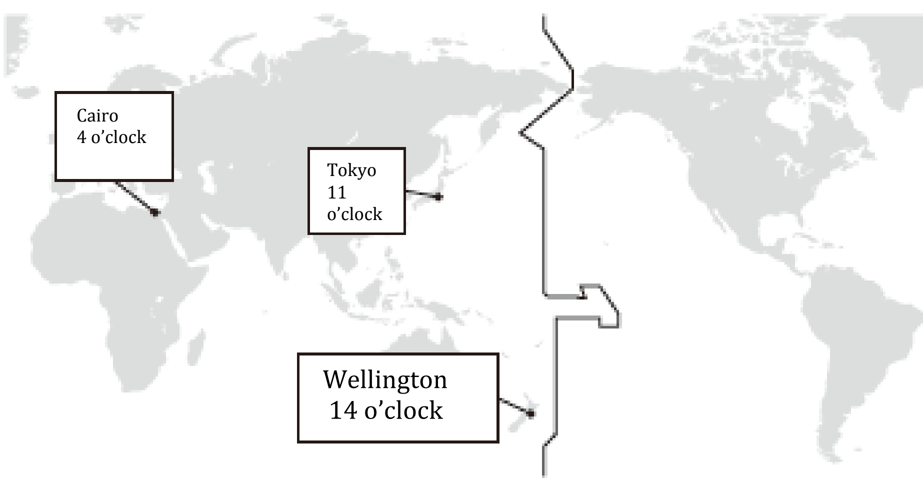
(b) $a - b$

(c) $a \times b$

(d) $a \div b$

(4) The picture below shows the time in Cairo and Wellington when it is 11 o'clock in Tokyo. By using integers, the time difference between Tokyo and a city located to the east of Tokyo but not crossing the International Date Line can be expressed by a positive number, while the time difference between Tokyo and a city located to the west of Tokyo can be expressed by a negative number. For example, the city of Wellington is located to the east of Tokyo, the time difference between Tokyo and Wellington can be expressed as +3 hours.

Using the time in Tokyo as the base, express the time difference between Tokyo and Cairo.

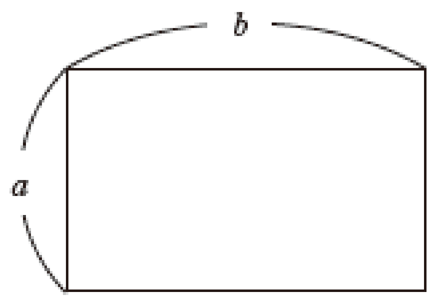


International Date Line

[2] Answer the following questions (1) through (4).

(1) Calculate $2(5x + 9y) - 5(2x + 3y)$

(2) There is a rectangle whose vertical length is *a* and whose horizontal length is *b*, as shown below.
What about this rectangle does the expression $2(a + b)$ represent? Select the correct answer from (a) through (e) below.



(a) the area of the rectangle

(b) double the area of the rectangle

(c) the perimeter of the rectangle

(d) double the perimeter of the rectangle

(e) the length of the diagonal of the rectangle

(3) A wire that is *a* meters long weighs *b* grams. How many grams is the weight of 1 meter of this wire? Write your answer as an expression with *a* and *b*.

(4) The equation $2x + 3y = 9$ can be solved for *y* as shown below.

$$2x + 3y = 9$$
$$3y = 9 - 2x \quad \cdots\cdots ①$$
$$y = \frac{9-2x}{3} \quad \cdots\cdots ②$$

Which of the following statements (a) through (d) is the justification for transforming equation ① to equation ②? Select the correct answer.

(a) Even when 3 is added to both sides of equation ①, the equality remains true. Therefore, we can transform equation ① to equation ②.

(b) Even when 3 is subtracted from both sides of equation ①, the equality remains true. Therefore, we can transform equation ① to equation ②.

(c) Even when both sides of equation ① are multiplied by 3, the equality remains true. Therefore, we can transform equation ① to equation ②.

(d) Even when both sides of equation ① are divided by 3, the equality remains true. Therefore, we can transform equation ① to equation ②.

[3] Answer the following questions (1) through (3).

(1) Solve the linear equation, $3x + 7 = 9$.

(2) Select a solution of the equation, $2x + y = 6$ from (a) through (d) below.

(a) $x = 4, y = 1$

(b) $x = 2, y = 1$

(c) $x = 1, y = 4$

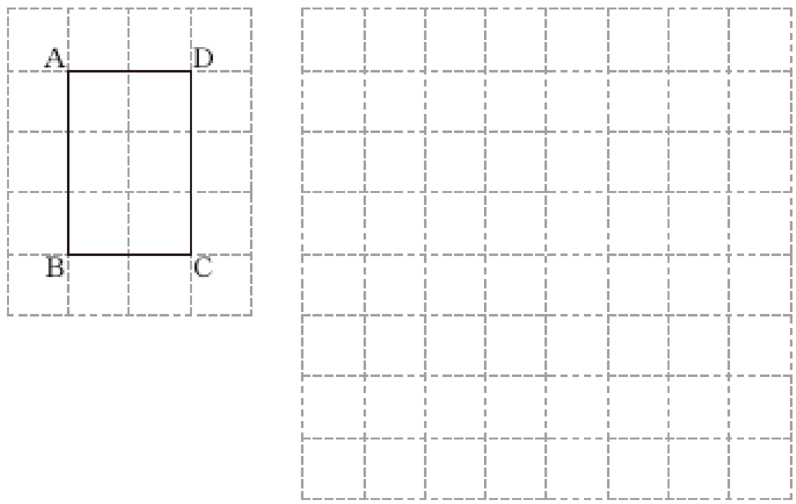
(d) $x = 1, y = 8$

(3) If you buy 3 notebooks and 2 pencils, the cost will be 460 yen. If you buy 4 of the same notebooks and 3 of the same pencils, the cost is 630 yen.

To determine the price of 1 notebook and the price of 1 pencil, write a system of equations by considering the price of 1 notebook to be x yen and the price of 1 pencil to be y yen. You do not have to solve the system of equations you write.

[4] Answer the following questions (1) through (3).

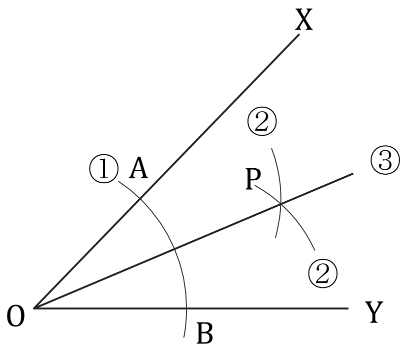
(1) Draw an enlarged figure of rectangle ABCD shown below using the grids on the answer sheet.



(2) We have drawn the angle bisector of ∠XOY using the following steps.

Construction steps.

1. Draw a circle of an arbitrary radius centered at O, and label the points of intersection with side OX and side OY as A and B, respectively.
2. Draw circles with the equal radius centered at points A and B, and label the point of their intersection as P.
3. Draw line OP.



The reason we can draw the angle bisector of ∠XOY using these steps is because quadrilateral AOBP has a particular property. The description of the quadrilateral is in the list of statements (a) through (e) below. Select the correct description.

(a) It is a line symmetric figure with line OP as the line of symmetry.

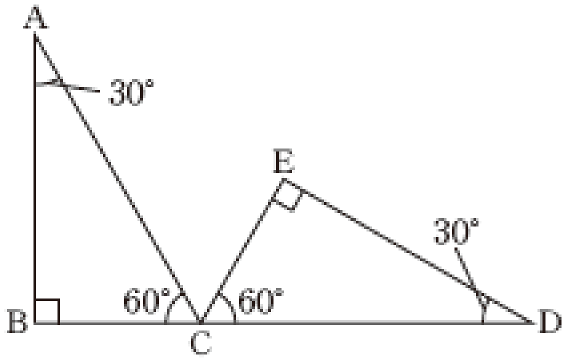
(b) It is a line symmetric figure with line OX as the line of symmetry.

(c) It is a line symmetric figure with line passing through points A and B as the line of symmetry.

(d) It is a point symmetric figure with point O as the center of symmetry.

(e) It is a point symmetric figure with the point of intersection of the line passing through points A and B and line OP.

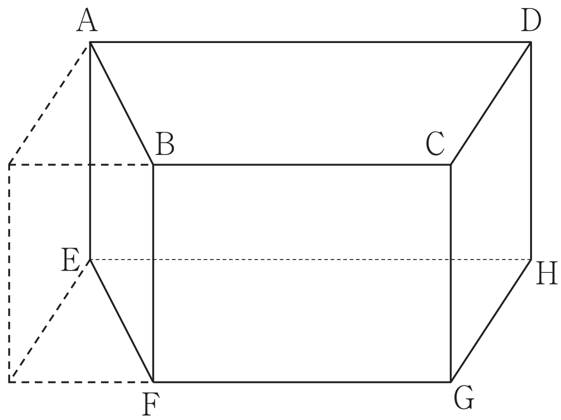
(3) As shown below, there is Δ ABC whose interior angles are 30°, 90°, and 60°, respectively, and Δ DEC that is congruent to Δ ABC. Points B, C, and D lie on the same line.



In order to have Δ ABC completely overlap Δ DEC by rotating Δ ABC around Point C, how many degrees does it have to be turned? Determine the measure of the angle of rotation.

[5] Answer the following questions (1) through (3).

(1) As shown below, there is a solid obtained by removing a triangular prism from a rectangular prism. From statements (a) through (d) below about the lines containing the edges of this solid, select the correct statement.



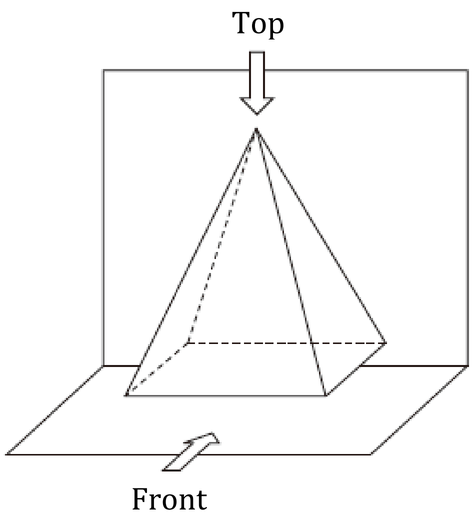
(a) Lines BF and DH will intersect.

(b) Lines BF and CG will intersect.

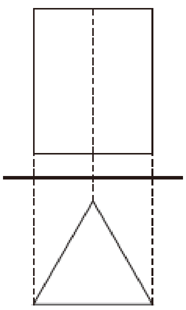
(c) Lines AB and EF will intersect.

(d) Lines AB and DC will intersect.

(2) The figure on the right is a sketch of a solid. The projection image of this solid is included in (a) through (e) below. Select the correct one.

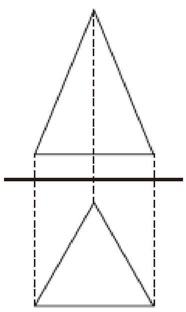


(a) Front view



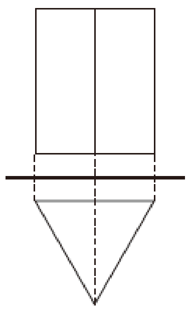
Top view

(b) Front view



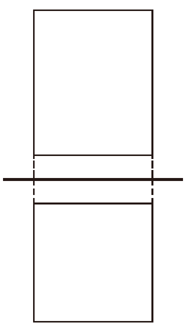
Top view

(c) Front view



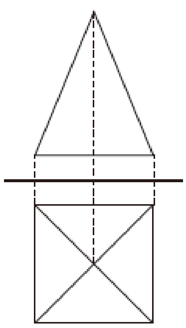
Top view

(d) Front view



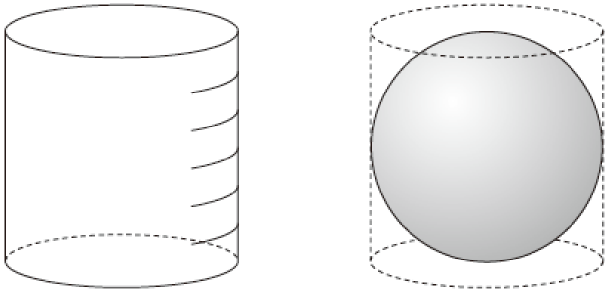
Top view

(e) Front view

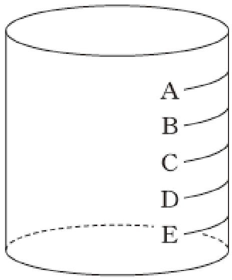


Top view

(3) As shown below, there is a cylinder whose base diameter is equal to its height and a sphere would fit exactly inside this cylinder. On this cylinder, there are marks that partition its height into 6 equal parts.



If we pour water whose amount is equal to the volume of the sphere, up to which mark on the cylinder will the surface of water reach? Select the correct answer from (a) through (e) below.



(a) Mark A

(b) Mark B

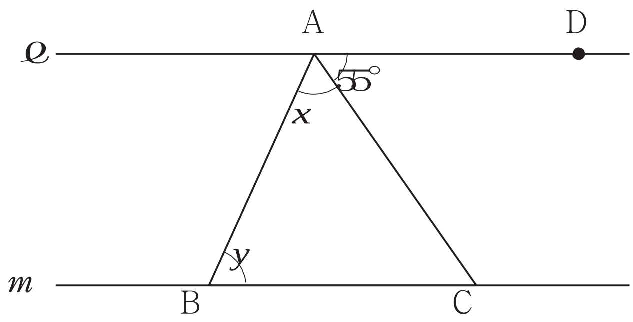
(c) Mark C

(d) Mark D

(e) Mark E

[6] Answer the following questions (1) and (2).

(1) In the diagram below, lines *l* and *m* are parallel. The measure of angle ∠DAC is 55°. How many degrees is $\angle x + \angle y$? Select the correct answer from (a) through (d) below.



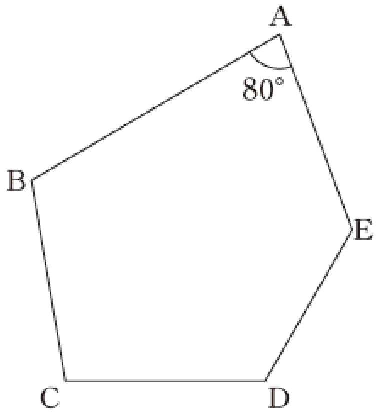
(a) 55°

(b) 110°

(c) 125°

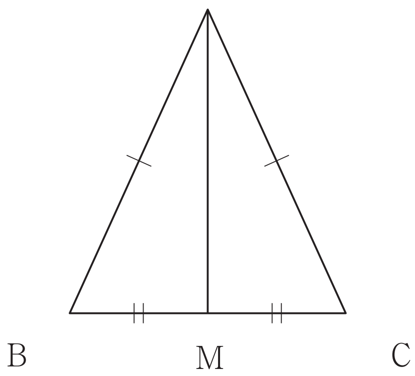
(d) 135°

(2) In pentagon ABCDE shown below, $\angle BAE = 80°$. Determine the measure of the external angle at vertex A.



[7] Answer the following questions (1) through (3).

(1) There is an isosceles triangle ABC where AB = AC.
M is the mid-point of side BC, and segment AM is drawn.
We prove below that ∠BAM = ∠CAM.



Proof

In △ABM and △ACM,
from what is given, AB = AC ·····①, and
BM = CM ·····②.
Since it is a common side, AM = AM ·····③.
From ①, ②, and ③, because [ ],
△ABM ≅ △ACM.
Because the corresponding angles of congruent figures are congruent,
∠BAM = ∠CAM.

In the proof above, select the appropriate congruence condition that will go into the [ ] from (a) through (e) below.

(a) three corresponding sides are congruent

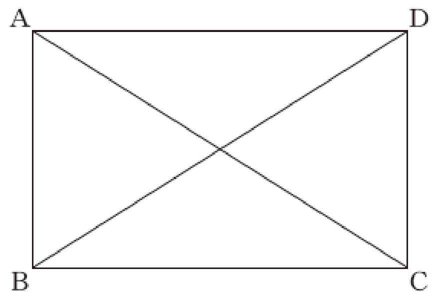
(b) two pairs of corresponding sides and the angles formed by them are congruent

(c) a pair of corresponding sides and two angles on their end points are congruent

(d) the hypotenuses and another pair of sides in right triangles are congruent

(e) the hypotenuses and a pair of corresponding acute angles are congruent

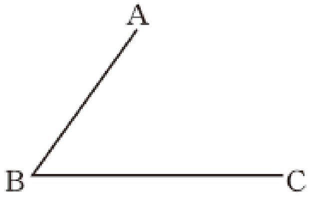
(2) In the figure below, quadrilateral ABCD is a rectangle.



Can we say that <u>the lengths of the diagonals are equal</u>?

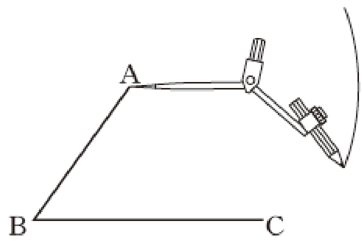
Express the underlined part of the sentence above using the letters of vertices and the = sign.

(3) As shown below, there are points A, B, and C. Points A and B are connected by a segment, and points B and C are connected by another segment.

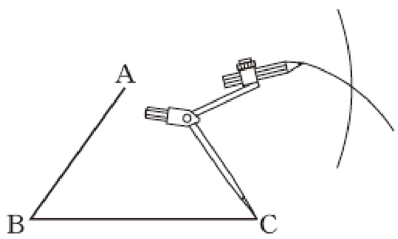


By following steps ①, ②, and ③ below, we will locate point D to draw parallelogram ABCD.

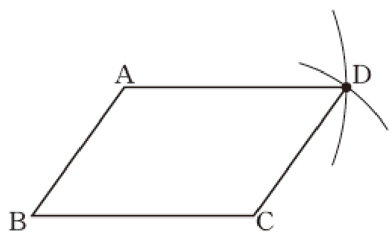
① Draw an arc centered at A with BC as the radius.



② Draw an arc centered at C with AB as the radius.



③ Let the point of intersection of the circles be D. Connect points A and D, and C and D.



In the steps ①, ②, and ③ above, what is the reason that quadrilateral ABCD is a parallelogram? From (a) through (e) below, select the correct answer.

(a) A quadrilateral with 2 pairs of parallel sides is a parallelogram.

(b) A quadrilateral with 2 pairs of congruent opposite sides is a parallelogram.

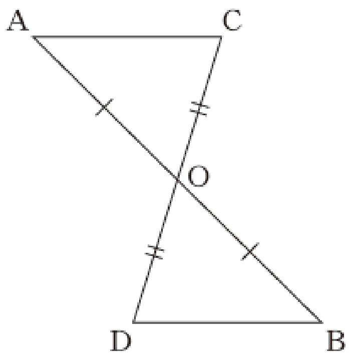
(c) A quadrilateral with 2 pairs of congruent opposite angles is a parallelogram.

(d) A quadrilateral with a pair of parallel and congruent sides is a parallelogram.

(e) A quadrilateral whose diagonals intersect at their mid-points is a parallelogram.

[8] Segments AB and CD are intersecting at their mid-point, O. In order to prove that AC = BD, a class used Figure 1 shown below.

Figure 1

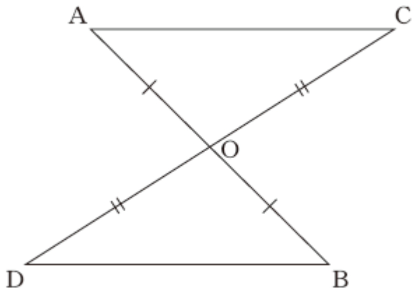


Proof

In △AOC and △BOD, from the given, AO = BO ·····①, and CO = DO ·····②. Since the vertical angles are congruent AM = AM ·····③. From ①, ②, and ③, because 2 pairs of sides and the angles formed by them are congruent, △AOC ≅ △BOD. Because the corresponding sides of congruent figures are congruent, AC = BD.

After the proof was completed, students drew a different figure, Figure 2, and thought ab

Figure 2



(a) The proof above has already shown that AC = BD in Figure 2 as well.

(b) We need to prove AC = BD again using Figure 2.

(c) With Figure 2, we need to prove AC = BD by actually measuring the length of their sides.

(d) In Figure 2, it is not true that AC = BD.

[9] In (a) through (e) below, there is a case where y is a function of x. Select the correct one.

(a) the area of the school yard, y m^2, for a school with x students

(b) the volume, y cm^3, of a rectangular prism with the base area, x cm^2

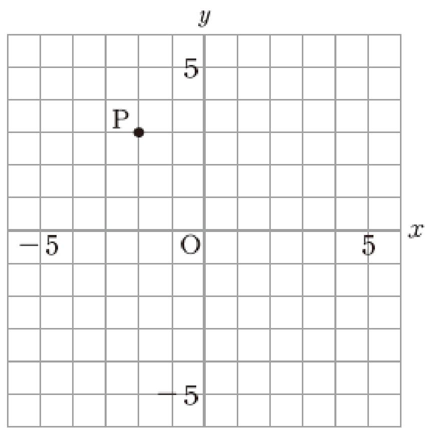
(c) the weight, y kg, of a person who is x cm tall

(d) a multiple, y, of a natural number, x

(e) the absolute value, y, of an integer, x

[10] Answer the following questions (1) through (4).

(1) In the figure below, write the coordinates of point P.



(2) From (a) through (e) below, select the equation that represent a proportional relationship with the proportional constant of 3.

(a) $y = 3x$

(b) $y = -3x$

(c) $y = 2x + 3$

(d) $y = -2x - 3$

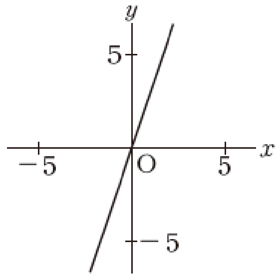
(e) $y = \frac{3}{x}$

(3) The table below shows that y is proportional to x.

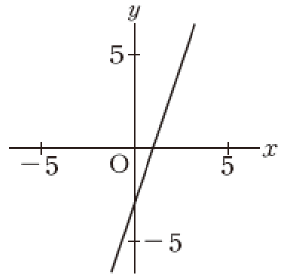
x	$\cdots$	1	2	3	4	$\cdots$
y	$\cdots$	-3	-6	-9	-12	$\cdots$

In (a) through (d) below, there is a graph that represents the relationship between x and y. Select the correct one.

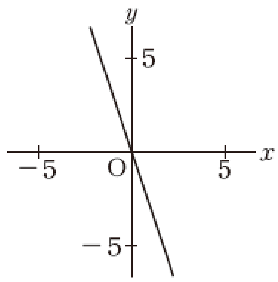
(a)



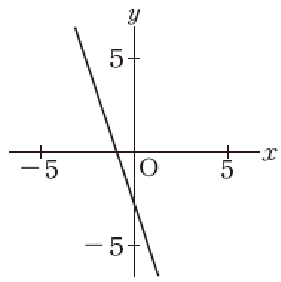
(b)



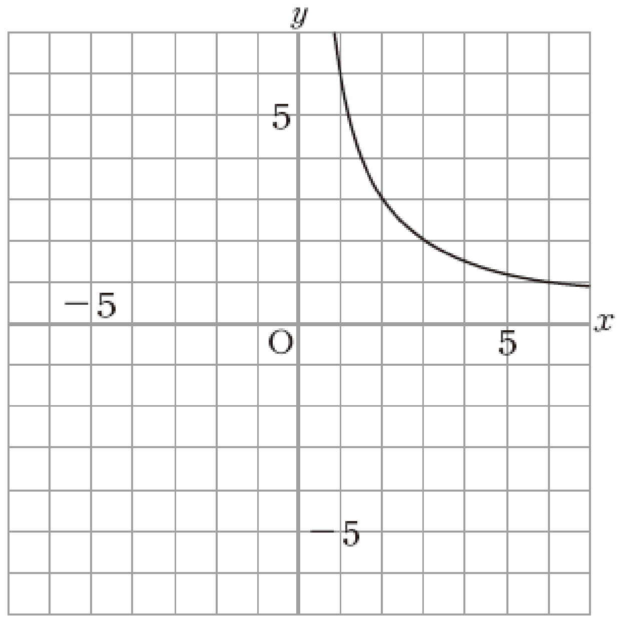
(c)



(d)



(4) The figure below shows part of the graph for the inverse proportional relationship, $y = \frac{6}{x}$. Complete the graph of this inverse proportional relationship.



[11] Answer the following questions (1) and (2).

(1) Suppose we are given the linear function, $y = 2x - 1$. Determine the value of *y* when the value of *x* is 3.

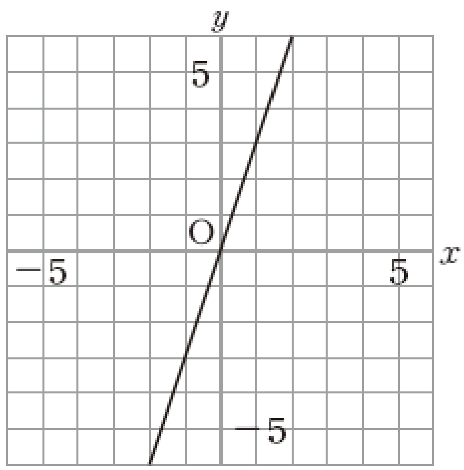
(2) The table below show a linear functional relationship between *x* and *y*. Determine the rate of change for this linear function.

x	$\cdots$	-2	-1	0	1	2	$\cdots$
y	$\cdots$	-9	-4	1	6	11	$\cdots$

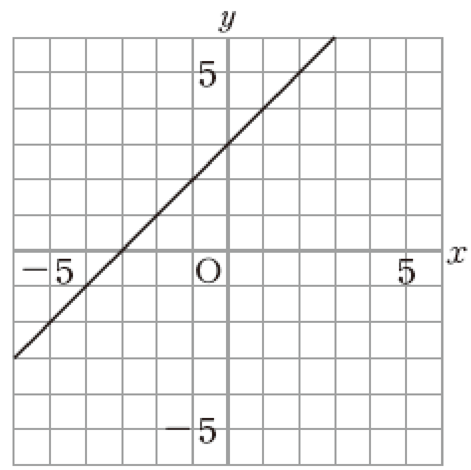
[12] There is a water tank with 5L of water. We will pour water into this tank at the rate of 3L per minute until the tank is filled. If y L is the amount of water at x minutes after we started pouring water into the tank, express y in terms of x.

[13] From graphs (a) through (e) shown below, select the graph of the linear equation, $y = 3$.

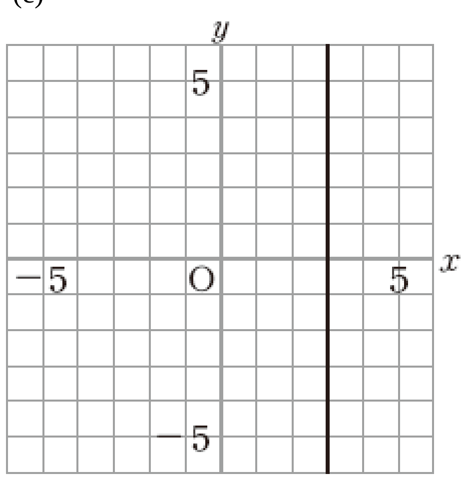
(a)



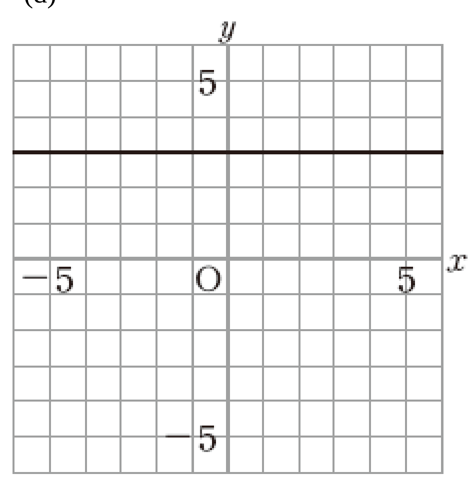
(b)



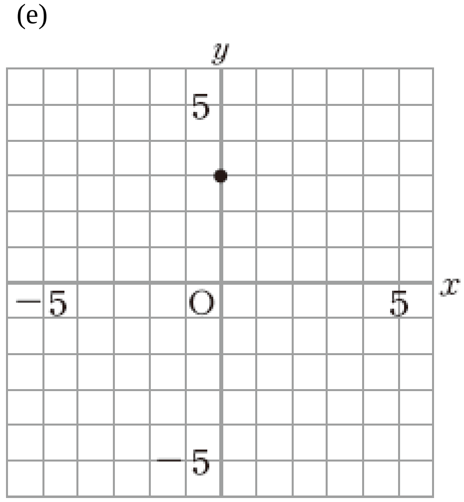
(c)



(d)



(e)



[14] Answer the following questions (1) and (2).

(1) A class of 35 students measures how far they can throw a handball. The mean distance of the 35 students is 21 m. From (a) through (d) below, select the one statement that is always true.

(a) Of the 35 records, the most frequent record is 21 m.

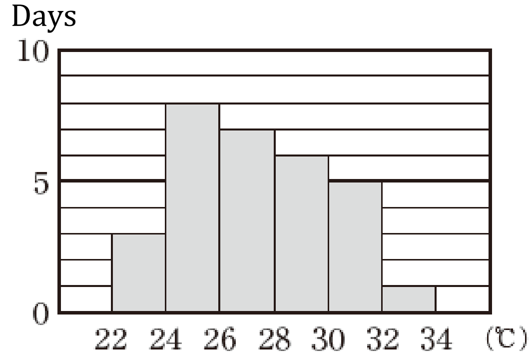
(b) If we divide the sum of all 35 records by 35, the result will be 21 m.

(c) The difference between the longest distance and the shortest distance is 21 m.

(d) If we arrange the records of the 35 students from the longest to the shortest, the 18^{th} record will be 21 m.

(2) The high temperatures during the period of June 1, 2012 through June 30, 2012, in a certain city are shown in the histogram below. From this histogram, we can tell that there were 5 days during this period in which the high temperature was greater than or equal to 30°C but less than 32 °C.

Distribution of High Temperature



Determine the relative frequency of the number of days with a high temperature greater than or equal to 22 °C but less than 24 °C.

[15] Answer the following questions (1) and (2).

(1) Suppose there is a fair coin – that is, if you toss the coin, it is equally likely to land on its heads as on its tails. We are going to investigate the relative frequency of the coin landing on its heads by tossing this coin many times. From the statements (a) through (d) below, select the correct description of the way the relative frequency will change.

(a) As the number of tosses increases, the variation of the relative frequency of the coin landing on its heads will diminish and its value will approach 1.

(b) As the number of tosses increases, the variation of the relative frequency of the coin landing on its heads will diminish and its value will approach 0.5.

(c) Even when the number of tosses increases, there will not be any variation in the relative frequency of the coin landing on its heads, and it is constant at 0.5.

(d) Even when the number of tosses increases, the relative frequency of the coin landing on its heads will continue to vary, and it will not approach any particular value.

(2) There are 2 dice of different sizes. Determine the probability that both dice will show 1 if they are tossed at the same time. Assume that both dice are fair—that is, the numbers 1 through 6 on each die are equally likely.